MATHEMATICAL,

GEOMETRICAL, AND PHILOSOPHICAL

DELIGHTS:

CONTAINING

ESSAYS, PROBLEMS, SOLUTIONS, THEOREMS, &c:

SELECTED FROM AN EXTENSIVE CORRESPONDENCE,

BY

THOMAS WHITING,

MASTER OF KEPPEL-HOUSE SEMINARY,

FULHAM-ROAD, *near* BROMPTON.

Editor of the SCIENTIFIC RECEPTACLE.—Author of SELECT EXERCISES.—

A KEY TO DITTO, &c. &c. &c.

LONDON:

PRINTED for Messrs. WHITING, DAVIS, GALE, LEYBOURN, and KEMMISH;

And sold by T. N. LONGMAN, Paternoster-Row.—1798.

P R E F A C E.

IT is generally expected when an AUTHOR offers any work to the public, he should give good reasons for so doing —and also point out the utility the publication is likely to be of to his readers.——When I introduced this MISCELLANY to the Mathematical Part of the Inhabitants of this Kingdom, it was done with a view to turn their attention from that torrent of Politics and Metaphysics which was overwhelming all the countries of Europe, and carrying down its rapid stream, the Reason, and more particularly the Happiness of all degrees of society.——Instead of seriously contemplating on Mathematical and Philosophical Subjects, our Youth were contending, or more frequently quarreling with their neighbours, about the nature of Government; and puzzling themselves about matters they could not understand for want of experience, and sometimes of sagacity.

THIS being the cafe, I wifhed to turn the attention of as many as poffible, from thofe idle and vain purfuits, to the more folid and convincing exercifes of Mathematics and Natural Philofophy; it being the opinion of the greateft men, that thefe kind of Publications produce more emulation, and confequently more Mathematicians than any regular fyftem of education whatever.——I flatter myfelf that I have, in a great meafure, brought about this defirable end; for not only emulation has been promoted among my liberal correfpondents, but feveral able Mathematicians have taken upon themfelves the editorfhip of fimilar Publications—though, I fear fo many of them may be prejudicial to each other.

I HAVE been for fome time paft, engaged in conducting a large SEMINARY, near LONDON, which occupies the greateft fhare of my time, fo that I have very little leifure to attend properly to this undertaking any longer, without a manifeft inconvenience to myfelf, I therefore intend to clofe this work with No. 11; hoping thofe gentlemen before alluded to, will go on with their Publications, and profecute the fame defirable purfuit—and I heartily wifh them fuccefs in their undertaking.

Keppel-Houfe,
Aug. 10, 1798.

Thomas Whiting.

Mathematical, Geometrical, and Philosophical

DELIGHTS.

ARTICLE I.

Mathematical Questions and Solutions.

Question 1, *from Ashby's Algebra.*

SUPPOSE a cask holds 81 gallons of wine when full, out of which a certain quantity is exhausted, and then the cask is filled up again with water, and so on 4 times (always filling the cask with water after every evacuation) there is at last found 16 gallons of wine left in the cask besides water. The question is, what quantity of wine was drawn out each time?

Solution, *by Mr. Thomas Whiting*, of Lambeth.

Arithmetically. Find 3 numbers in geometric ratio, between 81 and 16, thus $\sqrt{81 \times 16} = 36 =$ the second mean, $\sqrt{81 \times 36} = 54 =$ the first mean, and $\sqrt{36 \times 16} = 24 =$ the third mean; consequently 54, 36, 24, and 16 are the quantities of wine in the cask after each evacuation; hence $81 - 54 = 27 =$ the number of gallons drawn out the first time, $54 - 36 = 18 =$ the number of gallons drawn out the second time, $36 - 24 = 12 =$ the number of gallons drawn out at the third evacuation, and $24 - 16 = 8 =$ the number of gallons drawn at the fourth exhaustion.

Algebraically. Put $n =$ the ratio of the progression; then 16, $16n$, $16n^2$, $16n^3$, and $16n^4$, express the quantity of wine in the cask at first, and after each exhaustion. Hence $16n^4 = 81$, consequently $n = 1.5$. Therefore $16 \times 1.5 = 24$, $24 \times 1.5 = 36$, $36 \times 1.5 = 54$, and $54 \times 1.5 = 81$, the same number as found above.

Remark. The first method of solution does not succeed when the number of means is even, but a rule may be thus investigated. Suppose it was required to find 2 geometrical means between 2, and 16, put $x =$ the third term, then $\sqrt{2x} =$ the second, and as the product of the two means is equal to the product of the two extremes, $x\sqrt{2x} = 2 \times 16$, or $\sqrt{2x^3} = 32$, hence $x = 8$, and $\sqrt{2x} = 4$. Or thus, let the first term be a, and the fourth b, and x as before, then $\sqrt{ax^3} = ab$, squared and divided by a gives $x^3 = ab^2$, hence $x = \sqrt[3]{ab^2}$, a general theorem.

Question 2, *by T. W. from Davison's Repository.*

There are two houses, one at the top of a lofty mountain, and the other at the bottom; they are both in the latitude of 45°, and the inhabitants of the summit of the mountain, are carried by the earth's diurnal rotation, one mile an hour more than those at the foot.

Required the height of the mountain, supposing the earth a sphere, whose radius is 3982 miles.

Solution by the Proposer.

Let P (*Fig.* 1) be the pole of the world, C the earth's centre, A the bottom of the mountain, B the top, AD a perpendicular let fall from the bottom of the mountain upon the earth's axis, and BE another perpendicular let fall from the top to ditto. By trig. As radius : 3982 :: cos. lat. 45° : AD = 2815.69 and $2815.69 \times 2 \times 3.14159$, &c. = 17691.54 = the circumference of the given parallel of latitude, or distance the inhabitants at the foot of the mountain are carried in 24 hours. Then 17691.54 + 24 = 17715.54 the space gone over by the inhabitants at the top of the mountain. Hence as the lines AC and BC describe similar cones about CD and CE, we have this analogy, as 17691.54 : 3982 :: 17715.54 : 3987.45 = CB, hence 3987.45 − 3982 = 5.45 miles the height of the mountain.

Question 3, *by Mr. John Farey, of London.*

Required the nature of the path a person must describe in going between 2 fires, one of which is 3 (n) times as big as the other, and 20 (d) yards asunder, to feel an equal heat from each fire in every part of his journey.

Solution, *by Mr. Colin Campbell, of Kendal.*

Let the points F and f (*Fig.* 2) represent the two fires, and the line Ff their distance, M any point in the path. Let fall MP ⊥ to Ff, and put fP = x, and PM = y; then $FM^2 = \overline{d-x}|^2 + y^2$, and $fM^2 = x^2 + y^2$; but the heats received from the fires are directly as the fires, and inversely as the square of the distances; whence we have $\frac{n}{\overline{d-x}|^2 + y^2} = \frac{1}{x^2 + y^2}$, or $nx^2 + ny^2 = d^2 - 2dx + x^2 + y^2$, this reduced gives $y^2 = \frac{d^2}{n-1} - \frac{2d}{n-1} \times x - x^2$, and putting $y = 0$, $x = \frac{d}{n-1} \times \pm\sqrt{n-1} = 7.32$, or -27.32. Now as the unknown quantities x and y never rise above the second power, the curve or path will be a line of the second order; and these two values of x shew that the curve crosses the line Ff (produced) in two points, the affirmative root being at L, and the negative the contrary way at K: and as LK must be an axis, the curve is necessarily either a circle or an ellipsis. To determine which, find the ordinate fN when $x = 0$, viz. $y^2 =$

$\frac{d^2}{n-1}$, also $fC^2 = \frac{d^2}{\overline{n-1}|^2}$, the sum $= \frac{nd^2}{\overline{n-1}|^2} = CN^2$, or $CL^2 = \frac{nd^2}{\overline{n-1}|^2}$ ∴ the path is a circle whose radius is $= \frac{d}{n-1} \times \sqrt{n}$, and the distance of its centre from $f = \frac{d}{n-1}$. Thus if $d = 20$, and $n = 3$, $CL = 17.32$, $fL = 7.32$, $fK = 27.32$ and $FC = 10$.

Geometrically. Let the points Ff represent the two fires, which must be considered as luminous points, and let the heat of F : f :: n : 1, then on the given base Ff construct the triangle FMf such that the side FM : f M :: $\sqrt{n}$: 1, then M will be in the path. Draw CM meeting Ff in C, and making the ∠ CMf = the ∠ CFM, and the △s CfM, CMF are similar by construction. Therefore CF : CM :: CM : Cf, and as CF : FM :: CM : fM, and by transposition CF : CM :: FM : fM, therefore CM : Cf :: FM : fM :: $\sqrt{n}$: 1, by construction : consequently $Cf = \frac{CM}{\sqrt{n}}$; but as CF : CM :: CM : Cf, and $Cf = \frac{CM^2}{CF} = \frac{CM}{\sqrt{n}}$, and $\frac{CM}{CF} = \frac{1}{\sqrt{n}}$, or $\sqrt{n} \times CM = CF = Cf + Ff$, therefore $\sqrt{n} \times CM - Cf = Ff$, but $Cf = \frac{CM}{\sqrt{n}}$, and $\sqrt{n} \times CM - CF = \sqrt{n} \times CM - \frac{CM}{\sqrt{n}} = \frac{\overline{n-1} \times CM}{\sqrt{n}} = Ff$, and $CM = \frac{\sqrt{n} \times Ff}{n-1} =$ a constant quantity. And $Cf = \frac{CM}{\sqrt{n}} = \frac{Ff}{n-1}$ which is also a constant quantity. Therefore the distance Cf is given, and the distance CM is also given, consequently the curve is a circle whose centre is C, and radius CM. But Ff is $= 20$, and $n = 3$ per question, therefore $CF = \frac{Ff}{n-1} = \frac{20}{2} = 10$, then $CM = \frac{\sqrt{n} \times Ff}{n-1} = \sqrt{3} \times 10 = 17.32$. Therefore if from f, fC be set off $= 10$, and from C as a centre a circle be described with 17.32 as radius, this circle will be the required path.

Question 4, *by R. L. W.*

What is the worth of a quantity of wool, when the price per todd, weight, and the value of the whole, are in geometric progression, the ratio being 20.

Solution, *by W. C.*

Put $x =$ the price per todd, $y =$ the whole weight, then will xy be the whole value; then by geometric proportion $y^2 = yx^2$, and $y = x^2$; and per question $x^2 \div x = 20$, that is $x = 20$, hence $y = 400$, and $xy = 8000$.

Question 5, *by Mr. G.*

Suppose a person is to be entitled to 2 guineas, if he throws precisely 8 heads, at 6 throws, with 7 guineas. Required according to an equality of chance, the value of his expectation.

Solution, *by W. C.*

It is evident that the probability of throwing any number of heads, with any given number of guineas, in any given number of times will be the same as that of throwing the same number of heads, with as many guineas as are contained in the product of the given throws and guineas, in one time. Now putting h = the heads required to be thrown in t = any given number of throws, and g = any given number of guineas; then will 2g denote the variations in g, number of guineas, and $\frac{\text{comb. of h in g}}{2g}$ = the probability of h number of heads falling in one throw; consequently $\frac{\text{comb. of h in } g \times n}{2^g \times^n}$ will express the probability of h number of heads falling in t number of throws; that is, in the present case $\frac{\text{comb. of 8 in } 7 \times 6}{2^7 \times^6} = \frac{118030185}{349804651104} = \frac{1}{37262.048}$ = the probability of the thing required happening, whence it will easily be found, that his expectation is not more than 1—20th part of a farthing.

Question 6. *by W. B.*

A gentleman has a triangular fish-pond, the sum of whose three sides AB, BC, and AC, measure 159 yards, and the ∠'s A, B, and C, are 40, 80, and 60 degrees, I demand the length of each side seperately.

Solution, *by C. T.*

Let S.∠A(*Fig.* 3) = 40° = s, S.∠B = 60° = n, S. ∠ C = 80° = t. AB + BC + AC = 159 = b, AC = x. Then by Trig. n : x :: s : sx ÷ n = CB, and n : x :: t : tx ÷ n = AB. And $x + \frac{sx}{n} + \frac{tx}{n} = b$; whence $x = \frac{nb}{n+s+t} = 55.2201$.

Question 7. *by Mr. Abingdon.*

There are two pieces of silver coin, each .1 of an inch thick; and the diameter of the less piece, 1 inch; and it is observed, that when the centre of the greater piece is applied to the circumference of the less, one half of the area of one side of the less piece is covered therewith. Required the diameter of the greatest piece, and the weight of each of these pieces.

Solution, *by Merones Minor.*

Let AGBD(*Fig.* 4) be the less piece of coin, C its centre, & AFB part of the greater, D its centre. Draw AB, CB, DB, and DECF, then $DE = \frac{BD^2 + CD^2 - CB^2}{2CD}$, (Em. Geom. 2. 23. Cor.) $= BD^2$ in the given case. Now suppose DB $= .6$ then EF $= .24$ and by a table of segments, the area AEBF $= .16102627$, the area AEBD $= .25455055$, and their sum $= .41557682$, instead of $.39269908$. Wherefore I suppose DB $= .58$ first, $= .57$ second, and find the sum of the segments $= .39340112$, and $.38238216$; and then by trial and error, DB $= .579363$, the radius of the greater piece. Hence (Em. Mech. 8vo. 72. Cor. 2.) their weight $= .64296$, and $.47887$ parts of an oz. Avoirdupoise.

Question 8, *by Mr. Bromby*, of *Cottingham School.*

Required that triangle whose three sides, and perpendicular falling on the longest side, are in geometric ratio.

Solution, *by Mr. John Ryley*, of *Beeston.*

Take CD(*Fig.* 5) $= 1$, and n $=$ the ratio; then the sides of the triangle will be expressed by n, n^2, and n^3. Also, $BD = \sqrt{n^2 - 1}$, and $AD = \sqrt{n^4 - 1}$; therefore $\sqrt{n^4 - 1} + \sqrt{n^2 - 1} = n^3$; which squared becomes $2\sqrt{n^4 - 1} \times \sqrt{n^2 - 1} = n^6 - n^4 - n^2 + 2$; now if this equation be squared and reduced, there will arise $n^{12} - 2n^{10} - n^8 + 2n^6 + n^4 = 0$; which being divided by n^4, and the square root extracted it becomes $n^4 - n^2 - 1 = 0$, or $n^4 - n^2 = 1$. Hence $n^2 = (1 + \sqrt{5}) \div 2$ and $n = \sqrt{(1 + \sqrt{5}) \div 2}$. Therefore when the sides and perpendicular of a triangle are in geometric ratio, they will be expressed as follows, when CD $= 1$, $CB = \sqrt{\frac{1 + \sqrt{5}}{2}}$, $CA = \frac{1 + \sqrt{5}}{2}$, and $AB = \sqrt{2 + \sqrt{5}}$.

The same, *by Mr. Thomas Whiting.*

Put the perpendicular $= 1$, and the ratio $= x$, then x, x^2 and x^3 will denote the three sides of the triangle; now since the perpendicular $\times$ by the base or greater side is $=$ to the product of the other 2 sides, consequently the triangle is a right-angled one, having the greater segment of the base $=$ the least side, for $1 : x :: x : x^2$ per question, and $1 : x ::$ greater segment ($= x$) $: x^2$ per sim. Δs, now to find x we have (Eu. 47. 1.) $x^6 = x^4 + x^2$, or $x^4 - x^2 = 1$. This reduced gives $x = 1.27202$.

Corolory. All right angled triangles, whose 3 sides are in the above continued ratio, have also the less segment of the base, perpendicular, and 3 sides in geometrical progression: for the greater segment ($= x$) $: 1 :: 1 : 1 \div x =$ the least segment.

☞ *This question is in Newton's Universal Arithmetic, but as the above Solutions are both different from Sir Isaac's, we presume they will not be disagreeable.*

C

Question 9, *by Mr. T. Bulmer, of Thorp.*

A man has a field in form of a right angled triangle, whose area is 15 acres, and the sides are as 3, 4, and 5; now he wants to have a gravel walk round the outside of the said field, and to contain just one acre, and to be of equal breadth in all places, except the angular points. Required the breadth of the walk, and also what it will come to walling round the outside of the said walk, at 1s. 6d per yard?

Solution, *by Mr. John Ryley, of Beeston.*

As the area of the triangle ABC, (*Fig.* 6) and area of the walk are given $=72600$, and 4840 yds respectively; the area of the triangle nsi, is also given $=67760$ yds. Now if AB $=x$; then per question $3:4::x:4x\div 3=$ BC, therefore $2x^2\div 3=72600$ and $x=\sqrt{108900}=330$; hence BC $=440$, and AC $=550$. But the sides of similar triangles are to one another as the square roots of their areas, therefore $\sqrt{72600}:\sqrt{67760}::330::318.8013$ $(=ns)$ $::440:425.0805$ $(=si)$ $::550:531.3506$ $(=ni)$. Now put AB $=a$, BC $=b$, ns $=m$, si $=n$, ni $=r$, $4840=p$, and st $=x$; then will $Ar=a-m-x$, and $Cw=b-n-x$; therefore the areas of the trapeziums Amnr and Cviw, are $=$ to $ax-mx-x^2$, and $bx-nx-x^2$; also the areas of the parallelograms nv, sw, and the square ut are $=$ to rx, nx, mx, and x^2 respectively, which areas being collected, become $ax-mx-x^2+bx-nx-x^2+rx+nx+mx+x^2=p$; or, $ax+bx+rx-x^2=p$; hence $x=\dfrac{a+b+r-\sqrt{\overline{a+b+r}|^2-p}}{2}=$ 3 yds. 2 ft. 2.64 in. Otherwise, the breadth of the walk may be found nearly, by taking half the sum, of the 3 sides of each triangle for a divisor, and the area of the walk for a dividend, and the quotient will be 3 yds. 2 ft. 2.276 inches, nearly the same as before.

Question 10, *by Astronomicus.*

Supposing the right ascension and declination of a star to be given, also the right ascension of another star; it is required to determine the declination of this last, so that the difference of their velocities in azimuth may be the greatest possible, when they are upon the same vertical circle, in a given latitude.

Solution, *by Nauticus.*

Let ZPH represent an arch (*Fig.* 7) of the meridian, where Z is the zenith, P the pole, and H the point of the horizon which is of the same name with the latitude. Moreover let HA be the horizon, S the place of the given star, and * that of the required one; and Z* SA the vertical circle they are on when they pass each other in azimuth. Then by the question, the difference of the fluxions of the angles SZP and * ZP must be a maximum. Now, by Theo. 21, of Cotes's tract De Estimatio Errorum, &c. in the spherical triangle SZP, P:

Z :: R × ſin. ZS : ſin. PS × cos. S; and by ſubſtituting in this proportion for ſin. PS, ſin ZS, and coſ. S their equals, derived from the principles of ſpherical trigonometry, we ſhall have $\dot{p} : \dot{z}$ :: R^3 : coſ. PZ × R^2 — ſin. PZ × coſ. Z × cotan. SZ. In like manner we may derive $\dot{p} : \dot{z}$ :: R^3 : coſ. PZ × R^2 — ſin. PZ × coſ. Z × cotang. * Z. Hence if radius be taken equal to unity, the fluxion of the angle PZS will be equal to $\dot{p}$ × coſ. ZP — $\dot{p}$ × ſin. ZP × coſ. Z × cotan. SZ; and the fluxion of PZ * will be equal to $\dot{p}$ × coſ. ZP — $\dot{p}$ × ſin ZP × coſ. Z × cotan. * Z; and, conſequently, their difference or $\dot{p}$ × ſin. ZP × coſ. Z × cot. ZS — $\dot{p}$ × ſin. ZP × coſ. $\overline{\text{Z} \times \text{cotan. Z}}$ * muſt be a maximum; or, becauſe $\dot{p}$ × ſin. PZ is conſtant, coſ. Z × $\overline{\text{cot. ZS} - \text{cot. Z*}}$ will be a maximum. Now as the coſine of Z can never exceed unity, and as the difference of the cotangents of ZS and Z *: that is, the difference of the tangents of A and AS will be infinite when the required ſtar is in the Zenith; it is manifeſt that the coſ. Z × $\overline{\text{cot. ZS} - \text{cot. Z*}}$ will be a maximum when the declination of the required ſtar is equal to the latitude of the place.

Queſtion 11, *by Mr. William Kay.*

To determine a point in a given Hyperbola which is neareſt to any given point in the oppoſite Hyperbola.

Solution, *by Mr. James Webb.*

Let CE (*Fig.* 8) be the given hyperbola, IQ the oppoſite hyperbola, and I the given point in it. From I, as a centre, conceive a circle CEF to be deſcribed, which touches the hyperbola CE in the point C; this point, it is evident, is that which is ſought, and may be determined as follows: let OP be the ſemi-conjugate axe of the two hyperbolas, = c, the ſemi tranſverſe EO = OQ being = 1: draw IC, and produce it to meet the axe produced in G, and draw IR and CD perpendicular to EQ produced. Let OD be put = x; IR = a, and OR = b; then by the properties of the hyperbola, $1^2 : c^2 :: x : c^2x$ = DG, $1^2 : c^2 :: \overline{1+x} \times \overline{x-1} : c^2 \times \overline{x^2-1} = DC^2$; and by ſimilar triangles $c^2x : c^2x + x + b :: c\sqrt{x^2-1} : x$. Conſequently, multiplying means and extremes, and reducing the equation $\frac{b}{ac} = \frac{x}{\sqrt{x^2-1}} - x \times \frac{1+c^2}{ac}$; and if x be now conſidered as the ſecant of an arc, the radius of which is unity, $\frac{x}{\sqrt{x^2-1}}$ will be the coſecant; and the exceſs of the coſecant, above the ſecant drawn into a given quantity is known. Hence x is readily found by the method pointed out at p. 470. Philoſoph. Tranſact. for 1781.

Cor. Had the ſemi-tranſverſe EO, inſtead of being equal unity, been taken equal to any given quantity t; the final equation would then have been $\frac{tx}{\sqrt{x^2-t^2}} - x \times \frac{t^2-c^2}{ac} = \frac{t^2b}{ac}$; and the only difference that would have ariſen in finding the value of x, by the method pointed out above, would have been in taking x as the secant of an arc, the radius of which is the given quantity t.

Answered also by Mr. James Eastwood, and the Proposer.

Question 12, *by Rusticus.*

Given the area, one of the ∠'s, and the difference of the including sides of a plane triangle to construct it.

Solution, *by Nauticus.*

Construction. In the given ∠ H (*Fig.* 9) constitute the parallelogram HGAI, = to twice the given area; and in HI produced, take IK equal to the given difference of the including sides, also KE, so that KE × IE may be equal IA × AG. Through E and A draw EF meeting HG produced in F: then completeing the parallelograms, as in the figure, and joining BC; BAC will be the triangle required.

Demonstration. The angle BAC is = the opposite vertical angle GAI (Euc. I. 15) = the given angle GHI (Euc. I. 34); and the triangle BAC = half the parallelogram ABCD (Euc. I. 34) = half the parallelogram HIAG (Euc. I. 43) = the given area, by construction. Lastly, since IE : AG :: AI : KE, by construction, and GF : GA :: AI : IE by sim. triangles, *ex æquo perturbato,* IE : GF :: IE : KE. Now the antecedents being here the same, the consequents must be equal: that is GF (=AB) = KE; and AC (=IE) exceeds AB (=KE) by IK, the given difference. Q. E. D.

Mr. George Sanderson, and the Proposer also gave elegant constructions to this question.

Question 13, *by Caput Mortuum,*

To surround a fish pond of a given area, in the form of a given trapezium, with a walk of a given area, and of the same breadth every where, by a geometrical construction.

☞ *This is Prob. 9, Newton's Universal Arithmetic.*

Solution, *by Mr. Isaac Dalby.*

Analysis. Suppose the thing done, and let ABCD (*Fig.* 10) be the fish pond, and EFGHDCBAD the walk of equal breadth surrounding it; through A, D, C, B, draw Em, Hm, Gn, Fn, which it is evident, will bisect the angles of all the trapeziums, EG, AC, &c. whose sides are parallel, and whose angles fall in these lines, and consequently equally distant all round. Make the trapezium IKLM so that its sides may be ∥ to the sides of the given trapez. and produce EH, AD, and IM; take Hg = HG, Dc = DC, and Ml

= ML, draw gcl, which must be a right line because HDM and GCL are right lines, and (letting fall the perp. ch, Dt. Dw) Dw = Dt = ch; hence it follows from Euc. I. 36, 37, 38, that the quadrilaterals DHgc = DHGC, MlcD = MLCD, therefore HMlg = HMLG; and consequently, if gl was produced to meet Hm, the △ formed thereby on the base Hg would be equal in area to the △ HZG.

In like manner, if there be taken gf = GF, cb = CB, lk = LK, and fe = FE, ba = BA, ki = KI, the quadrilateral lgfk = LGFK, and kfei = KFEI, and therefore the locus of the points E, A, I, &c. when transferred to the points e, a, i, &c. is a right line, and the quadrilateral EAae = the area of the walk (by hypoth.) hence this easy.

Construction. Take IKLMI ∥ and equidistant from ABCDA, and draw AI, DM, produced both ways at pleasure; produce AD, IM, and take Da = DC + CB + BA, also, Mi = ML + LK + KI, that is Aa, Ii = the perimeters; through a, i, draw aR to meet AI produced, then is the △ aRA given; make the rectang. AO = △ aRA, and the rectang. aN = △ aRA + area of the walk, take aP a mean proportional between aA and aS, erect the perpendicular PE to meet IA produced, and PE is the breadth of the walk required; and drawing Ee, ∥ Aa, the quadrang. Ae is its area: For rectang. AO (△ aRA) : rectang. aN :: aA : aS :: △ ARa : △ ERe (Euc. VI. corol. 19.) therefore rectang. aN = △ ERe = △ aRA + area of the walk (by construction) therefore EAae = area of the walk.

Corol. I. *This method of solution holds good in any polygon, regular or irregular, or consisting of any number of sides.*

Corol. II. *When the given figure is a regular polygon, the points m, Z, n, R, fall in its center.*

The same answered by the Rev. Mr. Hellins, Teacher of the Mathematics and Natural Philosophy.

Analysis. Let abcd represent the fish-pond, & ABCD (*Fig.* 11) outside bounds of the walk. Then, since the walk is of the same breadth on every side, it is evident that if any two corresponding angular points be joined, the line which joins them will bisect those angles. Thus the line Aa bisects the angle DAB, and the line Bb bisects the angle ABC, &c. If now Aa and Bb be produced, till they meet in e there will be given the triangle aeb. In like manner the three other triangles bfc, cgd, and dha become known. The problem then is reduced to this:

To four given triangles to add as many spaces, by producing the sides of those triangles untill they meet four right lines drawn parallel to their bases, which four spaces taken together, shall be equal to a given space: to facilitate the construction of which I shall premise the following.

Lemma. If through two triangles of equal bases, and between the same parallels, a line be drawn parallel to their bases, it will cut off equal spaces from those triangles.
This is sufficiently evident from Euclid I. 38. and Ant.

Corollary. If the sides of two triangles, having equal bases and altitudes, be produced to two lines, drawn parallel to, and at equal distances below their bases, the spaces added will be equal to each other.

Construction of the Problem. Let the two indefinite lines (*Fig.* 12) ls, ln, form a right angle at l; in ls take lm = ab, the base of the triangle aeb in fig. 11, and draw mn so that the altitude of the triangle lnm may be equal to the altitude of the triangle aeb. In like manner, in the same right line, take mo, oq, qs, respectively equal to the bases of the other three triangles, making the altitudes of the triangles mpo, orq, qts, formed on them, equal to the altitudes of their corresponding ones in the given trapezium. Produce st until it cut ln in v. Produce sl, and make the triangle lxu, by Prob. IV. p. 218 of Simp. Geom. similar to the triangle lvs, and equal to the given area of the walk. Join xv, and in vl, produced, take vy = vx, and ly will be the breadth of the walk.

Demonstration. Produce vs till it cuts a line drawn through y, parallel to ls in z. Then (Euc. VI. 19) the areas of the similar triangles yvz, lvs, and lxu are as the squares of the sides yv (= xv) lv, and lx; and since $\overline{yv}^2$ ($= \overline{xv}^2$) $= \overline{lv}^2 + \overline{lx}^2$; it is evident the area of the triangle yvz is equal to the sum of the areas of the triangles lvs and lxu; and consequently that the quadrilateral ylsz is equal to the triangle lxu. Produce now nm, po, and rq to meet the line yz in the points, w, i, k, respectively; then by the foregoing Lemma, the quadrilateral ylmw is equal to its corresponding space AabB; and so are the others, mwio, oikq, qkzs to their corresponding ones in the first figure, and their sum, or the quadrilateral ylsz, is therefore equal to the area of the walk. Q. E. D.

Scholium, 1. It is evident that this construction may be used for any multilateral.

Scholium, 2. If a circle be described, the diameter of which bears the same proportion to the perimeter of the given trapezium that radius bears to the sum of the four cotangents to the four half angles of it; and if a tangent be drawn to this circle, equal to the side of a square which has the same proportion to the area of the walk that radius has to the sum of the said four cotangents; the difference between the radius of this circle and the secant to that tangent will be the breadth of the walk.

Another Answer to the same, by Mr. George Sanderson.

Analysis. Suppose the thing done, and ABCD, (*Fig.* 13) the pond, EFGH the outward boundary of the walk, AIEK, BLFM, &c. trapezia, made by perpendiculars from the angles of the pond on the sides of the walk. Then because the perpendiculars are equal, the trapezia, as well as the angles E, F, G, and H, are bisected by the lines AE, BF, &c. but the angles are given, therefore the ratios of the perpendiculars to the corresponding sides of the trapezia are given. Moreover it is manifest that a rectangle on one of the

equal perpendiculars, and the ſum of the ſides IE, LF, &c. of the trapezia, together with a rectangle under the ſame perpendicular, and the ſum of the ſides of the pond is equal to the area of the walk: whence the following

Conſtruction. Make ak equal to the ſum of the ſides of (*Fig.* 14) the pond, and let R be the ſide of a ſquare that is equal to the area of the walk. To ak draw the indefinite perpendicular kr, on which take kg to ak in the given ratio of IE to AI, and gh : ak :: LF : LB (IA); hf : ak :: NG : CN and fr : ak :: PH : DP, join ar, and on ak take ae a third proportional to ak and R; then by Problem 3, Book I. of Mr. Wales's Deter. Section, cut ea in o, ſo that the ſquare on ao may be to the rect. contained by eo and ak in the ratio of ak to kr; and having erected the ⊥s AI, BL, CN, and DP (Fig. 13) each equal to ao, through the points I, L, N, P, draw EH, EL, FG, and GH parallel to the ſides of the pond, meeting in the points E, F, G, and H, and the thing is done.

Demonſtration Join ag, ah, af, and draw oq parallel to kr, cutting them in the points l, n, p, q. By ſimilar triangles, and the conſtruction ao : ol :: ak : kg :: AI : IE, but AI = ao by conſtruction; therefore IE = ol. And by the ſame reaſoning, LF = ln, NG = np, and PH = pq. And becauſe o and AIE are right angles, a rectangle on ao and ol is = to a rectangle under AI and IE = trapezium IEK and a rectangle under ao and ln (twice triangle lan) = trapezium BLFM, ∴ a rectangle under ao and oq is equal to the ſum of the trapezia AIEK, BLFM, &c. Again ak : kr :: ao^2 : oe × ak (by conſt.) = $\overline{ae - ao}$ × ak = ae × ak − ak × ao; but ae × ak = R^2 by conſtruction therefore ak : kr :: oa^2 : R^2 − ak × ao :: ao : oq :: ao^2 : oq × ao, wherefore R^2 = ak × ao + oq × ao, but ak is equal to the ſum of the ſides of the pond by conſtruction and ao = AI = BL, &c. Therefore, the rectangles AL, BN, &c. together with the ſum of the four trapezia are equal to R^2 the given area of the walk required.

ARTICLE

ARTICLE II.

An Essay on the Aurora Borealis, by Amanuensis, in Answer to a Query in No. 2, of the Scientific Receptacle.

SUPPOSE the earth a great magnet.—That magnetic effluvia are constantly issuing in great quantities from its North Pole, and that these move from the north to the southward, in the direction of what is called the magnetic meridian—that these effluvia are of a martial or ferrugineous nature, no thing being magnetical but a substance of that kind, and vice versa.

Iron and sulphur, even in their gross bodies, mixed with a little water, are exceeding apt to take fire, much more so, when highly subtilized and attenuated.—The hot mineral waters, probably, arise from this, and so may volcanoes. All chemists know with what eagerness sulphur acts upon iron. Several volcanoes, or burning mountains, have been discovered of late years in and about the north-polar-regions, which cast up sulphurous vapours to an in immense height. There are springs near them the hottest in the world, their heat even equalling that of boilling water.—May not those sulphureous vapours blended with the magnetic or ferrugineous effluvia, catch fire and fulgurate?—

The vapour or fume of iron dissolved in spirit of vitriol is most readily set on fire.—May not the magnetic effluvia give them a kind of magnetic direction? We see, in fact, the lucid columns, or radiating flashes, of the Aurora Borealis, almost always shot off from the north to the south, correspondent in a great measure to the magnetic meridian.—And I have constantly observed the corona, concourse, or concentration, if I may so call it, of these lucid rays near the zenith so much to the east of it, as answered nearly to the western declination of the common magnetic needle; that is, a straight line, drawn from one to the other, would be nearly in the direction of the magnetic meridian.—I think I never observed the corona to the westward of it.—What seems not a little to confirm this notion is, that, during the appearance of a considerably great and vivid northern light, the magnetic needle suffers very great agitations; caused, probably, by the colluctation and explosion, of the sulphureous and magnetic effluvia. This is more particularly observed in Sweden and the north parts of Europe, as being near the source of these effluvia.

But, further, as we scarce ever see an Aurora Borealis, but when the wind blows from some point or other, between the east and west of the northern semi-circle, this also, may help to drive the sulphureous coruscating vapours southward.—And when the wind is very strong from E. N. E. or W. N. W. it may not a little alter their magnetic direction, or current. I have several times observed, when a strong north-easterly wind hath blown, some faint appearances of a northern light here and there, and abundance of small, lucid coruscating nubeculæ scattered up and down the hemisphere, now suddenly appearing, then disappearing; so that I imagined the wind had dispersed the fund of the luminous vapours;

for we see such lucid vibrating, broken, small clouds after the grand explosion, and at the end of a common Aurora Borealis.—Nay frequently, such small, bright, flashing clouds are seen up and down the heavens, without any other appearance of an Aurora Borealis, the lucid vapours being then but in small quantities, and much scattered: but it is remarkable, that these little, flitting, luminous clouds seemed always in a vibrating, tremulous motion, and moving very fast from north to south, though sometimes there was little or no wind.—These nubeculæ were so extremely thin, that even stars of the third or fourth magnitude were seen through them.

Those northern lights are seen vastly more frequent, more bright, more beautiful, and variously coloured, in the northern parts of Europe than here; and here much more than to the southward; because in the polar regions, the magnetic effluvia are vastly more strong and copious, and the neighbouring volcano's send up immense quantities of sulphurous vapours (which cannot but rise very high in such a dense, cold atmosphere) and these, as it were, fermenting with one-another, catch fire.—In Sicily, and the surrounding seas, they see luminous appearances, very near resembling those of the Aurora Borealis, when Vesuvius or Etna burn, and these rays are commonly of various colours, as these of the northern lights, viz. red, yellow, greenish, crimson, &c. Possibly both the one and the other are tinged by some mineral substance from the volcanoes. For though globules of rain may refract light of different colours, there seems to be nothing in the matter of an Aurora Borealis that is apt to do it.—Besides, it is unquestionable that the Aurora shines by its own light, and not from the sun, as well as the lights of Vesuvius.

We well know that different minerals will tinge flame of different colours.—May not then the diversity of colours of an Aurora be another argument that it arises from the exhalations of volcano's. The sulphureous vapours of volcanoes are shot up to an inconceivable height, (sometimes even great stones are thrown up from them to four or five hundred feet, and the ashes vastly higher, so as oftentimes to be carried by the wind fifty or a hundred miles, nay, leagues) so high indeed, that they may retain very little gravity, their centripetal force continually decreasing, as their distance from the earth increases—and their centrifugal will be much increased by their revolving about the axes of the earth in a very large circle.—But farther, sulphureous vapours have a kind of a vis centrifuga, and will rise in vacuo, whereas all other vapours sink.—It is certain, the fumes of gunpowder will rise to the top of a tall exhausted receiver, and even prove lucid, though the gunpowder itself doth not flash.—Thus, the vapours over Vesuvius are sometimes very lucid, though the crater-magnet at that time, actually belched out flame. We know sulphureous vapours are sometimes carried to an astonishing height, and collected into vast bodies of inflammable matter, far above the gross terrestrial atmosphere.—The great Dr. Halley, from very just observations, estimated the meteor of the 19th of March, 1718-19, (which cast such an amazing brightness, and made such a very loud displosion) to be very near seventy miles perpendicular above the surface of the earth; whence it was seen over a great part of Europe at the same time. Now if such a great body of sulphureous vapours as this could be sustained at the very top of our atmosphere, or even in the æther above it, how much higher may we suppose the mere subtile vapour of the lumen boreale to be carried?—This will account for the great height and distance some of the northern lights are ſeen at, without having recourſe to M. Mairan's zodiacal light, or Profeſſor Euler's repulſion of the ſun-beams. It is poſſible theſe vapours, when carried to ſuch a vaſt height, and in a medium ſo exceeding rare, may actually become lucid, eſpecially when mixed with the ætherial nitre, as the fumes of gunpowder mount and ſhine in vacuo.—Indeed this phenomenon of the gunpowder ſuggeſts to me, that a highly ſubtilized ærial nitre always enters the compoſition of an Aurora, (for it is every where diffuſed throughout the whole atmoſphere) and nothing is more like the vivid pearl-coloured flaſhes of an Aurora, than a deflagration of nitre and ſulphur; and the flame may be tinged with red, green, yellow, &c. by the addition of different minerals.—Certainly, nitre, ſulphur, and iron, are greatly diſpoſed to inflame and coruſcate.—The arctic regions abound with nitre.—The northern lights are vaſtly moſt frequent in cold ſeaſons, when the atmoſphere is greatly ſtocked with nitre.—It is ſcarce to be doubted, but that common lightning abounds with all theſe principles. Hence may ariſe another conjecture, that, as lightning is certainly of an electrical nature, ſo poſſibly may be the nature of an Aurora.—The incredible ſwiftneſs of its flaſhes, and the inſtantaneous propagation of its coruſcations through all the northern parts of the hemiſphere, ſeem to favour ſuch a thought.—May the luminoſity be conveyed on the magnetic effluvia as the electric on an iron wire? But this is, I fear, indulging too far in whimſey. The accounts we have had from Iceland, Greenland, and other places within the arctic circle, by the whale fiſhers, and others, that have been given by the Jeſuits, &c. who travelled to the N. E. parts of China and Tartary, aſſure us, that there are ſeveral volcano's in theſe parts, ſome of which have broken out within theſe few years.—Is it not then the freſh eruptions of ſome of theſe volcano's that have produced the northern lights ſo common of late years? Before this preſent century began, it is certain, they did not appear for many years even in Sweden, Norway, and Lapland, or at leaſt very ſeldom; whereas M. Maupertius ſays, they are now almoſt conſtant in theſe countries during the winter months.—Is it not then the more frequent and violent eruptions of theſe volcano's that make the Aurora Borealis more common and more illuſtrious? And is it not the ceſſation of theſe eruptions that puts a ſtop to theſe luminous phenomena, as we know they ceaſe for weeks, for months, for years; and that too when all other circumſtances ſeem to favour their productions?

They appear vaſtly more frequent and great in the moſt northern countries, as they lie near the ſource of the magnetic effluvia and ſulphureous vapours. Doth not an appearance of a kind of theſe nocturnal lights and coruſcations in Sicily, &c. on the eruption of Veſuvius, and ſometimes merely from the ſulphureous exhalations iſſuing from it, without actual flames, ſeem to confirm this opinion. Moreover it is not altogether improbable that ſulphureous exhalations from more ſouthern volcano's, ſwimming on the top of the atmoſphere, and revolving with the earth, round its axes, may be carried towards the poles, and contribute ſomewhat to the formation of an Aurora. Perſons, acquainted with natural hiſtory and philoſophy, will readily ſee on what principles I have advanced this theory.

ARTICLE III.

A *Eulogium on the Newtonian Philosophy.*

By NEWTONIENSIS.

AS an Encomium on Philosophy is my professed design, I shall set forth its excellence, through a series of speculations on its nature, subject, and end; and the utility, pleasure, and happiness which from thence accrue to mankind.

The nature of Philosophy is science itself; its essence is knowledge; and wisdom, in all its various branches, whose every attribute is founded on something superlatively great and sublime.

Nor am I here to be understood according to the common acceptation of the word Philosophy; for what vain imaginations, what whimsical conjectures, of every sort, have not at times been ushered into the world, and gilded over with this respectable name? The Stoics of old abused it to subvert the nature of things, and to persuade us out of our senses, which all conspire to convince us that *pain is an evil*. The Sceptic also takes this venerable word in the most abusive manner to inforce a doctrine directly repugnant both to common sense and reason, *viz. The uncertainty of demonstration itself.* Again, the Democratic school would make us believe, that particles, of inert matter, from their most chaotic state, could dance into form and order; compose harmonious systems of worlds establish laws of motion, and be productive of increase, life, sense, and soul, in all its various degrees, in themselves alone; and to this they were impious enough to give the term Philosophy. Such monstrous positions are an approbrium to the science; and against these only it is that the Apostle inveighs, when he dehorts from the Philosophy and vain deceit of the Heathen.

Yet farther, our modern Sceptics have improved upon the antients, and would have us doubt even of matter or substance itself. They tell us that all things may consist in phantasy and idea, without any such thing as real substance, or solid particles of matter in the world. Descartes has refined upon the old atheistical tenets of Lucretius, and makes a World a-la-Mode de Paris.

A Horse and a Clock, with him are only two different sorts of machines, and this they have presumed to term Philosophy.

Not such is the Philosophy of which we treat, but as different therefrom as reason from absurdity. It affronts not common sense, controuls no reasoning faculties, nor impresses our minds with impious and irreligious sentiments. On the contrary it is every way agreeable to the purest dictates of them all; a system of plain and genuine truth, and inspires such principles only as naturally tend to correct our senses, to improve our reason, to enlarge our underſtanding, to illuminate the mind, and raiſe the ſoul to the higheſt pitch of rational knowledge our nature will admit of in this terreſtrial ſtate.

From what has been ſaid, and from a conſideration of our own being, it evidently appears, that Philoſophy is the greateſt excellency, and the higheſt perfection of human nature. For what is our make or conſtitution, but body and mind? As to the firſt, 'tis plain we are not denominated men from thence; for body conſidered as matter only, is common to all things in this world; and body conſidered as endued with a power of *growth* or *increaſe*, is common to VEGETABLES and Foſſils. Again, body conſidered as animated or endued with *life* and *ſenſe*, is common to every ſpecies of ANIMALS in nature, even the *oyſter* enjoys theſe prerogatives.

As it is not in the matter of man's body that we are to ſeek for the excellence and dignity of our nature, ſo neither does it reſult from the *form* of man. For all that can be ſaid in this reſpect, is no more than this, That the *form* of our bodies is the moſt commodious, and beſt adapted to anſwer the purpoſes of human life; and as much as this may be ſaid for the form of every animal. Beſides, the difference between our bodies and that of a quadrupede is rather in the *poſition* than *form* of the body, for the parts are nearly the ſame in both; only they have a prone poſition, we an erect one.

Since, then, we have little to vaunt of on account of our *corporal qualities*, there muſt be ſome other principle in which the dignity and ſuperiority of human nature conſiſt; and that can only be the MIND. It is the MIND of man by which he excels all other earthly beings. This alone is that *image and likeneſs* of the DEITY in which he was firſt formed, and from whence all real *humanity*, is derived: Nor is it the natural, the rude, and undiſciplined mind, the thing we are ſpeaking of; this is no more than what we properly call *inſtinct*, and what is common to us likewiſe with the brute creation, according to their ſeveral kinds. But the mind which is properly and truly *human*, is that only which is well informed, and fraught with proper erudition, and the ſeeds and principles of natural knowledge; that is, with TRUE PHILOSOPHY.

PHILOSOPHY, therefore, is the only ſource of all true glory and greatneſs in the human mind, and the ultimate perfection of our nature. For herein does the life of man moſt properly conſiſt; without Philoſophy we cannot be ſaid to live a *human life*, which does not conſiſt of a *ſtate of ſenſation for ſo many years*; for this is a brutal or mere animal life; but is meaſured by a *ſucceſſion of rational and ſublime ideas, which paſs in a given time in the mind.* Thus he who revolves twice the variety of ideas in his mind, in the ſpace of one day, lives as much in that one day, as another does in two, whoſe conception is but half ſo extenſive. Conſequently, thoſe who have the moſt capacious minds, and comprehend the greateſt variety of noble and exalted ideas, enjoy life in the greateſt perfection, and in the higheſt and moſt exquiſite degree. As on the other hand, thoſe who are ſtrangers to Philoſophy, that is, who are actuated only by animal inſtinct, or ſenſation, and the vulgar ideas thence ariſing, can ſcarcely be ſaid *to live at all*; theſe ſhould be looked upon as ſo many abortions, or miſcarriages in forming the *genuine human mind.* Theſe are not ſo properly called *men*, as creatures in the *ſhape of men.*

As the body without the ſoul is dead, ſo the ſoul, without a philoſophic erudition is, in a manner, dead alſo; for thus it only animates the body, and degenerates into meer human instinct. Knowledge is to the mind what food is to the body; without food there can be no nutrition, no growth, no salubrity in the body; and without natural Science, or Philosophy, the mind or soul of man can receive no increment, no expansion. no perfection or consummation, but is little, low and weak.

Is knowledge, then, so necessary, so natural, so essential and important a principle to man? No wonder we find it so amiable and desirable to, so arduously sought after, and exquisitely delineated, by every wise and well-formed mind. Let us only observe in what sublime and figurative strains this sound Knowledge, or true Philosophy, is characterised and recommended by the ancient eastern sage, under the person of divine wisdom.—— PROV. Chap. VIII.—*Doth not wisdom cry, and understanding put forth her voice?—Unto you, O men, I call, and my voice is to the sons of men. O ye simple understand wisdom, and ye fools be of an understanding heart.—Receive instruction, and not silver; and knowledge rather than choice gold. For wisdom is better than rubies; and all the things that may be desired, are not to be compared with it. Riches and honour are with me.—I lead in the way of righteousness, in the midst of the paths of judgment.—The Lord possessed me in the beginning of his way, before his works of old. I was set up from everlasting, or ever the earth was.—When he prepared the heavens I was there.—When he established the clouds above.—When he gave the sea his decree.—When he appointed the foundations of the earth. Then was I by him, and I was always his delight.—Now therefore hearken unto me, O ye children.—Hear my instructions and be wise.—Blessed is the man that heareth me, watching daily at my gates.—For whoso find me findeth life, and shall obtain favour of the Lord. But he that sinneth against me wrongeth his own soul: All they that hate me love death.*——What a noble allegory is here of this divine science? And how emphatically is it recommended to mankind? How great the blessings which attend it? And the wretchedness of those who slight and neglect it.

The SUBJECT of Philosophy is that which, next to its nature, enobles it, and gives it the pre-eminence of all other sciences. Every science derives the greatest part of that which makes it amiable to mankind from its subject. For tho' all knowledge is, in itself, valuable and desirable, and tends some way or other to a general good, yet every branch is not equally pleasant and agreeable in the pursuit, nor attended with equal advantage to the student. But what science can compare with Philosophy, in regard to the exquisite and ineffable pleasure and transport that never fails to arise in a well-formed mind from its subject, *(i. e.)* from a contemplation of the manifold and wonderful works of God.

The business of this science is to enable us, in a proper manner, to consider the HEAVENS, that is, the infinite space, the interminable void, the *Universe*, of all created worlds, the *Sun* and *Stars* which God has ordained; it shews us that Suns and Stars are synonymous, and must equally imply those immense fountains of light and heat, which, in form, govern and animate a system of revolving planets: And thus it astonishes the mind with a certain and indubitable proof and prospect of an infinity of worlds, and creates an idea every way worthy of, and adequate to the notions we ought to entertain of an infinitely wise, perfect, and powerful Being.

PHILOSOPHY, in the degree it is imparted to us, can, indeed, contemplate only one of those innumerable worlds particularly; but then, from this alone, it fills our minds with sublime and august ideas; ideas strange and incredible to the uninstructed unphilosophic mind. The *Virtuoso* is hereby taught, that the SUN is made not only to rule the day, or enlighten one earth, but many; that it is not a small but an immensly large body of concentrated fire, the constant emanations of which give light, life, and motion to every creature in each revolving world.

For by this science we farther learn, that the *Sun* informs a beautiful and harmonious system of planetary bodies, which constantly revolve about him, governed by one equable and universal law. That these planetary orbs are not small points of shining light, as they seem to the uninstructed eye, but large, very large and opake bodies, which have no light in themselves, but shine with the borrowed light of the Sun: and among these Philosophy shews us our EARTH, the terrestrial ball on which we live; it convinces us that it revolves about the Sun like the rest, but in a smaller orbit, and at a nearer distance than many of them: It shews us that (whatever high opinion we may have of our native globe) it is very small indeed, yea, almost inconsiderable, in regard of some of the other planets, one of which is at least a thousand times as large, and others at such a distance, that so small a planet as our earth quite vanishes from their sight.

Philosophy discovers to our view a scene of secondary worlds, or systems of secondary planets, circulating about the primaries as they do about the sun, and accordingly by one and the same universal law. And as we know one of the primary planets (*viz.* our earth) to be the habitation of infinite species of animals, and productive of as great a variety of plants, minerals, fossils, earths, *&c.* so by analogy we are taught to understand the same of all the rest. Thus also the Philosopher, by descrying various mountains, hills, valleys, and extensive plains in the globe of our moon, justly concludes the same things are to be observed in those of *Jupiter* and *Saturn*, by the inhabitants of those planets: and hence infers the various globes, which compose our system, are so many particular habitable worlds; and that, therefore, every system, proper to each solar star, through all the infinity of space, are so many habitable worlds, of numberless different beings, possessing every degree of animal and vegetable life; and this creates in our minds a most sublime and august idea of the works of creating power and wisdom, highly worthy of the divine Author, and deserving our serious contemplation and curiosity.

By this divine science, we are adduced to a yet more intimate knowledge of the nature of these remote and (to the vulgar) unknown bodies. We hereby discover not only their motions about the sun, but, in many of them, a regular and uniform motion about their own axes; and not only that, but also the times in which those motions are performed; so that of course we are hereby taught that they have alternate *Day* and *Night* (as well as ourselves) throughout their years respectively. Not only this, but even the different lengths of days and nights in those planets, are demonstrated by philosophical discoveries: And, again, we are hereby assured they enjoy a *Variety* or *Vicissitude of Season*, in some a greater, in others a less degree than what we find in our own globe.

Yea, so exquisite are the researches of this science, that the same motions we have been describing in the planets, are discovered in the sun itself. For the center of the sun is constantly describing an orbit about the common center of gravity of the system, and at the ſame time the body of the ſun uniformly revolving about its own axis; which two wonderful motions of this great luminary, are to be known only by the votaries of Philoſophy.

Farther, we are hereby ſhewn, that not only in the moon, but in all the celeſtial bodies in general, there are different kinds of matter in their compoſition; the greater part of which will reflect light, and ſhew them luminous; but the other will not, by which means, in thoſe parts, they appear dark, and are variegated with ſpots. Nor is there any of thoſe bodies ſo remarkable in this reſpect as the ſun itſelf; for though it be the very eſſence of fire, it has, nevertheleſs, ſome very dark parts, of a ſurprizing magnitude, which emit no light, and are variable as to their form, bulk, and appearance.

Beſides the celeſtial ſcenes hitherto mentioned, Philoſophy preſents us with a view of one more, tremendous, indeed, to the mind, devoid of ſcience, but the greateſt inſtance of wiſdom, order, and harmony that can be found for curious contemplation. The BLAZING COMET cannot more ſurprize and terrify the peaſant, than it delights and gratifies the mind and ſight of the Philoſopher. Charmed with the rare phænomenon, he bids, for a while, adieu to indifferent things, and attends, with tranſport, the novel object of the ſkies. Conducted by Philoſophy, he views, with teleſcopic eye, the wondrous ball, contemplates its rapid and accelerated courſe towards the ſun, its near approach to the ſun's body, the amazing lengths of its aſcending rarified atmoſphere of kindled vapour, which he is taught to meaſure, and finds it to extend many millions of miles among the ſpheres of neighbouring planets. He views, intrepid, the mighty flame, oft whirled with impetuous force acroſs the path of the earth: He follows the departing devious planet, as far as ſight will permit; and can afterwards eaſily attend him, in his mind, to its moſt diſtant retreat, in the dark and empty regions of ſpace: He ſees it diminiſhing it heat, contracting its tail, and retarding its courſe to the laſt.

By Philoſophy we are taught to underſtand the nature and cauſe of ECLIPSES; and enabled to fore-tell, for any time to come, when theſe deficiencies of light will happen in either luminary. Alſo, what the quantity thereof will be, the preciſe time of their continuance, when viſible or inviſible to us, or to any part of the globe. So that what the vulgar mind is unexpectedly ſurprized with, the Philoſopher naturally looks for, and knows muſt happen, in conſequence of the eſtabliſhed laws and order of the heavenly motions.

From celeſtial regions we deſcend to the neighbourhood of our own globe, which we find every way ſurrounded with a *fine inviſible Matter*, called the ATMOSPHERE, or AIR. And here what wondrous ſcenes of ſublime and uſeful knowledge does Philoſophy diſcloſe! We here learn the true origin, the generation, the various properties and qualities of this ſubtile, ethereal matter; we are taught the reaſon of its *Weight* and *Preſſure*, and how to eſtimate the quantity thereof, ſo amazing to the unexperienced mind. We are further inſtructed into the cauſe of the *Elaſticity* or *Spring* of the air, and know from hence its fitneſs for the *Propagation of Sounds*, for *Reſpiration in Animals*, for *Vegetation in Plants*, and for various other purpoſes in the œconomy of the world. The various *Denſity* or *Rarity* of the air, is a matter which Philoſophy ſets in the cleareſt light; and hence we ſee the reaſon of the *Riſe of Vapours*, and the *Formation and Suſpenſion of Clouds* to various heights therein. We hence learn how it becomes the proper medium for the generation and production of all kind of *Meteors*, as the *Aurora Borealis*, *Lightning*, *Hail*, *Snow*, &c. We are also made acquainted with the particular manner in which it *detains the Light* to render all things visible about us; and the absolute necessity thereof, for the existence of *Fire and Flame*. These and many more, are the noble topics of philosophic erudition, in this part of nature's handy works.

Philosophy now conducts us to the exquisite contemplation of light, and her lessons on this subject are beyond all expectation clear, subtile, and sublime. We are here convinced of the true nature of that sort of matter which enlightens the whole mind, and renders all things visible to the eye. We now see it clearly proved that light is not a *Quality of Bodies*, but a *real Body*; or specific substance, which is found to possess all those properties and qualities which are common to all sorts of matter. We find in it *local Motion*, and are assured of the true *Velocity of its Motion*, to a surprizing degree of exactness. We are at the same time made to see, that the velocity of light incredibly exceeds that of any other sort of body: We hence learn also the exceeding tenuity or smallness of its particles, and how by this means it is fitted for the medium of vision. Lastly, we are taught the particular manner in which light acts upon other natural bodies, and they upon it; and from hence are enabled to account for the principal phænomena of nature.

Thus we are shewn, by true Philosophy, how the particles of light, by their extreme velocity, actuate the parts of all bodies, (which they enter by reason of their smallness) and thereby produces all the different degrees of *intestine Motion*, and consequently of COLD, WARMTH, HEAT, FIRE, and LUMINOSITY, which depend thereon. And as these qualities of bodies are produced by light, so they produce an infinity of others, and are the cause of most of the operations of nature. Hence we see how light comes to be the most universal agent of nature, dispensed through all the system from the *Magazine or Receptacle of Fire* in the sun's body.

By Philosophy we also learn the particular manner in which *Bodies act upon Light*, viz. by *reflecting*, *refracting*, and *inflecting* the rays thereof; also, the determinate laws by which each of those operations are performed, together with the various powers in different bodies which produce those effects. And hence the foundation and principle of the most delightful science of OPTICS are clearly understood.

By the PHILOSOPHY *of Light*, it is that we arrive at the true *Doctrine or Cause* of COLOURS: We are thereby taught how they all arise from the different action of the particles of light, on the expansion of the optic nerve in the eye; and that these result wholly from the different magnitudes of the particles of light, which is demonstrated from the *Analysis of Light*, by Experiments of the *Prism*. Hence the reason why one and the same body is susceptible of any colour whatsoever, and will appear of one colour by *Reflection*, and of another by *Refraction*, Hence the doctrine of *Composition* and *Transmutation of Colours*: Hence lastly, the cause of the *Variety and Inversion of Colours* in the *celestial Bows*; of *Haloes*, *Parhelia's*, &c. becomes clear and evident to the strictest mathematical demonstration.

By PHILOSOPHY we learn the true cause of *Transparency* and *Opacity* in bodies, and know it to be just contrary to the vulgar opinion: for bodies are *pellucid* on account of the smallness of the parts and pores; and *opake*, when they are large: The most opake body becomes transparent by a sufficient diminution of its parts; and the most transparent will become opake by enlarging its pores. And the reason of all this is evident on the principles of this science only.

Philosophy not only searches out and demonstrates the *Laws of Motion*, in the *grand* machinery of mundane systems, but likewise explores and settles the same with respect to the mutual action of all bodies upon each other, whether *mediately*, by any intervening power, or *immediately*, by *Contact or Percussion*. It is this science alone that can give us any proper ideas of *Motion* and *Rest*; all we can have without are absolutely uncertain in their quantity, and false in their direction. Or, in other words, no man can tell whether a body does really move or not, nor which way, nor with what degree of velocity, unless inspired with Philosophy. The *Quantity of Motion* or the *Percussive Force*, in any case of striking bodies, is not to be estimated or understood but by the axioms of Philosophy: And that *Action* and *Re-action* between bodies are equal, or that the moving cart acts upon the horse, as much as the horse does upon it, never sounds absurd but to the unphilosophic ear.

The *Doctrine* and *Science* of MECHANICS is purely philosophical, as it depends entirely on gravitation, and the laws of motion. While, therefore, PHILOSOPHY unfolds to us the *Nature* and *Laws of centripetal and centrifugal Forces*, and the manner in which bodies affect each other thereby, we evidently see all that relates to the *Gravity* or *Weight* of bodies, and to the various *Momenta* or forces with which they act on each other by means of instruments, at different distances, and with different quantities of matter: And in this consists the whole *Theory of mechanic Science*. Hence the particular properties, and a just estimation of the forces of all the simple mechanical *Powers*, (as they are called, *viz.* the LEVER, the PULLEY, the AXIS IN PERITROCHIO, the INCLINED PLANE, the WEDGE, and the SCREW, (together with all kinds of BALANCES) at once become known; and from thence the structure and powers of all *compound Engines* and *Machines*, as *Cranes*, *Mills*, *Clock-work*, &c. become facile to the PUPILS of *Philosophy*.

From the more obvious powers of nature, we are led by Philosophy to a view of her more secret and amazing scences of action: We have here opened to our minds the *wonderous Laboratory of Nature*, and the stupendious processes therein carrying on, unheeded and unthought of by the vulgar. This part of Philosophy is the *Microscope of the Mind*; we hereby view all the small particles of matter, endowed with a *mighty Power of Action*, by which they are constantly actuating each other by ATTRACTION or REPULSION; and hence ensues, that variety of *Properties*, *Qualities* and *Phænomena* depending on the *Figure*, *Size*, *Motion*, and *Action* of the corpuscles, or constituent parts of bodies.

Thus we are shewn, that those particles, by attracting each other, do cohere together with various degrees of firmness, according as they touch by a greater or less quantity of surface; and thus constitute all variety of bodies, with different degrees of consistence, from the *Hardest* to the Softest, and from the most *fixed* to the most *fluid* bodies in nature.

On the other hand, we are taught by the precepts of this science, that when the particles of matter are separated beyond the sphere of attraction, there commences a *repulsive Power*, by which they equally and mutually repel each other, and by this means acquire what is usually called their *Springiness* or *Elasticity*. Hence the prodigious force of all

G

elastic fluids, as air, heated vapour, *&c.* is easily accounted for on this principle of a *centrifugal Force*, actuating the separated parts of matter.

On this PHILOSOPHIC THEORY depends the solution of all the phænomena attending the various processes of chemistry: We hereby see the reason why, upon a mixture of different sorts of matters, there often ensues a violent intestine motion in the mixture, by which various degrees of heat, and sometimes *Flame*, are instantly produced, attended with great *Ebullition* and *Colluctation* of the parts. We see how *solid Bodies dissolve* in various *Menstruums*; and *fluid ones* become *fixed* and *hard*. We see why *heavy Bodies* are suspended in *lighter Fluids*, and the opakest rendered by solution the most transparent: And the methods of analysing all kinds of natural bodies, and examining their component parts or principles, are clearly pointed out. Yea, so far does Philosophy proceed in the powers of nature, as almost to create any body required, from the given principles of matter, and their known laws of action.

Furthermore, Philosophy conducts us to the interior recesses of the earth, and there shews us the mighty operations of nature; suggests to us the manner how *Metallic Ores* are generated; how earths concrete into various forms of *Stone*; how *Sulphur, Salts*, and other principles of natural chemistry, produce the variety of mineral waters; why some are hot and others cold; the cause of *Earthquakes*, and the *formidable Eruptions of Volcanos*, are no longer secrets in the school of Natural Philosophy.

The *Nature* and *Laws* of FLUIDS are ascertained by no other science but Philosophy: We here see all that is necessary to constitute matter a *fluid Substance*; and as such we see the different manner in which they act from solid bodies, and thence learn every thing relating to their absolute and specific gravities, the quantity and force of pressure, the reason why any thing *sinks* or *swims*, the nature and use of the *Hydrometer*, the *hydrostatic Balance*, and every thing else in the compass of that part or branch of the science, called HYDROSTATICS.

Again with respect to the *Motion of Fluids*, (or the science of *Hydraulics*) how excellent is the service of PHILOSOPHY? Before this science enlightened the literary world, with what uncertainty did we grope after the true origin of *Springs* and *Fountains*? How poor were our notions of the *Motion of Fluids* in general! We knew nothing of the theory of *Aquæducts*, or the reason why water rose in *Pumps*: The suspension of quicksilver in the *Barometer*, was a mighty *Mystery*; nor could they ever account for the action of so much as that most simple instrument the *Syphon*, or common *Crane*: Much less could they estimate the force of *spouting* fluids, or say what the action of the air must be to move the sails of a mill. As to the doctrine, or *Theory of the Tides*, that was indeed vulgarly adjudged to be the effect of the moon, but how, and according to what laws it is effected, was a matter too deep and difficult for any but *Wisdom's* ELDEST SON to investigate and explain.

Then as to the *Doctrine* of WINDS, Philosophy accounts for all their phænomena on the plainest principles; and shews why some are *constant* and *invariable*, why others are *periodical* and *alternative*, and why, in all great latitudes, the winds are *uncertain*, both as to their *immediate cause*, and also as to the *Course or Point of the Compass* from whence they blow.

The NATURE and THEORY of SOUNDS, and consequently what may be properly called the *Science of Harmony*, was never understood till Philosophy brought it to light; and this was not till *Newton*'s days. From him we learn the true cause of sounds, and trace them from the tremulous body, through all the elastic ærial *Undulations*, to the curious *Structure* and *Mechanism* of the *Ear*. From him we are taught wherein their various differences consist; why some are *loud*, and others *low*; some obtuse, and flat, others sharp or acute; some more agreeable, and others less so. Hence all the grounds of HARMONY, MELODY, and MUSIC, are derived: The *Rationale* of *musical Proportions*, the *harmonical Division* of lines, the structure of *Organs*, *Harpsichords*, and other musical instruments, are all the natural result of PHILOSOPHY.

If we look into the *vegetable World*, what amazing scenes doth Philosophy present to our view! Here nature annually unfolds itself, vegetates and grows into *Plants* and *Trees*. The GENERATION of PLANTS is a mysterious and inconceivable thing; but Philosophy acquaints us with the wonderful manner thereof. It shews us each plant in its *Embryo* state, in the pre-existent seed, and thereby convinces us of a truth incomprehensible and incredible to vulgar minds, *viz*. That every plant, of every kind, was compleatly in all its parts, included in the seed of each preceding plant; and so the whole tribe were all contained and included in infinite miniature, in one respective original seed.

Philosophy next apprises us of the curious and exquisite apparatus of parts for the production of the Embryo-plant. The scene here lies in the *Flower*, whose delicate attire is destined not only for *Beauty* and *Fragrance*, but principally for the purposes of generation.

By our philosophical researches we have been enabled to make great improvements in the knowledge of the *Make* and *Structure* of the bodies of plants and trees: We see the wondrous system of attracting capillary vessels, which imbibe and draw up the SAP, or nutritious juices of the earth, by means of the *Roots*, and which is constantly perspired off by the *Leaves*. Besides these, we find other vessels destined to supply the plant with air; and astonishing it is to consider how each annual system of air and sap-vessels (which makes the *Annulus* or ringlet of wood, by which the tree does each year encrease its bulk) unravel and expand itself from the *Bark*, in which all the bulk or liguous part of the tree is originally contained. These and many other curious and engaging speculations in BOTANY, we owe entirely to the invention of *optical Glasses*, and consequently to our favourite science PHILOSOPHY.

But in nothing is the excellence of Philosophy so conspicuous as in its sublime discoveries relating to the nature and structure of animal bodies, and the use of the several parts. By this science we are taught the *divine Laws* of *animal Mechanism*; not in the low nonsensical notion of the *Cartesians*, who consider *Animals* as meer machines, devoid of life or sensation: On the contrary, true Philosophy represents an animal fabric as one of the noblest works of God, in which *dead Matter* is made to *live*; inert matter is rendered capable of action and motion; matter absolutely devoid of any sensitive faculty, endowed with various powers of sensibility, in different modes, and almost infinite degrees. But above all, to consider how this inanimate, inert, insentient substance should be constructed with faculties rendering it capable of mind and thought, is the most mysterious and amazing speculation! This fixes the bounds to philosophical enquiries; hitherto can we go, but no further. Bold presuming man may as well pretend to make an animal, as to account for its powers and functions. These are all the works of infinite wisdom, whose *Judgments are unsearchable*, and *Ways past finding out*.

But however inscrutable the origin of an animal may be, the laws by which the several *animal Functions* are governed, and the *vital Actions* performed, are the proper subjects of Philosophy; and though the cause, the manner, and intimate texture of most parts of animal bodies, are latent and incomprehensible, yet it is great satisfaction to think we are admitted to the knowledge of the offices, uses, and ends of the several parts, and the *general Oeconomy* of animal nature; which is one of the most agreeable and sublimer lessons of Philosophy.

Thus we are shewn the nature, make, and disposition of the BONES, and how they give *Firmness*, and *Stability* to the body. We are next taught the structure and use of the *Muscles*, for giving motion and strength to the parts; though the *Modus Agendi* or muscular motion be among the number of nature's *Arcana*. We have lately been instructed in the true use and design of that noble organ the HEART, the *Primum Mobile* of animal nature; from hence we learn the origin and use of that wonderful system or compages of vessels we call ARTERIES and veins for circulating the blood and animal fluids through every part of the body, for the grand and final purpose of NUTRITION. Besides these, we find another wonderful apparatus of vessels or parts we call *Nerves*, which have their origin from the BRAIN and *Marrow*, and are appointed by nature the *instrumental Cause of Sensation* to animals. Thus the *Optic Branch* is destined for VISION; the *Auditory Nerves*, for HEARING; the *Olfactory Pan* for SMELLING; the *Nerves* spread over the tongue and palate for TASTING, and all the other nerves, minutely ramified through all the body, for the general sense of FEELING. But the immediate cause of this nervous sensation, whether by means of a *fine subtile Fluid*, called *Animal Spirits*, passing through the hollow *Fibrillæ* of the nerves, or whether by means of a *subtile ethereal Spirit* acting upon the solid *Capillamenta*, or whether this great work of nature be any otherways effected, is as yet a matter concealed from human intelligence.

But whatever be the cause thereof, it is, without all doubt, derived from the noble *Viscus* the BRAIN: For the brain is manifestly of the *Glandulous Kind*; and the use of the GLANDS is to secrete the various juices destined to serve the various purposes of *animal Life*. Thus the *Liver* secretes the *Bile*; the *Pancreas* the pancreatic juice; the *Kidneys* strain off the *Urine*; the *Breasts* collect the *Milk*; the *Testes* screen and prepare the *Semen*; and other glands the *lymphatic Liquor*. By such wonderous contrivances are the operations of life carried on, and the animal functions perfected through the determined period of duration for each respective species.

These, and such like subjects, enoble the science of Philosophy, and give it ineffable merit and praise: Nor is this all; the extreme usefulness of this science, and its universal and indispensible service to all mankind, command our highest regard for it. I have already observed, that it is an essential quality of human nature; *Man is not Man without it*: It gives us all the pre-eminence and dignity due to our species; every thing besides being of a meer animal and sensual nature: But to be a little more particular.

In Theology the absolute necessity and importance of Philosophy is most observable: No man can have any certain, any natural, any just ideas of the *Divine Being*, or Deity, but what he is obliged to this science for. For the invisible things of him from the creation of the world (or from the works of creation) are clearly seen, being understood by the things that are made, even his eternal Power and Godhead: So that they are without excuse who pretend to know God, and discourse of his attributes from any other principles than those of Philosophy; which can only be esteemed a genuine *Commentary on the Bible of Nature*, by which alone we are to be directed in forming every rational Article in the *Creed of that Faith which is according to Knowledge*.

Ethics or Morality has all its foundation in Philosophy: Are not our manners and behaviour proportioned to our knowledge and understanding? Do we expect virtue of the same lustre in a little mind as in a great one? And can vice appear to any so enormous, deformed, and detestable as to those who best understand the natural rectitude of things? Are we to wonder if those who understand not the reason of the laws of *Right and Wrong*, should unconcernedly transgress them? If, therefore, we would have mankind be virtuous, and act aright, let their minds be early formed and embued with the principles of *Philosophy*, i. e. of wisdom and knowledge.

In Astronomy we owe every great improvement to Philosophy; yea, the whole science itself: We hereby know the nature of circular and elliptic motion, and the laws which govern bodies moving in these or any other orbits: We hence learn all the *Anomalies* of motion in a system of bodies; and can settle the *Theories* for calculation. Hence the places, position, *Aspects*, *Transits*, *Occultations*, *Eclipses*, and other affections of the heavenly bodies become known for any given time, past, present, or to come.

In Chronology we are guided by the unerring hand of Philosophy. We thence get a true idea of Time, and the only just methods of measuring it, and dividing it in a natural and proper manner. By this means our *Periods* and *Cycles*, our *Years* and *Days* become constant and certain; which would otherwise be vague and unsettled things, and induce a world of confusion in our accounts, and thereby disturb the occurrences of life.

In Navigation and Geography, great and manifold are the uses of *Philosophy*. From thence we learn the size, dimensions, and figure of the earth; and by the discovered properties of the wonderful stone, are enabled to navigate the spacious seas, with much certainty and safety. Hence a communication and commerce, with other nations and people, is opened unto us; we are hereby made, as it were, proprietor as well as inhabitants of the earth: And most of the wealth and commodities of life are owing to this philosophical improvement of the natural properties of *Wind* and *Water*.

In Mechanics, who does not know that every axiom, every principle, every process, depends upon, and is deduced from one Catholic Proposition of Philosophy, *viz. That Action and Re-action are equal*; *and that the Action, or Force of Action, is compounded of the Quality of Matter and Velocity conjointly*, in every moving body. On this single principle we account for all the effects of every mechanical power or machine, whether simple or compound. For not only the lever, the pulley, the wheel and axle, the wedge and screw, but the action and effect of almost every instrument for moving heavy bodies, and every edge-tool for dividing bodies, have their theory and rationale in the principles of *mechanical Philosophy*.

Yea, GEOMETRY itself is but the Philosophy of the magnitude and dimensions of natural bodies, and their various proportions and relations to each other on that account: And no one who understands any thing of the *modern Newtonian Mathesis*, can deny, that its very first principles, (*viz.* The *Doctrine* of FLUXIONS) consist in the doctrine of motion, and velocity of the generating powers of bodies; and therefore every mathematical science is, in its general nature, purely philosophical: And it would be very easy to shew, that some of the most perplexed propositions of Geometry, are demonstrated with the greatest ease by Philosophy; and that some problems, impracticable by the *Geometrician*, are solvable with the greatest facility and exactness by the Philosopher.

In HYDROSTATICS nothing can be done to any good purpose without the aid of Philosophy: No man could construct the HYDROMETER, or the *Hydrostatic Balance* in the best manner, nor direct their uses to so many great purposes, as those who understand the grounds of this science. Who could have investigated or computed the *Center and Quantity of Pressure* against any *Pen*, *Dam*, or *Sluice*, but a person skilled in *mathematical Philosophy?* And the perfection of *Mill-work* is well known to depend on a thorough skill in the theory of spouting fluids, since only one certain ratio of the velocity of the water, and that of the wheel, can be admitted to answer that end.

In HYDRAULICS, the construction of all kinds of *Pump-work*, *Water* and *Fire Engines* entirely depend on the theory of the *Motion of Fluids*. Hence the art of *Levelling*, the draining of marshes, the making of *Fountains*, or *Jet-d'Eaus*, the praxis of *Reservoirs* and *Aqueducts*, the building of *Bridges*, *Locks*, *Sluices*, and an hundred other necessaries of life, owe their origin, and their ultimate perfection.

In OPTICS, what a variety of the most curious inventions and structures of instruments has of late flowed in upon us? Scarce a year or month can pass not pregnant with optic discoveries and contrivances: And yet none of these inventions, none of those machines, owe their origin to any other source than PHILOSOPHY. 'Tis this science only can discover not only why a microscope can assist the eye to discern small objects, or a telescope distant ones, but it enables the artist to give the best form to his glasses, and to dispose them in the best manner, in the structure of these and other instruments to answer the ends proposed. And who can say to what limits this growing science may yet extend, under the conduct and direction of PHILOSOPHY.

I need not say that PERSPECTIVE, DIALLING, or the *Art of Shadows* in general, is purely philosophical. These arts consist only in the various representations and optical views of nature: And to represent things under the same appearance and respective relation which they have to each, requires no small art or skill in Philosophy. How little do we esteem a meer mechanic Diallist, who knows nothing of the reason or Philosophy of his art? Who sets the *Stile of a Dial* pointing to the *Pole*, for no other reason but because he cannot make it shew the hour in any other position.

PAINTING, as it consists in an *exact Imitation of* NATURE, by a judicious mixture of *Colours*, and a proper disposition of various *Tints*, *Lights*, *Shades*, &c. must be pronounced a *philosophic Art*, whose theory depends on the most refined principles of this science. A person by a thorough skill in the *Doctrine* of *Light and Colours*, might almost make a picture a *Priori*: How natural, genuine, and excellent must that portrait be, which is executed by a hand whose every motion is directed by the dictates of presiding science?

As to MUSIC, I have already observed, that Philosophy is the very soul of that science; and though it may be learned as an art, by meer mechanic practice, and a good ear, (as it is called) yet I believe if any MUSICIAN was to join the *Theory with the Practice*, his compositions and their airs would be thereby greatly improved; and the pleasure, sweetness, and harmony of sounds would be exquisitely heightened, even to his own sensation. And who does not know that a Mathematician, by the bare dint of Philosophy, can compose a piece of music, without any assistance from either art or ear? Of how much more service then must it be to those who happily possess both?

GUNNERY, or the *Doctrine of* PROJECTILES, is perhaps, the only art whose theory is purely philosophical throughout, and that yet has received little or no advantage from this all-perfecting science. Till Sir *Isaac Newton*'s time, all that was wrote on this subject was errant jargon: Since him we have had many pieces on the *parabolic Hypothesis*, whose theories are founded in *Vacuo*, and vacuous theories they are indeed: Their authors not understanding *true Philosophy*, could not instruct mankind in the *Principles of Gunnery*; and this is but too well known an instance of the fatal consequences that attend either the ignorance or neglect of Philosophy, in the momentous affairs of life. However, something considerable has already been done, and more may soon be expected, to give the engineer all the advantages he can possibly have from the present *Mathesis* and *Philosophy*.

CHEMISTRY, considered as an art, has its theory wholly dependent on the philosophic doctrine of attraction and repulsion: And I need only mention how much the *chemical Doctrine of* ELEMENTS has been of late improved and refined by philosophic discoveries and disquisitions. How gross their notion of *four Elements!* how imperfect their number of *three!* how absolutely ignorant were they of the most considerable of the real constituent parts of natural bodies, I mean *Air in its fixed State?* Again, how vulgar and unphilosophical are their notions of *elementary Fire?* And to say truth, it appears from their writings, that there is nothing which they seem so little to understand as the true nature of their most familiar element *Fire*; and in which they stand in so much need of the light of the *Newtonian Philosophy*, which alone gives a rational account of that and every other chemical element.

In PHYSIC and SURGERY the whole field of Philosophy, in its utmost extent is concerned: For, on the one hand, if we consider the human body as a system of *Solids* and *Fluids in Motion*, this will require, at once, a thorough knowledge in all the *Laws of Motion*, of *Action* and *Re-action*, of *Attraction* and *Repulsion*, of every *mechanical* principle and power, the *hydrostatic* and *hydraulic Laws of Fluids*, and every other principle of nature's agency in one, who has the care of such a noble machine to keep it in order, and to rectify it when out. On the other hand, with respect to the *Materia Medica*, it is evident the utmost skill in the *philosophical Principles of Chemistry, Botany, Pharmacy, &c.* is required to render those arts of the greatest service to mankind, in the cure of numberless disorders to which they are liable.

And what shall I more say? For the time would fail me to speak of *Anatomy*, and of *Botany*, and of *Agriculture*, and of *Gardening*, and of every *mechanic and manual Art and Trade* also, even down to *Brewing* and *Baking*; whose professors and artists, by the various improvements and precepts of Philosophy, have been enabled to explain to us the *animal Oeconomy*, the nature of *Vegetation*, the *Culture of Plants*, the *Improvement of Land*, the *Manufacture of Goods*, and meliorating the methods of procuring and preserving our *Bread and our Meat*, our *Beer and our Wine*, and it is in my power to shew, that a man in every vocation, in every employment of life, has occasion enough for the assistance of this science; and that in every occupation, no artist can execute and succeed so well as he that keeps close to nature, and best understands her operations, which as I have shewn, is all that we are to understand by PHILOSOPHY.

If then all I have said be true, (and who will say it is not?) if PHILOSOPHY be of that importance to mankind, as I have shewn it is, we need not wonder to see the wise and knowing part of mankind, in every age, have so great an opinion of such a science, and so desirous of being initiated into its mysteries. How ardent were the pursuits of *Plato*, *Pythagoras*, *Socrates*, *Aristotle*, *Seneca*, and other sages of antiquity, after Philosophy, even in its *Infant State*? But to see and enjoy it in its present glory and perfection, what studies would have been too arduous, what voyages too dangerous, what climates too distant for those champions of wisdom not to have undertaken, with the greatest alacrity and pleasure; How are all the great genius's of every age endeavouring to eternize their memories by inventing new systems of Philosophy? Yea, how frequently do we observe persons destitute of all genius, and scarce entitled to common sense, anxiously aspiring to the honours of this science? So great are the charms of knowledge, even to the *Eunuchs of Science* themselves!

It is very remarkable, that whereas other arts and sciences give only a polish to mankind, and make them expert and ingenious, this of Philosophy, in a peculiar manner, confers not only the highest delight, and the most transporting pleasure to the mind, but even HAPPINESS itself. The attribute of Philosophy is FELICITY by general consent: Thus the inspired penman.—HAPPY *is the Man that findeth Wisdom*. Thus *Virgil* too, FÆLIX *qui potuit Rerum cognoscere Causas*.

After discanting so largely in the praise of a science it may be expected I should say something of the best ways and means of attaining to it; but though this be no part of my design, yet it will be expedient to hint, that for those who would make any tolerable proficiency in the study of Philosophy, there are three several methods for that purpose, *viz.*

First, *Attentive reading*, or a diligent perusal of the best books that are wrote upon the subject, especially those in the systematical way, if only a general notion be proposed. But to be a master of the science, requires an *universal Mathesis in the Newtonian Stile and Manner*.

Secondly, a small *Apparatus of Instruments*, for making observations on nature, as *Microscopes*, *Telescopes*, an *Air-pump*, *Hydrostatic-balance*, *Barometer*, *Hermometer*, &c. But

But especially the first of these, *viz.* the MICROSCOPE, both for the rocket and for the *Camera Obscura:* For this one instrument will discover more of the secret sciences of nature's operations, than all the others of the optical kind put together. They are that sort of *Spectacles* which every wise man should wear.

Thirdly, the most ready and easy means of attaining to a general knowledge of this science, is a *Course of Lectures and Experiments* on the various subjects thereof. And this will readily appear if we consider only the design of it, which is to represent the principal appearances of nature to the view of the audience, and to illustrate the nature and truth of them by experiments, on a large and general apparatus of instruments: So that such a course of experiments is, in reality, but a general and distinct view of nature in miniature; and therefore is of the same use to the mind, as a telescope to the eye. It brings the remote, confused, and distant scenes of nature near to our sight; and gives a glorious inspection of the manner and rationale of most of the operations carried on therein. To say the truth, there is no other way but this by which one can acquire any tolerable or adequate notions of the real principles of this science. A verbal account of this science, in books only, avails little more than *Don Domingo*'s account of the world in the moon. If to books we join an apparatus of proper instruments, we shall more rationally and successfully conduct our enquiries and researches into nature: But who is there that will be at the expence of a general apparatus, much less of a very particular one, for this purpose? Again, who have leisure, and if they have, will be at the trouble of a constant series of experiments, to explore by themselves the endless mysteries of nature? Very few, indeed: Nor can we expect it should be otherwise; for there is required for this purpose a most peculiar and critical genius, attended with a natural impulse to such disquisitions; and not only that, but what is still more rarely found, a capacity for a *physical Mathesis* in a very high degree; for unless a man be qualified in all these respects, he can never make a good proficiency by himself in this science, nor be any ways fit for instructing others as a *Professor*.

From what has been said we may fairly make the following inferences to compleat the praise or just *Encomium* of Philosophy.

First, I have already observed in general, that it is by this kind of knowledge only that we attain to the true refinement and perfection of our nature, which does not consist in the *Matter*, *Form*, or *Animality* of our bodies, but in the powers and faculties of the mind. The more a person, therefore, is imbued with the principles of this science, the more properly he may be said to be *Human*, or approach more nearly to the ultimate perfection and essence of human nature: Philosophy is, therefore, the grand characteristic of man.

Secondly, the great service of this science for improving the NATURAL SENSE, is from hence most evident. How great are the natural blessings of sense! And how miserable do we think ourselves when destitute of any one! when blind, when deaf, &c. How anxious are we to preserve our natural sight, hearing, &c. and at what great expence of money and pain do we endeavour to retrieve them when lost? If these things then are so precious and important to our happiness, how highly should we esteem that science which affords such vast improvements to each of them? Nature confines your prospect within narrow limits, but Philosophy expands or enlarges the *Sphere of Vision* near *ten Million* of times by the telescope. And by nature we are permitted only to view the more gross and coarse part of her works; and yet in these what pleasures do we find? But when Philosophy presents the microscope to the eye, what wondrous scenes appear! what numberless objects before unseen; what endless variety of species; and what amazing beauty and most exquisite perfection ravishes the eye, in its survey of this infinite new creation; And yet, after all, with what indifference do we treat, and how little do we regard this surprizing improvement of our senses?

Thirdly, I have shewn, that all our *moral Sense*, and *religious Sentiments*, must arise from the principles of this science; and here I shall add, *That Philosophy is greatly subservient to Revelation, especially that of the Christian Religion, and easily accounts for or removes most of the Difficulties and Disputations about it.* For by acquainting us with the manner in which primary and secundary causes act, the first absolutely and independently, the last mechanically and consequentially, we are brought to see that the first may interpose to produce any of the phænomena of nature, without interrupting the course of her operations in the ordinary way. The power which first produced an acorn, might, at any time, create the oak which bears it: The infinite wisdom that first established the order of the generation of animals in the common way, may, at any time, produce a man, either adult, as the first *Adam*, or *in Utero*, from a special or peculiar animalcule, as in the incarnation of Christ: So that, in general, we hereby see that there is nothing absurd, unreasonable, or inconsistent with the nature of things, in the *Doctrine of a miraculous Power*; and consequently all those supernatural effects, which are said to have been produced by Christ and his Apostles, are no ways unworthy of our belief. 'Tis evident St. *Paul* often appeals to *natural Philosophy*, to illustrate and inforce the doctrines of revealed religion, particularly in the case of the *Resurrection*, (1 *Cor.* xv.) the most important of all others. Again, we find in the large field of Philosophy, several surprizingly analogous representations of the different states and life of man: Thus the various state of the caterpillar, chrysalis, and fly, (all different forms of the same creature) seem plainly to refer to and typify the present, the mortal, and the future glorified states of man. I might here shew how readily most of those fruitless and perplexing disputes, which have so much and so long distracted the Christian scheme, admit of a thorough decision from the principles of Philosophy, particularly those relating to the soul, the intermediate state, &c. But here I must stop for fear of giving offence: Too much light blinds the eyes, and puts men strangely out of humour. The day is not ushered in at once, but dawns upon us by degrees.

Fourthly, We hence learn of what inestimable service Philosophy is to mankind, in utterly *destroying the very Foundation of Enthusiasm, Superstition, and all Kinds of Imposture.* And it is evident, that nothing but true Philosophy can do this; because these things are wholly founded in ignorance, and are truly works of darkness; but at the approach of this science, ignorance retreats with shame, and impostors, conscious of their villainy, skulk in corners. What glaring instances of this truth has this last century produced? Where are now the wizards and necromancers, the pseudo prophets, the demoniacs, the wonder-working relicts, and the group of omnipotent priests that formerly swarmed in this island? Why, at the feet of Philosophy they bowed, they fell, they lay down; where they bowed, there they fell down dead. So let all her enemies perish, O Lord; but let them that love her be as the sun when he goeth forth in his might.

Fifthly, it appears from what has been said, that it is highly for the honour and interest of every individual to understand Philosophy more or less; as it is essential to the perfection of our natural, moral, and religious sense, we cannot neglect or despise it, without doing violence to reason, yea, to our very nature; and consequently thereby incur the greatest dishonour and shame. If we consider it in regard to mens callings, and business, we shall find it greatly conducive to their interest to have a general notion of this science: For it has been shewn, that it is the foundation of almost every art in life, and gives the rationale thereof: Can the advantage of understanding it then be doubted by any artist whatsoever? Can it be thought a man of theory and science, *i. e.* who understands the true nature and qualities of the subjects of his art, should not be able to manufacture and improve them to a much greater advantage, than the *meer Mechanic* that knows nothing but by practice? This is so far from being a question, that we see it verified every day, in every profession of life, agreeable to *Solomon*'s observation.

Seest thou a Man diligent (or dexterous) in his Business? He shall stand before Kings, he shall not stand before obscure People.—The Wise only shall inherit Glory, and Shame shall be the Promotion of Fools.

Sixthly, Philosophy alone is the source of all *true, solid, or real Learning: For* what is learning but an *Attainment to the true Knowledge of Things?* And if so, by what other means can this knowledge be acquired, than that of experimental Philosophy? By no other, certainly. For what can philology do? Only acquaint us with the knowledge of words, and that is meer *verbal Learning*. What can *Metaphysics* do? Nothing to the purpose, for want of *Data* and experiments. No true knowledge can result from hypotheses, however so ingeniously contrived or disguised. What can *Logic* do in this respect? Only teach us how to digest or methodize the principles of knowledge which we acquire by Philosophy, and reason from them in a proper manner. As to *Poetry*, it is so far from being the source of any learning, that, on the contrary, it has, for its, subject, *pure Fiction*, which is quite its opposite: If *Wit* and *Fancy* be your taste, read *Poetry*; if *Wisdom* and *Learning*, attend on Philosophy. *Criticism*, notwithstanding, all its high pretensions, has nothing in it worth the name of *Learning*. As to those we vulgarly call the *learned Professions*, viz. *Law*, *Physic*, and *Divinity*, I appeal to any man's judgment if there be any thing in the two last by which they can merit that distinguishing epithet, which is wholly due to that Philosophy which is founded on observation, experiment, and mathematical ratiocination. I take no notice of the *Law*, it being a sort of learning *Sui Generis*, and therefore does not come under my cognizance. In short, let men be ever so ambitious of being esteemed learned, yet while they are unacquainted with the *Newtonian Physico Mathesis*, their learning must be extremely superficial, and fit only for meer nominal *Masters of Art*.

Seventhly, It is very manifest from what has been premised, *That the Honour, Commerce, and Wealth of a Nation, bear a high Proportion to the Culture and Improvement of Arts and Sciences, and consequently to Philosophy, which is the Foundation of all.* The truth of this inference nobody will deny: For what honour or renown can be any how possible to an illiterate and barbarous people? In what contempt do all mankind hold the *Chinese*, the *Tartars*, the *Indians*, &c. for their pride, their ignorance, their brutality and inhumanity, which with all other enormous vices, proceed from their want of erudition, and the study of the sciences? Do not they who undertake to polish and civilize a savage and rude people, do it by introducing the study of the arts and sciences? What an illustrious experiment of this sort have we seen tried with success, on the wild *Muscovites*, by *Peter* the Great? And what immortal honours has he thereby procured to himself and his empire? And, to look nearer home, how different a face does our own nation bear at this time, from what it did a few centuries ago? When ignorance of science, slavery, and superstition in religion bore sway here, what infamy attends the history of those times! But when Liberty came, and introduced Philosophy and true Religion, how greatly did we rise in reputation, and how justly renowned for learning above all the kingdoms of the earth! And I think we may truly affirm, That it is more *Honour to be King of the learned* British *Nation, than Emperor of all the World besides.*

In the Eighth and last place we may justly infer, That *Natural Philosophy* is, in a most peculiar manner, the gift of Heaven; the greatest blessing and ornament to mankind; the universal parent of all arts and sciences; and therefore superior to them all in dignity and honour. That it is a science which merits the highest regard, and also meets with it, from all the truly great and wise among men: That it stands upon the eternal foundations of truth, and must therefore endure for ever: That as to its theory or *Rationale*, it is the most sublime and arduous: It is a mystery that has been hid from ages, and from generations; but is now made manifest, to all nations, by the divine writings of the immortal Sir Isaac Newton.

ARTICLE IV.

A

Stricture on the Anti-Newtonians.

By NEWTONIENSIS.

NO more let poets boaſt their golden age,
Nor Halcyon-day employ the metre'd page:
Let times of darkneſs in oblivion lie,
And tune your voices by philoſophy.
The age of ſcience let the muſes ſing,
And to great NEWTON's ſhrine their offerings bring.
His *Manes* let each grateful ſage adore,
Who taught us more than all men knew before.
Whoſe *Genius* moves o'er the chaotic mind,
And gives thereto the human form deſign'd.
His beaming diction did each truth diſcloſe,
And error fled affrighted when it roſe.
In paths of ſcience NEWTON leads the way,
So clear, that ſcarcely the perverſe can ſtray.

Ye ſophs ſuperb, lay pride and ign'rance by,
And learn of humble NEWTON—*not to lie.*
Let him thro' your dark ſouls transfuſe the light,
And ſet your aukward wry neck'd ſpirits right.
World-mongers vain, your elements forego,
Nor how the orbs were made, preſume to ſhow.
Inſcrutable are the works of deity,
Read *Euclid* firſt and learn your A, B, C.

Ye theoriſts too, who impiouſly pretend
God's handy-works to criticiſe and mend:
Retract your errors, learn from common ſenſe,
All pleaſures in variety commence.
Your earths devoid of mountains, rivers, ſeas,
The wiſe would ſhun, and you alone they pleaſe.
Let delug'd worlds no more your wits employ,
To ſhip-wreck nature, and our faith deſtroy.

Ye ſons of ſable night, whoſe wretched ſtrife
Is to defraud the brute of ſenſe and life:
Allow them ſpeech, they ſoon would change the ſcenes,
Prove *they* have reaſon, *you but meer Machines*.
Much bolder yet, and blaſphemous the pen
That dare to fix on free-born ſons of men
The horrid yoke of fate, that ſtrives to blind
By *Laws of Mechaniſm* the heavenly mind.
What reaſon dictates, *virtuouſly* we chuſe,
Nor are we *vicious*, if we muſt refuſe.
Who this denies, muſt needs himſelf belie,
And charge his failings on the deity.

Ye mitred chiefs of error's wildeſt band,
Who can the ſov'reign power of truth withſtand.
Who even demonſtration can beguile,
May triumph over reaſon for a while;
But you at laſt, reluctant ſhall obey
The voice of nature, and be forc'd to ſay,
The *Sun's* at *Reſt*, and bears o'er all the ſway.

And thou, proud *Polignac*, shalt last defame,
And treat with scandal great Sir ISAAC's name;
Inspir'd, I pray, by what delinquent muse,
Dare you, the *First and King of Men* abuse?
The *Nine* revere him, and with pride combine,
To *sing the Writings and the Man divine.*
Presumptuous bard? what mortal could surmise
That *Poets* should on NEWTON criticise!
That *Fiction-mongers* should in learning rule,
And dictate to the sage of NEWTON's school:
With equal scorn and ridicule we hear
From thee a panegyric, or a sneer.
You prove a *Plenum*, sooth; and 'tis as plain
You prove at last a *Vacuum* in the brain.
Extol *Cartesius*, let him be your theme,
And o'er his *Vortices* supinely dream.
But the *Principia* spare, nor treat with scorn
What you, to understand, were never born.
Shall bards, low halting, on pathetic feet
Assail great NEWTON in his high retreat?
Shall wittlings, void of mathematic skill,
Say what are *Nature's Laws* or *Nature's Will?*
Mathesis' boasted chief, direct to weigh
Number and measure what he shall essay:
Forbid it, Heavens! and blast the impious strain
That takes so oft your hallow'd name in vain.
Against *Lucretius* let the verse be writ,
Inferior much in judgment as in wit.
Here stop, meer bard, for know the learn'd and wise
An *Anti-Newton* from their hearts despise:
What NEWTON writes, admits of no contest,
This Popish tongues, tho' pad-lock'd, have confest.
Tho' *Hecatombs of Scribblers* him assail
Yet great is Truth, and ever shall prevail.

Not so elate, methinks grim *Zoilus* cries,
Your sanguine hopes excite my deeper sighs:
Ill-fated incidents each day presage
The direful advent of a gloomy age;
Reflect how oft the most refulgent day,
Precedes a night of horror and dismay:
Your SUN is set; and each revolving year
Some *Stars of primal Order* disappear.
And grant I may, for once conjecture right,
These times appear the twilight of the night.
For see your boasted science set at nought,
How cheap is wisdom, yet how little sought,
Her schools decline, her pupils fall away
And sicken with the small remains of day.
See dulness' minion now the first in fame,
And stars and garters urge him to declaim;
See judges, prelates, and the courtly fair,
To hear the matchless orator repair:
See *Nature's Senate*, erst of great renown,
Now dwindling into foplings of the town.
See how their great law-giver they despise
And to *Batavia's* idol sacrifice:
See learning's disregarded sons appear,
Scarce known to fame, in *Diaries* once a year.
See *Hocus-Pocus*, puppet-shews and plays,
The gay polite diversions of our days:
See time demolish'd with egregious skill,
By chiefs—at E, O, billiards and quadrille.
View yonder, mankind crowding in the lump,
To see the *Conjuror* in a bottle jump;
How willingly in *Folly's Noose they're led*,
To see the *Necromancer* raise the dead:
Lo, there your great, your wise, your worthy ones!
How justly cries *Britannia, O my Sons!*
Such are the omens———

Enough, forbear on all mankind to rail,
Your omens and predictions nought avail:
Nor more alarm us with a gloomy night,
The glorious sun of learning still shines bright:

And as in fields of Æther, 'twill be here,
The *old* extinct, *new Stars* shall still appear.
The *Bats* and *Owls*, and other birds of night,
Do not abhor but *can't endure* the light:
So 'tis with us; the weak and feeble mind,
With learning's mighty blaze would be struck blind.
You ne'er could wisdom's votaries *many* call,
There must be a *great Vulgar* and a *small*;
Nor dare that awful synod to deride,
Where eruditions favorite sons *preside*;
Where *true Nobility* expands her sphere,
And lustre adds, and greatness to the peer,
Where kings inhaunce the glory of their crown,
And Charles, by founding it, acquir'd renown.
There *British Genii*, with unrivall'd skill,
In Newton's glorious cause employ the quill.
What if some wealthy noodle now and then,
To shew his want of sense, employs his pen,
Inspir'd by dulness, who his cranny fills
With all the weight of lead in Mendip-hills:
Shall we from thence conclude the age to blame?
No, let such write, and curse themselves with fame.
What, tho' so few attend in wisdom's school,
'Twere rash to say, that every man's a fool;
Each future age a *Pope* or *Locke* may yield,
Perhaps a *Richmond* too, and *Chesterfield.*
A little remnant we shall always find,
Endu'd with sense, with spirit, and with mind,
In every rank of high and low you'll view,
A race distinguish'd, and a chosen few:
How blest their eyes, which nature's beauties see!
How blest their ears, which hear her harmony!
How blissful those who understand her laws,
And of *each great Effect can know the Cause.*
Thrice happy all whom Newton can inflame
To seek, by science and by virtue, fame.

Ye sons of art, great Newton's worth unfold,
To endless ages let his deeds be told:
To distant echoing worlds exalt his name,
And in eternal *Pæans* sound his fame.
No laurel crowns, or monuments prepare,
(Such trophies mortal kings and heroes wear.)
In *Works of Genius* shall his altars rise,
And glory build his temple in the skies.
Where he, by wisdom sceptred, from her throne,
Shall reign o'er all superior, and alone,

ARTICLE V.

THE DESCRIPTION AND USE OF

A New Surveying Quadrant.

By Mr. JOHN BICKFORD, of *Westminster.*

MATHEMATICAL Instruments are so essentially necessary in several branches of arts and sciences, that every attempt to improve them, I presume, will not be unacceptable to the ingenious mathematician. The mensuration of angles in all positions, with the greatest ease, expedition, and accuracy, claims the highest place in practical mathematics. It is not my intention in this place, to describe the several ingenious methods, and curious instruments invented for this purpose; but only, to describe an instrument intended principally for the use of the Land-Surveyor; which for ease and accuracy in operation, will be found to equal, if not excel, every angular instrument now in use.

The DESCRIPTION. *(Fig. 1.)*

The instrument is a quadrant or quarter of a circle, having a moveable index A round the centre C, upon the index at C is placed a large speculum B, about the size of the large one on Hadley's quadrant; this is placed truly perpendicular upon the centre of the instrument, and in a right line with the middle of the index. Likewise upon the top of the speculum at d, is a large opening to view distant objects through when the instrument is used; from the centre of the index at C is placed a hair or fine wire truly perpendicular as C d; that part of the index which cuts the degrees, &c. on the limb, is divided by Nonius's divisions, so as to shew every 5 or 3 minutes of a degree, according to the size of the instrument, (which if it be about 12 or 15 inches radius will be a very good size) and this will be sufficiently near for the purpose of land-surveying. The limb EE is divided into 180 equal parts or degrees, which is divided into these number of parts from the properties of reflection, and must be numbered accordingly; at E is placed a perpendicular sight to guide the eye to the speculum in the time of using the instrument; the rest of the parts are evident by the figure.

The USE.

1st. To measure an horizontal angle Q P R, *(Fig. 3)*

Hold the instrument in an horizontal position at the angular point P, and direct the side of the quadrant E C in the direction P R, by looking through the sights E & d, then move the index on the limb till you see the object Q on the speculum exactly coincide with the hair on the glass when the hair is in the direction of P R, then the index will shew the measure of the angle on the limb.

2d. To measure a vertical angle T S V, *(Fig. 4.)*

Hold the instrument in a vertical position at the angular point S and direct the sights E d in the horizontal line S T, then move the index till the object V on the glass exactly coincide with T, then will the index shew the angle on the limb.

OBSERVATION.

By a little variation in the construction of this instrument, it may be made to answer many important purposes in Astronomy and Navigation; for by adapting a spirit-level near the sight E, the sun's altitude may be taken with ease and accuracy when the horizon cannot be seen, which is sometimes the case at sea even when the sun shines. This instrument will be found preferable to the sextant in measuring the distance of the moon from the sun or a fixed star, for the purpose of determining the longitude of a ship at sea: For by this instrument, not only a larger field is taken in by the speculum, whereby the objects are caught sooner by the eye; but, the angle being only a single reflection which renders the objects on the speculum almost as distinct as the objects themselves; whereas on the common sextant the angle is a double reflection, which must of course render the objects less distinct.

ARTICLE V.

AN

Essay on the Longitude,

By Mr. Charles Clark, of Millbank-Row, near Grosvenor-House, Westminster.

Mr. Whiting,

YOUR laudable plan of publishing half-yearly, Mathematical and Philosophic Questions, Essays, &c. is highly deserving attention; and, as a well-wisher of these Sciences, it gives me pleasure to find there is such an opening for Mathematical Correspondence; which may not only prove a delightful amusement, but also be productive of great improvements in Scientific Knowledge: even hints from indifferent capacities, when considered by persons of superior abilities, and more extensive knowledge, may be so improved upon, as to become beneficial to society, and the World at large. This being the case, I beg through the medium of of your excellent Publication, to offer my thoughts on a method of obtaining the difference of time between two Meridians, by means of the moon's right ascension; and which I think may be accurately enough determined, by observations at sea, to answer this purpose.

The Nautical Almanack published yearly, under the Direction of the Royal Astronomer, and the Commissioners of Longitude, contain tables accurately calculated, of the moon's right ascension and declination, at noon and midnight, for every day, throughout the year, adapted to the latitude and longitude of Greenwich Observatory, and likewise, the exact time she passes that Meridian. It will be necessary for putting my proposed method into execution, that the Navigator should be furnished with these tables, and also with good tables of the right ascension and declination of all the principal stars in the Zodiac; likewise two persons, capable of making correct observations with an Hadley's octant or sextant, of which two of the best kind should be ready and carefully adjusted. Let one of the observers take the moon's Meridian altitude, and the other the altitude of any such star in the Zodiac, as may happen then to be on, or very near to the Meridian; then will that ſtar's right aſcenſion as found in the tables, be the right aſcenſion of the moon; and if you add or ſubtract (as may be neceſſary) the difference of the Meridian altitudes of the moon and ſtar (when corrected for dip, refraction, and parallax) to, or from the declination of the ſtar, you will have the declination of the moon; by means of which, and her Meridian altitude, the latitude of the place of obſervation may be determined; then with the moon's right aſcenſion, examine the tables in the Nautical Almanack, and you will find at what time at Greenwich, the moon had that right aſcenſion:- the difference between this time and the time of your ſea obſervation (determined by the uſual method) will be the difference of time between thoſe two Meridians, and conſequently their difference of longitude will be determined.

I am perfectly ſenſible that many difficulties may be ſtarted againſt this method, but which I think may in a great meaſure be removed. Firſt it may be ſaid, that the horizon by moon-light is not eaſily ſeen; this objection I would obviate by having an artificial one. Next it may be objected that the moon and ſtar might not be on the Meridian at the ſame inſtant of time; conſequently a difficulty would ariſe to get at the moon's right aſcenſion correctly; this I would correct by having a third perſon, with a good going watch, for about half an hour, to time the moon and ſtar's increaſe and decreaſe of altitude, before, at, and after they may be on the Meridian, in order to get at the true place of the moon (when corrected for dip, refraction, and parallax) at the time the ſtar is on the Meridian; and which may be further corrected, by comparing the latitude found by the ſtar and moon's Meridian altitudes, ſeperately; and, the declination of the moon found by obſervation, correſponding with the tables in the Nautical Almanack, againſt her right aſcenſion.

In the third place, a great objection may be made to the moon's ſlow and unequal progreſs, ſo as to cauſe her right aſcenſion, in ſome parts of her orbit, to change only about thirty ſeconds motion in a minute, (four minutes of time being equal to a degree of longitude) but even this, where the greateſt difficulty appears, may, in a great meaſure, be ſurmounted by good inſtruments and correct obſervers, and the right aſcenſion and declination of the moon to be calculated in the tables of the Nautical Almanack, for every three hours.

If I find this method has the approbation of any of your Scientific correſpondents, and they will favor me with a line, I will, at a future time, ſend you a few examples for our Navigators to put in practice, whereby I am inclined to think, not only the longitude, but the latitude alſo, may be obtained by one and the ſame obſervation of the moon, when on the Meridian; the longitude by her right aſcenſion, the latitude by her declination, and by leſs calculation than any method hitherto made uſe of.

ARTICLE VI.

THE

Computation of a solar and lunar Eclipse,

By Mr. Thomas Milner, of Catterick, Yorkshire.

TO find the time of the sun's eclipse, Sept. 5th, 1793, at London. The elements computed from Mayer's tables, farther improved, are as follows:

	D	h	m.	s
1. The true time of conjunction, is Sept.	4	23	52	32
Longitude of the sun and moon —	5ˢ	13°	16	34
Latitude of the moon North, ascending	—	—	40	21
The moon's horizontal parallax —	—	—	54	3
Her apparent diameter —	—	—	29	28
The sun's diameter — —	—	—	31	45
The moon's horary motion —	—	—	29	48
The sun's horary motion —	—	—	2	26
The horary motion of the moon from the sun.		—	27	22

2. Let HZO be the Meridian of the place, *(Fig. 7)* HO the horizon, GB the equinoctial, GEB the ecliptic, Z the zenith, P the pole, S and M the places of the sun and moon in conjunction, PSF the sun's Meridian. Having found the sun's distance from the node ☊ S = 7° 37′ 39″, take ☊A to ☊ S, as the sun's horary motion to the moon's horary motion; then SA = 6° 59′ 52″. Draw AM; then in the spherical triangle ASM, right-angled at S, are given AS, and SM; to find the ∠ AMS = 84° 32′ 18″; MA being the moon's way from the sun; but because the eye of the observer is in motion, therefore the apparent motion in the moon caused thereby will be in the line CS. And to determine the position of CS in respect of MA, the ∠MNS must be known, which occasions the resolving several spherical triangles as follows:— In the right-angled triangle BFS, are given SB, and the ∠ B, to find FS = 6° 35′ 3″, and the ∠ BSF = PSA = 67° 24′ 58″. And in the triangle ZPS, are given PZ, PS, and the ∠ P, to find ZS = 44° 58′, and the angles PZS = 177° 22′ 32″, and ZSP = 1° 38′ 34″. Then ZSP and ASP being known, ZSA will be known. And ASM being a right angle; therefore ZSM = 20° 56′ 28″. In the right-angled triangle CSD, are given CD and DS, to find CS = 88° 8′ 45″, and the ∠ CSD = ZSN = 88° 8′ 42″. Then NSZ and ZSM being known, therefore NSM = 67° 12′ 14″. And AMS being known, its supplement NMS = 95° 27′ 42″, and therefore the ∠ MNS =

17° 20′ 46″. Then to find the quantity of the apparent motion along SN; that along MA is already known. Put a = the moon's horizontal parallax; then to the log. .259a add the cosine of latitude, and the sine of CS. Then the sum abating twice the radius, is the log. 0.939419 = 8′ 41″ the apparent horary motion along SN. Then this motion is to be compounded with the motion along AMN, as follows:

Let AS be a part of the ecliptic, (*Fig.* 10) MA the moon's way from the sun, SN the way of the apparent motion. Draw MR parallel to SN; and let MR be the horary motion along SN = MR, and MK the horary motion of the moon from the sun. Then compleat the parallelogram RMKW; draw the diagonal MWQ, which is the direction of the motion compounded of the observers, and the moon's motion, and MW is the total apparent horary motion; supposing the observer at rest. Then in the plain triangle MWK, are given MK, and KW = MR, and the ∠ MKW = MNS; to find the ∠ WMK = 7° 45′ 43″ and MW = 19′ 13″ the absolute horary motion. Now the angles WMK and KMS being known, therefore the ∠ SMW = 92° 18′ 1″.

3. To find the moon's parallax of altitude, say, as rad. : to cosine DS (the sun's altitude) :: the moon's horizontal parallax : to her parallax Mm = Vt = 38′ 11″ (*Fig.* 11) Then in the right-angled spherical triangle Mnm, are given Mm, and the angle mMn, to find mn = Ss = 13′ 39″ her parallax in longitude, which is increased so much; and Mn = [redacted] = 35′ 30″ her parallax in latitude; then Sn = 4′ 43″ her remaining latitude.

4. Draw GS for the ecliptic; (*Fig.* 12) at any point S, erect the perpendicular SM = 4′ 43″, the moon's latitude; through M draw the moon's way ☊ MQ, making the ∠ SMQ = 92° 18′ 1″. Draw SP ⊥ MQ. From the center S, with the radius SB = 15′ 52″, describe the circle BDC for the sun. And with the radius PA = 14′ 44″, and center P, describe the circle DAC for the moon.

5. Then P is the place of the moon's center in the middle of the eclipse. Make SH and SL = to the sum of the radii of the sun and moon; and the moon's center will be at L when the moon first touches the sun, or the beginning of the eclipse, and H the moon's center at the end of it. Then in the triangle PSL, are given SL, and SP; to find PL = PH = 30′ 14″; and the time of moving thro' PL or PH, at the rate of 19′ 13″ an hour is 1^h 34^m 23^s, for half the duration.

6. Then to find no, or the digits eclipsed; we have no = Sn + Po − PS, and 6no + Sn = 9° 47′ 13″ the number of digits eclipsed; and by the position of that part of the ecliptic, her parallax in longitude advances her so much forward, viz. 13′ 39″. And therefore she is 43^m 37″ past the apparent conjunction.

7. Whence, on the 5th of Sept. 1793, the apparent time at London, of the

Beginning is	9^h	37^m	15^s	morn.
Middle	11	11	38	
End	12	46	1	
Duration	3	8	46	
Digits eclipsed	9°	47′	13″	

To compute an Eclipse of the Moon, which will happen on the 25th day of February 1793, for London. The elements computed from Mayer's Tables, are as follows:

1. The true time of opposition, at London,		10^h	49^m	28^s
And the apparent time is		10	36	14
2. The sun's place is	11^s	7	51	43
The moon's place	5	7	51	43
The place of the ascending node	5	15	28	37
Her latitude south			40	12
The sun's horary motion			2	31
The moon's horary motion			30	11

3. Let ☊B be a part of the ecliptic, AD the moon's apparent orbit, (*Fig.* 14) S the center of the earth's shadow, ☊ the place of the ascending node, and D the center of the moon, when in opposition. Hence the moon is approaching her ascending node, and ☊S = 7° 36′ 54″. Then to find the ∠ SDA, take ☊A, to ☊S, as the sun's horary motion to the moon's, hence AS = 6° 58′ 49″. Then in the right-angled spherical triangle SAD, are given AS, and SD, to find the ∠ ADS = 84° 32′ 41″.

4. Let fall SM ⊥ to AD. Now since the arches SD, SM, MD, are very small, they may be taken for right-lines; and the triangle SDM, for a plain triangle. Then having SD, and the ∠ MDS given, to find MD = 3′ 50″ and MS = 40′ 1″. Also, the time of the moon's moving through DM will be 8^m 18^s; and hence the time when she is at M, or the middle of the eclipse is known.

5. The sun's apparent semi-diameter	16′ 9″
His horizontal parallax	0 12
The moon's apparent semi-diameter	14 50
Her horizontal parallax	54 25

6. Hence the radii SB = 38′ 28″, MH = 14′ 50″.

7. Then in the plain triangle SGM, are given SG, and SM, to find MG = MK = 38′ 12″; and therefore the time of passing through MG = 1^h 16^m 20^s; and the whole time from G to K = 2^h 32^m 40^s.

Again hn = Sn + Mh − SM; and the digits eclipsed = 6hn ÷ Mh = 5° 22′ 22″.

Whence on the night of the 25th of February,

The eclipse begins at	9^h	28^m	16^s
Middle	10	44	36
Ends at	12	0	56
Duration	2	32	40
Digits	5°	22′	22″

M.

ARTICLE VII.

Mathematical Questions and Solutions.

Question 14, *from Page* 343, *Emerson's Algebra.*

THERE is a cask of rum, out of which was taken 27 Gallons, and filled up with water, and the same repeated three times more. At last there was found by the proof, to remain 16 gallons of rum in it. What was the content of the cask?

Solution, *by Mr. Thomas Whiting.*

Put n = the ratio of the progression, b = 27, and c = 16. By question the first of this book cn^4 = the content of the cask; then $\frac{cn^4}{cn^4 - b} = n$ = the ratio; this solved gives n = 1.5, therefore the content of the cask is 81 gallons.

Question 15, *by Mr. Timothy Simpson, of Papplewick.*

There is a rectangular piece of land ABCD, whose length AB, is a, and breadth AC, b chains. The quality of the land varies regularly in such a manner, as make the values of any several infinitely small portions thereof, in the subduplicate proportion of their distances from the side AB: but an infinitely small portion thereof, taken any where, at the distance of c chains, from the side AB, is worth d pounds per chain. From these premises it is required to divide the land into two equal values, by a right line drawn from the angular point B.

Solution, *by Mr. Joseph Garnett, jun. at Mr. Rodham's Academy, Richmond, Yorkshire.*

The method I have taken in my solution to question 637 Gents Diary (which see) of comparing the value of a field, &c. to the content of a solid body, may be applied with advantage to this question, as well as to several others of the like kind. Let G (*Fig.* 15) be any point in the side AC, and suppose GH and CI (⊥ to AC) to represent the values per chain at G and C respectively; then since per question $GH^2 : CI^2 :: AG : AC$, the curve AHI is a common parabola, and the value of the whole field is = to the solidity of the prism ACIKBD, the end whereof is the semi parabola ACI and length AB, and the question is reduced to this, viz. to cut from the said prism ACLMLA, by the plane ALM (⊥ to the plane ABCD) passing through A, whose solidity shall be to that of the whole body, as m to n, i. e. 1 to 2 in the present case.

To find a general expression for the solidity of any part ACIMLA, put $z = CL$, and $x = AG$, then per sim. Δ's $AC : AG :: CL : GN = zx \div b$, and per the parabola $\sqrt{AC} : \sqrt{AG} :: CI : GH = \frac{d\sqrt{b}}{\sqrt{c}} \times \frac{\sqrt{x}}{\sqrt{b}} = \frac{d\sqrt{x}}{\sqrt{c}}$ $\therefore$ GN. GH. $\dot{AG} = \frac{dzx^{\frac{3}{2}}\dot{x}}{bc^{\frac{1}{2}}} =$ the fluxion of the solid AGHONA, the fluent whereof, z being constant, is $\frac{2dzx^{\frac{5}{2}}}{5bc^{\frac{1}{2}}}$, which, when $x = b$, becomes $\frac{2dzb^{\frac{3}{2}}}{5c^{\frac{1}{2}}} =$ the solidity of the part ACIMLA. Now, per question, $\sqrt{c} : \sqrt{b} :: d : \frac{d\sqrt{b}}{\sqrt{c}} = CI$; consequently the solidity of the prism $= \frac{d\sqrt{b}}{\sqrt{c}} \cdot \frac{2ab}{3} = \frac{2adb^{\frac{3}{2}}}{3c^{\frac{1}{2}}} =$ the value of the whole field; $\therefore$ per question we have only to put $\frac{2dzb^{\frac{3}{2}}}{5c^{\frac{1}{2}}} : \frac{2adb^{\frac{3}{2}}}{3c^{\frac{1}{2}}} :: m : n$, whence $z = \frac{5m}{3n} \times a =$ (in the present case) $\frac{5}{6} \times a$.

Otherwise, by the common method.

Put $z = CL$, and $x =$ any part (AG) of AC, then per question $\sqrt{c} : \sqrt{x} :: d : \frac{d\sqrt{x}}{\sqrt{c}} =$ the value per chain at G, consequently $\frac{d\sqrt{x}}{\sqrt{c}} \times$ (fluxion of the area) $a\dot{x} = \frac{ad\sqrt{x} \times \dot{x}}{\sqrt{c}} =$ the fluxion of the value, whose fluent is $\frac{2adx^{\frac{3}{2}}}{3c^{\frac{1}{2}}}$, which, when $x = b$, becomes $\frac{2adb^{\frac{3}{2}}}{3c^{\frac{1}{2}}}$ the value of the whole field. Again, to determine the value of any Δ ACL, per sim. Δs. $AC : AG :: CL : GN = \frac{zx}{b}$ and the fluxion of the area is $\frac{zx\dot{x}}{b}$ which multiplied by $\frac{dx^{\frac{1}{2}}}{c^{\frac{1}{2}}}$ (the value per chain at G) gives $\frac{dzx^{\frac{3}{2}}\dot{x}}{bc^{\frac{1}{2}}}$, whose fluent (z being constant) is $\frac{2dzx^{\frac{5}{2}}}{5bc^{\frac{1}{2}}}$, which, when $x = b$, becomes $\frac{2dzb^{\frac{3}{2}}}{5c^{\frac{1}{2}}} =$ the value of the Δ ACL; $\therefore$ per question $\frac{2adb^{\frac{3}{2}}}{3c^{\frac{1}{2}}} : \frac{2dzb^{\frac{3}{2}}}{5c^{\frac{1}{2}}} :: n : m$, hence $z = \frac{5m}{3n}$. a, as before.

Question 16, *by Mr. A. Bucanon, jun. of Sedgfield.*

What sum of money in pounds and shillings is that, whose half is just the reverse?

Solution, by Mr. Timothy Simpson, of [illegible].

It is evident the number of pounds must be odd, for which put x; and the number of shillings even, for which put y; then by the nature of the question, $\frac{1}{2}x - \frac{1}{2} = y$, $\frac{1}{2}y + 10 = x$, by equating the values of y, from these two equations, we have $2x - 20 = \frac{1}{2}x - \frac{1}{2}$; hence $x = 13$, and consequently $y = 6$; therefore the sum sought is 13l. 6s.

Question 17, *by Mr. James Ashton, of Harrington, near Liverpool.*

In a triangular room, the sides of which are 14, 12, and 10 feet respectively, there stands a circular table, the diameter and height of which are each 2¼ feet, and the distances from the center of the table to the three corners of the room are 9, 6¼, and 5 feet respectively. It is required to find the height and position of a candle, placed on the said table, so that the periphery of the table's shadow, may fall upon each corner of the said room.

Solution, *by the Proposer.*

Let the small circle (*Fig.* 8) represent the table, c its centre, L the place of the candle, ABD the room, and C the center of the shadow, the periphery of which passes through ABD. By plane trig. we get Ap = 8.5714285 and Bp = 5.4285715; hence 12 : 1 (radius) :: Ap : sine ∠ ADp = .7142857 = 45° 35′ 4″; and 10 : 1 :: Bp : .5428572, = the sine of the ∠ BDp = 32° 52′ 42″; their sum is = ∠ ADB; which is = the ∠ ECB at the center = 78° 27′ 46″; then AB : sine ∠D :: AD : .8398071 = the sine of the ∠ ABD = 57° 7′ 11″; and as sine ∠ FCB, = .979775 : 1 (radius) :: BF = 7 : CB = 7.1445 = the radius of the circle made by the shadow on the ground. Now $\sqrt{CB^2 - BF^2}$ = CF = 1.43, and CB : rad. :: CF : sine ∠ FBC = .2001539 = 11° 32′ 45″. In the triangle DcB, BD = 10 : Bc + cD = 11¼ :: Bc − cD = 1¾ : 2.05625, then $\frac{10 + 2.05625}{2}$ = 6.028125 = BP; then Bc : rad. :: BP : sine ∠ BcP = .8930555 = 63° 15′ 35″ its complement = ∠ cBP = 26° 44′ 25″; then ∠ cBP + ∠ FBC subtracted from the ∠ ABD (57° 7′ 11″), leaves ∠ cBC = 18° 50′. Now having the two sides CB, cb, and the included ∠, we get the ∠ cCB = 79° [illegible] 8″; and sine ∠ cCB : Bc :: sine ∠ cBC : Cc = 2.3064 = the distance of the centers on the ground, viz. from the center of the shadow to the point on the floor perpendicularly under the center of the table.

Then $\sqrt{(2.3064)^2 + (2.5)^2}$ = 3.4 the part of the axis of the oblique cone, from the center of the shadow to the center of the table. Put a = 7.1445 = the radius of the shadow, b = the radius of the table = 1.25 c = 3.4 and x = cL, the other part of the axis; then a : b :: c + x : x ∴ ax = bc + bx, whence $x = \frac{bc}{a-b}$ = .721 feet, and the whole axis = 4.121 feet. Now by similar triangles 3.4 : .721 :: 2.3064 : .4892 = the distance from the center of the table to the centre of the bottom of the candlestick; and 3.4 : .721 :: 2.5 : .53015 of a foot = 6.36 inches = the height of the flame of the candle from the plane of the table, supposing the flame a point.

Question 18, *by Mr. Olinthus Gregory, of Yaxley.*

A gentleman has a fish-pond, in form of a rhombus, its perimeter being 80 yards, and superfices 384 yards, but he means to have it enlarged into an ellipsis; the conjugate diameter of which shall be the shortest diagonal of the rhombus, and the foci to be at the ends of the longest diagonal. He wishes to know what the part wanting to form the ellipsis will cost digging, at 6d. per yard on the surface.

Solution, *by Mr. Joseph Garnett, jun. at Mr. Rodham's Academy, Richmond, Yorkshire.*

Let ABDC (*Fig.* 13) be the rhombus, put a = AB = 20 yards. c = 384, and x = AE, then $\sqrt{a^2-x^2}$ = EB, and $2x\sqrt{a^2-x^2}$ (AC × ½DB) = c, hence x = $\sqrt{\frac{a^2 + \text{or} - \sqrt{a^4-c^2}}{2}}$, where the sign + or − gives $\left\{\begin{matrix}16 = AE \\ 12 = BE\end{matrix}\right\}$.
Now by the property of the ellipse 2AB = the transverse = 40, and per mensuration 2AB × BD × .7853 &c. − c = 369.98 yards = the area of the part wanting to form the ellipsis, which at 6d. per yard, amounts to 9l. 5s. nearly.

A direct calculation may be made as follows, without algebra, but it is not so general; thus the area (384) divided by AB (80 ÷ 4) is equal to DF, and $\sqrt{AD^2 (= AB^2) - DF^2}$ = AE, then AB − AF = FB, and $\sqrt{DF^2 + FB^2}$ = DB = 24 the conjugate; whence the area &c. as before.

Question 19, *by Mr. John Barrow.*

The chapel at Bolton (standing in the latitude of 54° 38′ N.) is an ancient building; over the north window is an inscription, in Saxon characters, importing that the building was finished on the 20th of May, when the shadow of the steeple bearing N. E. by E. 6° 41′ ¼ E. was in ratio to its height as 54 to 44, and the angle included by the steeple and shadow, was 95° 3′ : required the time of the day when the observation was made, and how long it is since the chapel was built?

Solution, *by Mr. John Farey, Norwich-Court, Fetter-Lane.*

Let AB (*Fig.* 9) represent the chapel, standing on the declivity AC, BC a ray of light from the sun, casting the shadow of the top on the point C, then we have given the angle BAC = 95° 3' and the ratio of AC to AB, as 54 to 44, whence per Trigonometry we find the angle BCD = 42° 11' 16.3" the sun's apparent altitude, from which subtracting 15' 50.6" for the sun's semi-diameter, and 1' 3.2" for refraction, and adding 6.2" for the sun's parallax, we have his true altitude = 41° 54' 28.7". In the spherical triangle ZSP (*Fig.* 4) let ZP represent the co-latitude = 35° 22', ZS = the sun's co-altitude = 48° 5' 31.3" and the angle SZP, the sun's azimuth from the North = 117° 3' 30". Then by Trigonometry we find the hour angle ZPS = 45° 00' 13" or 3^h 00^m 0.9" PM, and also the side ZS, whose complement is the sun's declination = 20° 24' 34.2" N. Now the year in which the sun had this declination on the 20th of May, at 3 hours PM, must be found by repeated trials from the tables of the sun's motion, the best for which purpose, are Mayer's. In this search we may be assisted, by considering that the Saxons (inhabiting this Island) were converted to christianity about the year 600; it is not likely that Mr. Burrow's ancient chapel, was built before that time; nor is it likely they were long after that period without chapels; accordingly, by a few trials, we find that AD 611. the declination of the sun on the 20th of May, at 3 hours PM, was 20° 24' 34.6" N. agreeing with the declination found above, within $\frac{2}{5}$ of a second. Therefore, this ancient chapel was finished in the year of our Lord, 611.

The *Editor* finds the hour and declination nearly the same as above, and then proceeds as follows, viz. The sun's declination on the 20th of May 1790, old style, is = 21° 59' 18"; the difference between this and the declination found above is = 1° 34' 44"; this difference answers to 9^d 0^h 3^m the time the sun takes in making the above change. The difference between the common and solar year, is 11 minutes and 3 seconds; hence 11m. 3s. : 1 year :: 9 days and 3 minutes : 1173 the years since the chapel was built; therefore, according to this method, it was finished about the year of our Lord, 617. But this question is of such a nature, that a very minute error will cause a large disagreement in the solution; therefore the above found answer cannot be depended upon for the following reasons. First, the difference between the solar and julian year, may not be exactly 11m. 3s. Secondly, the unequal change of the sun's declination, makes it difficult to determine the exact time (without much trouble) the sun is in making the above change. Thirdly, the change in the obliquity of the ecliptic will effect the solution. Hence it is necessary to obtain the true time, by making trials as Mr. Farey has done; but this method of approximating, is of excellent use in questions of this nature, as it points out a year not very far from the true one.

Question 20, *left unanswered in the S. Repository.*

Suppose three-fourths of the solidity of a bason, in form of a half sphere, whose diameter is 40 inches, standing with its base parallel to the horizon, to be filled with rain water; if a cylinder of dry oak be laid upon the surface of the said water, its own weight is just sufficient to sink it till each end of the cylinder just rests on the sides of the baton, and at the same time raises the water up to the brim. Required the dimensions of that cylinder?

Solution, *by Mr. John Ryley, of Leeds, Yorkshire.*

By having the diameter of the hemisphere given = 40 inches = AB; (*Fig.* 16) the content is found to be 16755.2 cubic inches; the fourth whereof is 4188.8 cubic inches = the content of that part of the cylinder which is immersed in the fluid. (per question) Now the specific gravity of water being to that of dry oak as 1 : .915 hence .915 : 1 :: 4188.8 : 4577.9235 = the content of the whole cylinder. Therefore 4577.9235 − 4188.8 = 389.1235 = the content of that part of the cylinder which stands above the surface. Now put d = Ec; v = ec; r = AC; a = 4577.9235 and b = 389.1235. Then by rule 5th, page 105, of Dr. Hutton's Mensuration $\frac{4}{3}v\sqrt{dv - \frac{3}{5}v^2}$ = the area of the segment abc; therefore $\frac{b}{\frac{4}{3}v\sqrt{v-\frac{3}{5}v^2}} = EF = \frac{a}{cd^2}$ the length of the cylinder: hence $\frac{25c^2b^2d^4}{16a^2v^2} = dv - \frac{3}{5}v^2$; or (putting $n = \frac{25c^2b^2}{16a^2} = .03418$) $nd^4 = 5dv^3 - 3v^4$. Again by the property of the circle $\sqrt{\left(r+\frac{a}{2cd^2}\right)\times\left(r-\frac{a}{2cd^2}\right)} = Ee = d-v$; hence $v = d - \sqrt{r^2 + \frac{a^2}{4c^2d^4}}$; or putting $\frac{a^2}{4c^2} = m = 8493667.914359$) $v = d - \sqrt{r^2 - \frac{m}{d^4}}$. Now from the above equation, viz. $nd^4 = 5dv^3 - 3v^4$, by substitution we obtain $nd^4 = 5d \times \overline{d - \sqrt{r^2 - \frac{m}{d^4}}}\Big|^3 - 3 \times \overline{d - \sqrt{r^2 - \frac{m}{d^4}}}\Big|^4$; this equation solved by trial and error gives d = 13.08 hence v = 2.6 = ec and EF = 34.08.

Question 21, *by Astronomicus.*

On what day in 1793, will the time between the moon's southing and setting be a maximum at London?

Solution, *by Mr. Thomas Whiting.*

This happens on December the 18th. the moon's declination north, being considerably greater than at any time in the year. But if there had been another month wherein the declination was equally great, the moon's hourly motion must have been found, and that which gave it the greatest would have been the answer. Remark. The horizontal parallax may also effect the solution a little.

Question 22, *by Mr. Thomas Whiting.*

A person standing at an unknown distance from 2 fires, (not in a line between them) of equal size; found the ratio of their heat as 2.5 : 3, but walking 10 yards nearer to the farthest fire, their ratio was as 2.9 : 3. Required his distance from each fire, in both situations; the distance of the fires being 25 yards?

Solution, *by the Proposer.*

Let A and B be the 2 fires, (*Fig.* 17) C the first and D the last station. Then as the intensity of heat is as the square of the distance from whence it is propagated; the sides AC, CB, also the sides AD, BD are as the square roots of the given ratios; hence by really extracting, and reducing them to their least terms we get AC : BC :: 1 : 1.095445115 and AD : DB :: 1 : 1.0170952554 for which substitute v and w. Put $a = 25 = AB$, $b = 10 = CD$, $x = AC$, $y = AD$, then $vx = CB$, and $wy = DB$. By page 113, Em. Trig. $\frac{a^2 + v^2x^2 - x^2}{2avx} = \frac{a^2 + w^2y^2 - y^2}{2awy}$ each side of the equation being equal to the cosine of the angle B. Also, $\frac{v^2x^2 + x^2 - a^2}{2vx^2} = \frac{b^2 + x^2 - y^2}{2bx}$, each side being equal to the cosine of the angle C. From the last of these equations $y^2 = b^2 + x^2 + \frac{ba^2}{vx} - \frac{bx}{v} - bvx$, this value of y^2 being substituted in the first equation, the values of x and y may be found with a sufficient portion of care and patience.

Question 23, *by Mr. Thomas Whiting.*

There is a cylinder, whose lenght is 100 yards, and diameter 2 inches; fixed in an horizontal position and moves freely about an axis passing through its centre; about it is a thread wrapped in such a manner that each round just toucheth the preceding one; the diameter of the thread .01 of an inch. How far must a person go in unwinding it, on condition that the thread be kept stretched at right angles to the cylinder, and that they go as far off as the extremity of the thread will permit.

Solution, *by Mr. John Bickford, of Westminster.*

Let AB (*Fig.* 18) represent the cylinder BC the thread when it is all run off, and AC the path the person goes when the thread is unwinding. First, AB = 100 yards = 3600 inches, and 2 × 3.1416 = 6.2832 the circumference of the cylinder, then 3600 ÷ .01 = 360000 the number of rounds of the thread on the cylinder, and 6.2832 × 360000 = 2261952 = BC the whole length of the thread; then (by 47 Eu. 1) having AB and BC given AC is easily found = 2261954 inches = 35 miles, 1232^y 0^f 2^i as required.

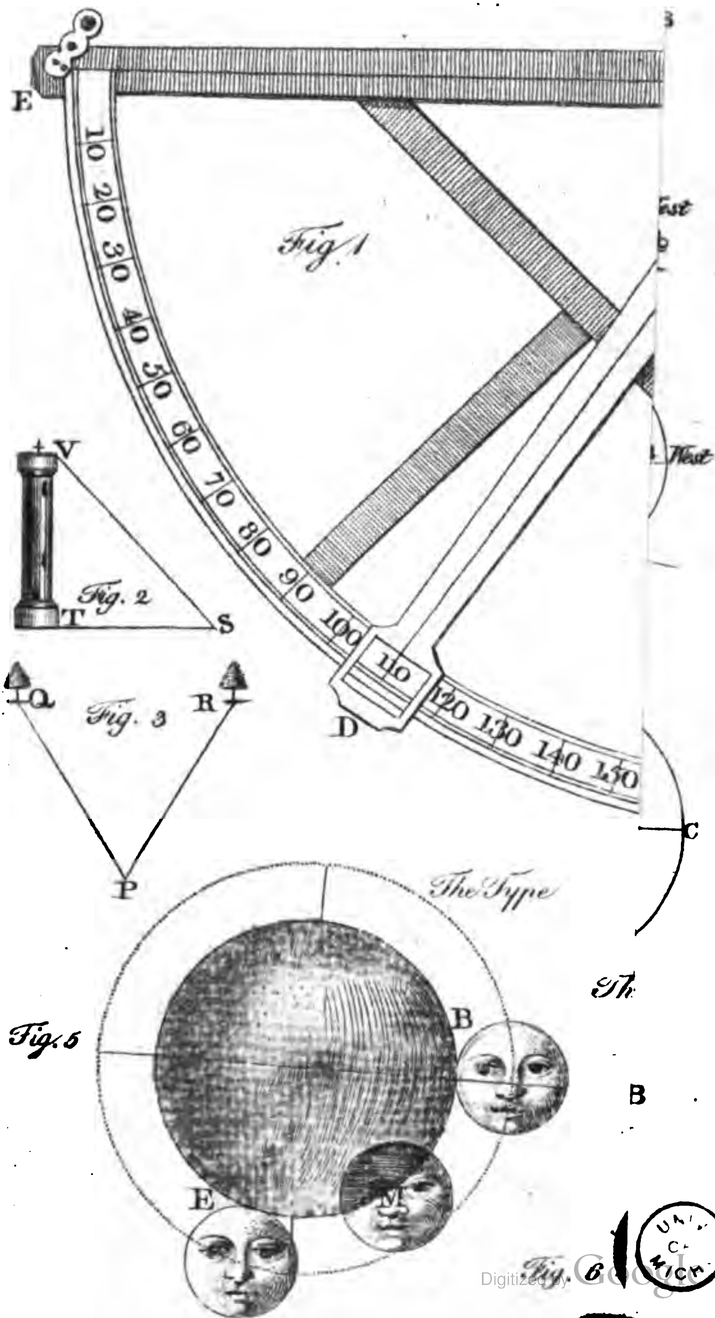


Fig. 1
E
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
D
C
West
West
Fig. 2
V
T
S
Fig. 3
Q
R
P
The Type
Fig. 5
B
E
M
Fig. 6
B

Question 24, *by Mr. Abingdon.*

There is a room whose breadth is 15, and length 20 feet respectively, and there is a slack small string fixed by its ends to the opposite corners of the ceiling, and there is a small weight fixed to the middle of the thread; and it is observed that when this weight is put in motion it swings 20 times in a minute. Required the length of the string?

Solution, *by Mr. Thomas Hewitt, of Spitalfields.*

By Eu. 47. 1. the diagonal of the room is 25 feet = 300 inches. And by the nature of pendulums $60^2 : 39.2$ inches :: $20^2 : 352.8$ inches = the length of the pendulum. Then we have (by Eu. 47. 1.) the square of half the diagonal = 22500 added to the square of the length of the pendulum = 124467.84 = to the square of half the length of the string; thus $\sqrt{124467.84 + 22500} = 383.36$, and $383.36 \times 2 = 766.72$ inches the length of the string required.

The same, *by Mr. John Bickford, of Westminster.*

Having the length and breadth of the room, the diagonal is found by Eu. 47. 1. to be 25 = AB. (*Fig.* 19, *Plate* 2) Then the length of a pendulum to vibrate 20 times in a minute, is found thus: (see Mr. Emerson's Mechanics, Prop. 28) divide 140868 by the square of the number of vibrations given, and the quotient will be the length of the pendulum; therefore, $140868 \div 400$ (=square of 20) gives 352.17 inches = DC, then in the right-angled △ BDC are given BD = 12.5 feet = 150 inches and CD as above to find BC which by Eu. 47. 1. is 382.65 inches = 31 feet 10.65 inches which doubled gives 63 feet 9.3 inches = ACB the length of the string required.

Question 25, *by Mr. R. Waugh, of Bushblades.*

Required the equilibrium power of an over-shot water wheel, at its circumference as arising from the weight of water only; having the following data. Diameter of the wheel 24 feet, number of buckets 60, revolutions in a minute 10, feeder of water for driving it 1200 hogsheads per hour; the water is thrown into the third bucket from the perpendicular, through and about the center, on the leading side of the wheel, and leaves the same five buckets before it arrives at the said perpendicular below; the momentum of the water arising from its stroke being allowed for the friction?

Solution *by Mr. John Bickford, of Westminster.*

Let ABCD (*Fig.* 20, *Plate* 2) be the wheel whose circumference is divided into 60 buckets, E is the third bucket from B, and the first that the water enters from the perpendicular BE, G is the fifth bucket from D, and is the distance the water leaves the wheel from the perpendicular ED.

First, 1200 × 14553 (cubic inches in a hogshead) = 17463600, then 60^m : 17463600 :: 1^m : 291060 the cubic inches of water issued every minute, and as the wheel goes round 10 times in a minute, having 60 buckets to be filled each time, therefore 291060 ÷ (60 × 10) = 485.1 inches of water in each bucket; by Robertson's Mensuration, the specific gravity of an inch of water is .036169 ∴ 485.1 × .036169 = 17.54558 lb. the weight of water in each bucket. Now 360° ÷ 60 = 6° the arc or distance between the centre of each bucket on the wheel: then by mechanics if the nat. cosine of the several arcs accounted from A, viz. of 6° 12° 18° &c. both ways towards B and D be multiplied severally into the weight of water in each bucket, the sum of the whole will be 311, 27042lb. the equilibrium power when applied to the circumference at A, as was required.

Question 26, *by Mr. Mayland.*

To what height above the City of London must a person be raised, on December the 18, 1792, at half past eleven at night, to see the moon's lower edge?

Solution, *by Mr. John Bickford, of Westminster.*

From Mayer's Tables corrected, the moon's place for the given time is 11^s 7° 37′; then from Mr. Emerson's Astronomy, p. 85, the depression of the moon's lower edge (when corrected for parallax, refraction, &c.) will be 13° 3′ Then as the cosine of depression 13° 3′ : Earth's semi-diameter 3982 miles :: rad. : 4087,56 miles ∴ 4087,56 − 3982 = 105,56 miles the height the person must ascend above the horizon of London to see the moon's lower limb as required.

Question 27, *by Don Quixotte.*

Given the base and least side, and the ∠ contained between the greater side and ⊥ drawn from the vertical ∠ and terminating in the base, to construct the triangle.

Solution, *by Mr. John Farey.*

Construction. Let AB (*Fig.* 1, *Plate* 3) be the given base, on A as a center, with the given least side as radius describe the circle DCB, and from B (per Simpson's Geometry, V. 7) draw the indefinite line BF intersecting the circle in C and D, and making the angle ABF equal the complement of the given angle contained between the greater side, and the perpendicular from the vertical angle on the base; or, which is the same thing, equal the least angle at the base, then joining AB, and AD, ABC or ABD is the triangle required; which is so evident from the construction that a formal demonstration is needless.

Limitation. When BF neither cuts, or touches the circle, the problem is impossible.

Question 28, *by Sancho.*

Given the base, and either of the angles at the base, also the radius of the inscribed circle, to construct the triangle.

Solution, *by Mr. John Farey.*

Construction. Let AB (*Fig.* 2) be the given base, parallel to which, at the given distance of the radius of the inscribed circle, draw the indefinite line EF, (per Simpson's Geometry, V. 9) and from A draw the indefinite line AD (V. 7) making the angle BAD equal the given angle at the base, bisect this angle by the indefinite line AG (V. 5) intersecting EF in H, on which as a center describe the given inscribed circle, and from B draw a tangent to the circle (V. 21) intersecting AD in C, and ABC is the triangle required.

Demonstration. By construction AB is the given base, CAB the given angle at the base, and IK the given inscribed circle; it only remains to prove that this circle is circumscribed by the triangle ABC. From H let fall the perpendiculars HI and HK; then by construction the side CB is a tangent to the circle, and the angles HAK and HAI are therefore equal; the Δs HAK and HAI are therefore equal and similar (I. 15.) and HK = HI = the given radius; therefore the sides AC and AB are tangents to the circle, in the point K and I (III. 6) and the triangle ABC circumscribes the given circle. Q.E.D.

Question 29, *by Sancho.*

Given the base, the perpendicular, and the least angle at the base, to construct the triangle.

Solution, *by Mr. John Farey.*

Construction. Let AB (*Fig.* 3) be the given base, parallel to which at the distance of the given perpendicular, draw (per Simpson's Geometry V. 9) the indefinite line DE, and from A draw (V. 7) AC intersecting DE in C, and making the angle BAC equal the given angle at the base, then joining BC, ABC is the triangle required, as is evident from the construction.

Question 30, *by F. R. S.*

It is required to find a figure contained within three circular lines that shall be equal in area to the quadrant of a given circle, purely geometrical.

Solution, *by Mr. John Lowry, of Houghton.*

Upon AC (*Fig.* 4) as a diameter, let the semi-circle ADC be described, and upon the radii AB, BC describe the semi-circles AdB, BbC, then is the space ADC bBd equal to a quadrant of the circle ADCR.

Demonstration. Circles are to each other as the squares of their diameters, therefore the two semi-circles upon half the diameter will be equal to half the semi-circle upon the whole diameter, and consequently the space ADCbBd, must be equal the remaining half; equal a quadrant of the circle ADCR.

Question 31, *by Mr. James Lamb, of Sproatley,*

Required a general rule for multiplying circulating decimals, without turning them into vulgar fractions?

Solution, *by Mr. Lamb, the Proposer.*

Multiplication of Repetends, or circulating Decimals, is easily performed by the following general Rule, viz.

If both the factors are pure repetends, or one a repetend and the other a finite number, multiply them together as common numbers, and point off the decimals in the product as usual.

But if one, or both the factors have any terminate parts joined with the repetends, subtract them from their own factors, and the remainders will be the new factors, which multiply together as before.

Then if only one factor contains a repetend,—divide the product by as many nines (accounting them decimals) as there are recurring figures in the repetend, continue the quotient till it either terminates, or becomes a single or compound repetend, and it will be the true product required.

But if both the factors contain repetends,—divide the last Quotient by as many nines as there are recurring figures in the other factor, and continue the quotient as before, which will be the true product required.

As the method of dividing by any number of nines is easily performed by addition, and perhaps not universally known, I shall therefore give the Rule, which is this:—Point the number to be divided into periods, consisting of as many figures each, as you have nines to divide by, set the first period under the second, add them together and set their sum under the third, add those together and put the result under the fourth, and so on as far as is necessary, if there is any thing to carry at the left-hand place of any period, set it under the right-hand place of the same period, and this number so carried must be constantly added to each succeeding period; lastly, draw a line underneath and add them together, and the sum will be the true quotient required.

Examples illustrating the whole.

Pure repetends:

Multiply .8
By .7

.9 | .56
.9 | .62

True product .691358024

Multiply .586
By .54

2344
2930

.999 | .316,440,000,
316 756

.99 | .31,67,56,75,
31 98 54
1 2

True product .319956

Multiply .36
By .6

.9 | .216

.99 | .24,00
24

True product .24

Repetends and finite Numbers.

Multiply .78549
By 35,6

471294
292745
235647

.99999 | 27,963,44400,
27963

27,963723

Multiply 49,64
By .6053

14892
24820
29784

.9999 | 30,04,7092,
3004
1

True product 30,050097

Repetends with terminate parts.

Multiply 582,347 by 8.
Subtract term. part 582

New fact. 582,765
8

,999 | 465,4,12,000
465877

4658,778

Multiply 251,43 by 8,74
7,87 — 87 Sub. ter. part

176001 — 7,87 New fact.
201144
176001

,9 | 1978,7541

Tr. prod. 2198,6156

Multiply 3,145 by 4,297.
Sub.ter. part 31 — 4

New factors 3,114 ----- 4,293
4,293

9342
28026
6228
12456

,999 | 13,3,684,020,000
133 817 837

,99 | 13,38,17 83 78
13 51 68 51
1 2

True product 13,5 16953 3

Multiply 45,13 by ,245
Sub. ter. part 451

40,62
,245

20310
16248
8124

,999 | 9,95,190,000
995 185
1 1

,9 | 9,96 18 6

True prod. 11,068735402

The above method of multiplying circular Decimals, will be found very easy with a little practice, and no ways burthensome to the memory as other rules are. Division is easily performed by this general Rule.

If the repetends in the divisor and dividend are both similar and conterminous, and have no terminate parts, divide them as finite numbers.

But if they are not similar make them to begin and end together; then if there be any terminate parts in the divisor or dividend, or both, subtract them, and the remainders will be a new divisor and dividend, which divide as finite numbers.

E X A M P L E S.

1st Divide $.\dot{6}9135802\dot{4}$ by $.\dot{8}$
Divisor $.88888888\dot{8}$ made sim.
$.\dot{8}88888888$ | $.6913580240$ | $.\dot{7}$ Q.
6222222216
Rem. same as div^d 691358024

Divide $.\dot{2}\dot{4}$ by $.\dot{3}\dot{6}$
.36 | .240 | $.\dot{6}$ Q.
216
24

Divide $.\dot{3}1995\dot{6}$ by $.\dot{5}8\dot{6}$
Divis. $.\dot{5}8658\dot{6}$ made si^r
.586586) 319956 ($.\dot{5}\dot{4}$ Q.
2932930
2666300
2346344
Same as dividend 319956

Divide $27{,}9\dot{6}372\dot{3}$ by 35,6
Divis. $35{,}6\dot{0}000\dot{0}$ made sim.
From $35{,}6\dot{0}000\dot{0}$ | $27{,}9\dot{6}372\dot{3}$
Sub. ter. p. 356 | 279
New div. 35,599644 | 27,963444 | $.\dot{7}854\dot{9}$ Q.
Rem. same as dividend 27963444

Divide $30{,}05\dot{0}09\dot{7}$ by $.\dot{6}05\dot{3}$
Divis. $.60\dot{5}36\dot{0}$ made sim.
$.60\dot{5}36\dot{0}$ | $30{,}05\dot{0}09\dot{7}$
Sub. ter. p. 60 | 3005
.6053 | 30,047092 | 49,64 Q.
Remains 0

$582{,}\dot{3}4\dot{7}$ | $4658{,}\dot{7}7\dot{8}$
582 | 4658
581,865 | 4654,120 | 8
Remains 0

$251{,}43\dot{0}$ | $2198{,}615\dot{6}$
25 143 | 219 8615
226,287 | 1978,7541 | $8{,}7\dot{4}$ Q.
Rem. 100572

Divide $11{,}\dot{0}6873540\dot{2}$ by $.\dot{2}4\dot{5}$
Term. part. sub. 11
Made sim. to dividend
.245245245 | 11,068735391 | $45{,}1\dot{3}$ Quotient.
9 80980980
1 258925591
1,226226225

326993660
245245245

81748415
735735735

81748415

Question 32, *by Mr. Thomas Crosbey, of York.*

A gun, whose angle of elevation is 45°, and impetus 30000 feet, planted at a place W, is able to carry a ball 60000 feet, to another place E, full East of the former: To what point of the compass, and at what angle of elevation ought a gun at M to be directed to, whose impetus is 44100 feet; and bearing from the first place E. S. E. 40000 feet, so that the two guns being fired at the very same instant, the balls may hit each other in the air.

Solution, *by Mr. Crosbey, the Proposer.*

As the two guns are fired at the same time, and the balls meet in the air, therefore the line of flight and the heights of the projectiles AL (*Fig.* 5) and BC will be equal; hence by the principles of gunnery, we have $\sqrt{44100} = 210 : \sqrt{30000} = 173.205$ :: 0.7071068 = sine of 45° : 0.5832116 = the sine of 35° 40′ 36″ the required elevation. Again, AL = $\frac{1}{4}$ WE the greatest amplitude = 15000 = BC; and by page 236 of Holliday's gunnery, we have this analogy. As tangent of 35° 40′ 36″ : radius :: 15000 : 20897.5 which being doubled gives 41795 = $\frac{1}{2}$ DM = BM or BD = half of the latter gun's amplitude. Now the time of flight being equal, and the paths intersecting in P, where the balls meet just over F, the horizontal point where the amplitudes cross each other, we have by a well-known property in gunnery, WF : FM :: WE : DM, or by plane trig. DM = 83590 : S. ∠ EWM = 22° 30′ :: WE = 60000 : S. ∠ DMW = 15° 56′ 36″ which added to ∠ EWM gives ∠ DFW = ∠ EFM = 38° 26′ 36″ or N. W. 6° 33′ 24″ westerly the point of the compass the gun ought to be directed to.

Question 33, *by Mr. Joseph Marshal.*

There is a cask of good sound heart of oak in form of the frustum of a cone, its external dimensions are as follow, viz. Slant height 54 inches, top diameter 24 inches, and bottom diameter 18 inches; it being filled with common water, weighed 647.64 pounds avoirdupoise. Required the thickness of the wood, admitting that a cubic inch of oak weighs .536569, and a cubic inch of water .578697 oz. avoirdupoise.

Solution, *by Mr. James Denningt.*

Put $a = 54$, $b = 12 + 9$, $c = 12 - 9$, $v = .536569$, $w = .578697$, $p = .2618$, and $x =$ the required thickness; then $\sqrt{(a^2 - c^2)} = 53.9166$ ($= d$) $=$ the perpendicular height of the cask. Now, per mensuration $3b^2 + c^2 \times pd = 18801.6673 =$ the whole solidity. Also $3 \times (b - 2x)^2 + c^2 \times p \times d - 2x =$ the solidity of the water. Hence $18801.6673\, v - (3 \times \overline{b - 2x}|^2 + c^2) \times p \times d - 2x \times v + (3 \times \overline{b - 2x}|^2 + c^2) \times d - 2x \times pw = 647.64 \times 16$ by the question. Hence by a cubic equation x is found $= 1$ inch, the thickness of the wood.

Question 34, *by Mr. Davis, of Overton.*

Given $x^4 - 4x^3 + 4x^2 = y^4 + 4y^3 + 4y^2$.
And $x^5 - 12x^3 - x^4 + 12x^2 = xy^4 + 12xy^2 - y^4 + 12y^2$.

Solution, *by Mr. William Armstrong.*

Extract the square root of the first, and divide the second by $x - 1$, and there will arise $x - 2x = y^2 + 2y$ and $x^4 - 12x^2 = y^4 + 12y^2$. Add 36 to the last equation, and extract the square root, it will be $x^2 - 6 = y^2 + 6$. By transposition $x^2 = y^2 + 12$. Add 1 to each side of the first equation; the square root of the sum is $x - 1 = y + 1$. This transposed and squared gives $x^2 = y^2 + 4y + 4 = y^2 + 12$; hence $y = 2$ and $x = 4$.

Question 35, *by Mr. Olinthus Gregory.*

Required my age in years, when it is known that the sum of the two digits which express it is 9; and if half my age be squared, that square will be equal to my age reversed.

Solution, *by Mr. William Armstrong, Pupil in Mr. Howard's School, Carlisle.*

Put $x =$ the digit in the tens place, $y =$ ditto in the units place; then $10x + y =$ his age; and $x + y = 9$; hence $\frac{100x^2 + 20xy + y^2}{4} = 10y + x$ per question, from the first equation $y = 9 - x$, this substituted in the other and reduced, gives $x^2 + \frac{2}{9}x = 1\frac{2}{9}$, whence $x = 1$, and $y = 8$. Consequently his age is 18.

Question 36, *by Mr. O. Gregory, of Yaxley.*

There is a common helix or spiral, (such as described at page 264 Fenning's Book of

Knowledge, 4th edition) whose length, when continued 10 times round is known to be 1319.46906. What is the distance of the two centers, from which the semi-circles that constitute the spiral were struck?

Solution, *by the Proposer.*

From the construction given in the book mentioned in the question, I have deduced the following rule for finding the length of the spiral, viz. Find the sum of an increasing arithmetical progression, whose first term is 1, common excess 1, and number of terms equal to the number of semi-circles struck from the two centers; this sum multiplied by the distance of the centers, and that product by 3.141593 will give the length of the spiral. Then to answer the question, I find the sum of an increasing arithmetical progression whose first term is 1, common excess 1, and number of terms 20, (because when the spiral is continued 10 times round, there will be 20 semi-circles) this sum will be 210, and then divide 1319.46906 by the product of 3.141593 into 210 (659.73453) the quotient 2 is the distance of the centers.

Queston 37, *by Mr. William Burden, Acaster Malbis, near York.*

There is a rectangular field ABCD (*Fig.* 6) through which parallel to the longest side AB or CD runs a ditch EF, over this ditch at G is a bridge at the distance of a chains from the angle A, and at b chains from the angle C, the whole field measures c square chains, and the sines of the angles AGE, and IGC are as m : n. Required the dimensions of the field?

Solution, *by the Proposer.*

Let mx and nx be the sines of the angles AGE, and IGC; then by trig. $1 : a :: mx : amx =$ AE, and $1 : b :: nx : bnx =$ CI. Also, EG $= a\sqrt{(1 - m^2x^2)}$ and CI $= b\sqrt{(1 - n^2x^2)}$. Then the breadth AD $= amx + bnx$, and the length AB $= a\sqrt{(1 - m^2x^2)} + b\sqrt{(1 - n^2x^2)}$. Then, $\overline{a\sqrt{1 - m^2x^2} + b\sqrt{1 - n^2x^2}} \times \overline{max + bnx} = c$, or, $a\sqrt{(1 - m^2x^2)} + b\sqrt{(1 - n^2x^2)} = \frac{c}{\overline{am + bn} \times x}$, from which the value of x may be determined, and thence the length and breadth of the field.

The same, *by Mr. John Ryley.*

As the ratio of the sines of the angles AEG, and FGC is given, the ratio of AE to FC, or BF to FC is also given: Therefore put BF $=$ AE $= mx$, and by the question FC $= nx$. By Eu. 47. 1. EG $= \sqrt{(a^2 - m^2x^2)}$ and FG $= \sqrt{(b^2 - n^2x^2)}$. Consequently $\overline{\sqrt{a^2 - m^2x^2} + \sqrt{b^2 - n^2x^2}} \times \overline{mx + nx} = c$; which being squared becomes $a^2 - m^2x^2 + \sqrt{(a^2 - m^2x^2)} \times \sqrt{(b^2 - n^2x^2)} + b^2 - n^2x^2 = \frac{c^2}{(m+n)^2 \times x}$, and if this be transposed and squared again, we get $4a^2b^2 - (4b^2m^2 - 4a^2n^2) \times x^2 + 4m^2n^2x^4 = \frac{c^4}{(m+n)^4 \times x^4} + \frac{(m^2+n^2) \times 2c^2}{(m+n)^2} + (m^4+n^4) \times x^4 - \frac{2c^2 \times (a^2+b^2)}{(m+n)^2 \times x^2} - 2 \times (a^2+b^2) \times (m^2+n^2) \times x^2 + (a^2+b^2)^2$. Now if $v = m + n$, $s = m^2 + n^2$ and $r = a^2 + b^2$ the equations after proper reduction become

$$\begin{aligned}4m^2n^2x^8 &- 4a^2n^2x^6 + 4a^2b^2x^4 - 2rc^2v^{-2}x^2 - c^4v^{-4} = 0.\\ -s^2 &\quad - 4m^2b^2 \quad - 2sc^2v^{-2}\\ &+ 2rs \qquad - r^2\end{aligned}$$

A final biquadratic equation with two positive roots.

Question 38, *by Mr. Thomas Crosby, of York.*

Ingenious Philomaths pray tell to me,
Height, age, and fortune of my charming she,
Which from the given equations* will appear,
I'll strive to do as much for you another year.

* $\frac{xz\sqrt{\frac{5}{13}x} + z}{x} = 1504\frac{8}{13} = m.$ $\frac{xz\sqrt{\frac{12}{25}z} + x}{z} = 780\frac{13}{60} = n.$

Where x represents her height in inches, $4-13x$ = her age, and z her fortune in pounds.

Solution, *by Mr. John Ryley.*

Put $\frac{5}{13} = a^2$, $\frac{12}{25} = b^2$; $u^2 = x$ and $u^2v^2 = z$: substitute these values in the room of their $=$s in the given equations and they will be transformed to $au^3v^2 + v^2 = m$, and $bu^3v^3 + 1 = nv^2$; hence $u^3 = \frac{m - v^2}{av^2} = \frac{nv^2 - 1}{bv^3}$, and after due reduction, we obtain $bv^3 + anv^2 - bmv = a$; or in numbers, $v^3 + \frac{46813\sqrt{5}}{24\sqrt{39}} \times v^2 - 1504\frac{8}{13} v = \frac{5\sqrt{5}}{2\sqrt{39}}$; which being solved v^2 is found $= \frac{60}{13}$; and from hence $x = u^2 = 65$ inches, her height; $u^2v^2 = z = 300$l. her fortune, and $\frac{4}{13}x = 20$ her age.

The same, *by Mr. Thomas Whiting.*

For z put xs, $a = \frac{5}{13}$, $b = \frac{12}{25}$, then the equations become $xs\sqrt{ax} + 1 = m$, and $xs\sqrt{bxs} + 1 = ns$, by transposition and involution we get $x^3s^2a = m^2 - 2m + 1$, and $x^3s^3b = n^2s^2 - 2ns + 1$. The first equation multiplied by bs, and the last by a, gives $x^3s^3ba = m^2bs - 2mbs + bs$, and $x^3s^3ba = n^2s^2a - 2nsa + a$. Consequently $m^2bs - 2mbs + bs = an^2s^2 - 2ans + a$. this put into numbers, and reduced gives $s = 4\frac{40}{65}$, therefore $x = \sqrt[3]{\frac{m^2 - 2m + 1}{s^2a}} = 65$, and $xs = z = 300$.

The same, *by Mr. Thomas Crosbey.*

Put $a = \frac{5}{13}$, $b = \frac{12}{25}$, $x = y^2$, and $y^2 \div v^2 = z$, then will the two given equations be reduced to $\frac{ay^5}{v^2} + \frac{y}{v^2} = my^2$, and $\frac{by^5}{v^3} + y^2 = \frac{ny^2}{v^2}$. From the first of these $y^3 = \frac{mv^2 - 1}{a}$, and from the second $y^3 = \frac{nv - v^3}{b}$, whence $\frac{mv^2 - 1}{a} = \frac{nv - v^3}{b}$, or $v^3 + \frac{bm}{a}v^2 - nv = \frac{b}{a}$, this brought into numbers and reduced $v = \frac{13}{60}$. Consequently $x = 65$ inches her height, $z = 300$l. her fortune, and $65 \times \frac{4}{13} = 20$ her age.

The same, *by Mr. James Denningt.*

It is manifest from the first equation, that the value of x must be such, that its submultiple of 13 being multiplied by 5, shall come out a square: Now 65 is the least number with this property, and the only one that can be of use, for 208 the next greatest is much too large for the present case. Therefore the equation becomes $65z\sqrt{25} + z = 65m$, or $z = \frac{65}{326}m = 300$; these values of x and z substituted in the last equation make it vanish: Hence the lady's height is 5 feet 5 inches, her age 20, and fortune 300l.

Question 39, *by Mr. John Harris, of London.*

At London on May 24, 1792, at 10 o'clock A. M. a roller lying on an horizontal plane, and placed at right angles to the sun, (or azimuth the sun was then on) cast a shadow to the distance of 5 feet 3 inches. Required the diameter of the roller?

Solution, *by Mr. William Davis, Overton.*

First there is given the latitude of the place, the sun's declination, and hour, to find the altitude of the sun $= 51° 32'$, which is equal to the angle TAB. Then in the $\triangle$ ABC *(Fig. 7)* there is given AB $= 63$ inches, and the angle BAC $= 25° 46' =$ half the angle TAB to find BC $= 30$ inches nearly; hence the diameter is $= 60$ inches $=$ 5 feet the answer.

Question 40, *by Mr. William Marsden.*

There is a piece of ground in the form of a plane triangle, the sides in arithmetic progression, whose common difference is 18 chains, and the vertical angle $= 134° 39'$: Required the sides and area?

Solution, *by Mr. John Ryley, of Leeds.*

Put s *(Fig. 3, Plate 1)* $=$ the sine of the $\angle$ ACB; $t =$ the tangent of half the said angle; a = 18; and BC = x; then per question AC = $x + a$, and AB = $x + 2a$. Now, radius (1) : t :: $\frac{3}{4} \times (x + a) \times (x - a)$: $\frac{3}{4} t \times (x^2 - a^2$ = the area of the Δ. And radius (1) : s :: $\frac{1}{2} x \times (x + a)$: $\frac{1}{2} sx \times (x + a)$ = the area of the Δ also. Consequently $\frac{3}{4} t \times (x^2 - a^2) = \frac{1}{2} sx \times (x + a)$; by division $3t \times (x - a) = 2sx$; hence $x = \frac{3ta}{3t - 2s}$ = 22.4767 chains. Therefore AC = 40.4767, and AB = 58.4767 and the area (found by either of the above expressions) = 32 A. 2R. 4.56P.

Question 41, *by Amanuensis.*

A gentleman, being on a morning walk, heard the sound of a clock from a Tower adjacent, bearing due East; and then the sound of another clock striking on a bell of equal size, from a Tower bearing due north; the intensity of the last sound was double that of the former; the distance between the two Towers 1 mile; and the wind blowing from the North-East. Required his distance from each Tower?

Solution, *by Mr. John Ryley, of Leeds.*

Let A (*Fig.* 8) represent the northern Tower, C the eastern, and B the place where the gentleman stood, when he heard the sound of the clocks. Put AC = 1 mile = a, BC = x, and by Eu. 47. 1. AB = $\sqrt{(a^2 - x^2)}$. Now if the intensity of sound be inversely as the square of the distance; and the ratio of the two given intensities as $m : n$; we shall have $a^2 - x^2 : m :: x^2 : n$, and from hence $mx^2 = na^2 - nx^2$. Wherefore $x = \sqrt{\frac{na^2}{m+n}}$ = (in the present case) $\sqrt{\frac{2}{3}}$ = .8164965809, and AB = $\sqrt{\frac{1}{3}}$ = .5773502689 parts of a mile as required.

Question 42, *by Mr. George Reynold's, a Member of the Prince of Wales's Academy, Norland-House.*

Suppose at the late Siege of Gibraltar, that a cannon being fired at the enemy from shore, the ball should just touch the top of the mast of a ship, 90 feet high above the level of the piece, which was at the distance of 200 yards, the ball continuing its course, struck the Admiral's ship, the instant he heard the report. Required the distance between the two ships?

Solution, *by Mr. John Ryley, of Leeds.*

Put AF (*Fig.* 9) = 200 yards = 600 feet = a; FE = 90 = b; $16\frac{1}{12}$ = c, 1142 = p, and AB = x : then by the property of the parabola BF : FA :: . E . ED = $\frac{ab}{x - a}$; therefore FD = $\frac{bx}{x - a}$, and by similar Δs AF : FD :: AB : BC = $\frac{bx^2}{a \times (x - a)^2}$.

But by the laws of projectiles, the time of descent through CB is = to the time of flight; Wherefore $\frac{bx^2}{ca \times (x-a)} = \frac{x}{p}$; which equation squared and reduced gives $x = \frac{ca^2 + bp^2}{ca}$ $= 12763.2$ feet $= 4254.4$ yards $= 2$ miles 734.4 yards; therefore the distance between the ships is 2 miles 534.4 yards as required.

Question 43, *by Mr. James Swift, of Cheetham.*

A heavy body projected from the ground in the direction of 32° above the horizon, fell at the feet of a person a certain distance off, the instant he heard the report of the piece: I desire to know the time of the ball's flight, and the distance of the person from the place of projection, allowing the velocity of sound to be 1142 feet in one second of time?

Solution, *by Mr. John Ryley, of Leeds.*

Put $t =$ the tangent of the angle of elevation; $a = 1142$ feet; $s = 16\frac{1}{12}$, and $x =$ the required amplitude. Then per Trig. $tx =$ BC *(Fig. 9)* and by the laws of falling bodies $\sqrt{(tx \div s)} =$ the time of flight; also the time wherein the sound moves from A to B, will be defined by $x \div a$. Therefore by the question $\sqrt{(tx \div s)} = x \div a$; whence $x = ta^2 \div s = 50670$ feet $= 16890$ yards $= 9$. miles 105 yards. Consequently $a : 1'' :: x : 44'' \ 22'''$ the time of flight?

Question 43, *by Mr. Thomas Keith.*

The area of an equilateral triangle is $25\sqrt{3}$, now, if lines be drawn from each angle to a certain point within the triangle, the areas cut off by them will be $\frac{25}{6}\sqrt{3}$, $\frac{25}{3}\sqrt{3}$, and $\frac{25}{2}\sqrt{3}$, respectively. Required the lengths of these lines, in as simple terms as possible, independent of algebra and decimals?

Solution, *by Mr. John Ryley.*

If the side of an equilateral triangle be 1, its perpendicular will be expressed by $\sqrt{\frac{3}{4}}$, and its area by $\frac{1}{4}\sqrt{3}$. Therefore, as similar triangles are to one another as the squares of their homologous sides $\frac{1}{4}\sqrt{3} : 1^2 :: 25\sqrt{3} : 100$ ∴ AB $= 10$. Now as the triangle ABC *(Fig 10)* is to be divided into three other triangles, whose areas are $\frac{25}{6}\sqrt{3}$, $\frac{25}{3}\sqrt{3}$. and $\frac{25}{2}\sqrt{3}$. if each of these areas be divided by 5 (half the side of the original triangle) the perpendiculars of these triangles will be expressed by $\frac{5}{6}\sqrt{3}$, $\frac{5}{3}\sqrt{3}$, and $\frac{5}{2}\sqrt{3}$: hence this

Construction. Perpendicular to AB and BC draw BG ($= \frac{5}{6}\sqrt{3}$) and BI ($= \frac{5}{3}\sqrt{3}$); also draw GO and IO parallel to AB and BC, and the point of intersection O, will be that to which the required lines are to be drawn.

Calculation. Draw the rest of the lines as per figure; then because of the parallel lines DK $=$ HG, KB $=$ OH, HI $=$ HE and KO $=$ BH; and by similar triangles CL : CB :: BG : BH :: OD : OK $=$ HC $= 5$: Therefore GH $=$ DK $= \sqrt{(BH^2 - BG^2)} = \frac{5}{2}$. Also CL : CB :: BI : BK :: OE : OH $= \frac{10}{3}$ and IK $=$ HE $= \sqrt{(BK^2 - BI^2)} = \frac{5}{3}$. Hence BK $+$ KD $=$ BD $= \frac{10}{3} + \frac{5}{2} = \frac{35}{6}$ ∴ DA $= \frac{25}{6}$. Also, BH $+$ HE $=$ BE $= 5 + \frac{5}{3} = \frac{20}{3}$ ∴ BC $= \frac{10}{3}$. Now as the lines AD, DO; BG, GO; CE, EO are given, the required lines will readily be found by Eu. 47. 1. as follows, viz. CO $= \sqrt{(EO^2 + EC^2)} = \sqrt{(\frac{25}{9} + \frac{100}{9})} = \frac{5}{3}\sqrt{7}$; AO $= \sqrt{(AD^2 + DO^2)} = \sqrt{(\frac{25}{4} + \frac{625}{36})} = \frac{5}{3}\sqrt{13}$, and BO $= \sqrt{(BG^2 + GO^2)} = \sqrt{(\frac{25}{4} + \frac{1225}{36})} = \frac{5}{3}\sqrt{19}$.

Question 44, *by Mr. Stephen Hartley.*

In a parabola, the parameter whereof is 20, if the fluxion of the area be divided by the abscissa, the quotient will be equal to the fluxion of the length of the curve. Required the abscissa and ordinate?

Solution, *by Mr. John Ryley, of Leeds.*

Put the parameter $= a$, abscissa $= x$, and the ordinate $= y$; then by the property of the parabola $ax = y^2$; and the fluxion of the area and length of the curve expressed in terms of y, will be $=$ to $\frac{2y^2\dot{y}}{a}$ and $\frac{\dot{y}\sqrt{(a^2 + 4y^2)}}{a}$ respectively. Therefore by the question $\frac{2y^2\dot{y}}{ax} = \frac{\dot{y}\sqrt{(a^2 + 4y^2)}}{a}$, which being reduced becomes $4y^4 = a^2x^2 + 4x^2y^2$. But by the equation of the curve $y^2 = ax$; therefore, $4y^4 = 4a^2x^2 = a^2x^2 + 4x^2y^2$; hence $x = \frac{3}{4}a = 15$ and $y = \sqrt{300} = 10\sqrt{3}$.

Question 45, *by Mr. John Bickford.*

If a cubical vessel, nine inches deep, be kept constantly full of water, and an apperture be made in the bottom that issues out three gallons per second, how much more will be issued out if a tube be fixed in the orifice, at the bottom of the vessel of the same dimensions and seven inches long?

Solution, *by Mr. John Ryley.*

If the constant depth of water in the vessel be 9 inches in the first case, and 16 in the latter; and heavy bodies descend by the force of gravity 193 inches in the first second of time: by the laws of falling bodies $\sqrt{193} : \sqrt{9} :: 2 \times 193 : 6\sqrt{193} =$ the velocity per second of the issuing water when the depth is 9 inches. Also, $\sqrt{193} : \sqrt{16} :: 2 \times 193 : 8\sqrt{193} =$ the velocity per second when the depth is 16 inches. Now as the quantities issued out of equal orifices in equal times are in the direct ratio of the velocities respectively, $6\sqrt{193} : 3$ gallons :: $8\sqrt{193} : 4$ gallons. Consequently $4 - 3 = 1$ gallon per second, the required difference. Otherwise. As the velocities and likewise the quantities of effluent water, at different depths; are as the square roots of the depths, $\sqrt{9} : \sqrt{16} :: 3 : 4$ as before.

Question 46, *by Philomathematicus.*

Supposing the law of attraction to be in the inverse ratio of the square of the distance, to find the nature of the solid of the greatest attraction.

Solution, *by the Proposer.*

It is plain in the first place, that the solid which attracts the given point A (*Fig.* 11) with the greatest possible force, of all solids under the same quantity of matter, must be a solid of rotation, since there can be no reason, why the matter should be distributed more on one side than on others. To find the nature of the curve which forms this solid by a rotation about its axis, let there be taken four equidistant and infinitely near ordinates, and let the lines MN, NO, O*m* be three indefinitely small sides of the curve; whose point M and *m* are supposed to be fixed, and the points N and O variable. Call the lines AP, AQ; AR, A*p*; $x, x'; x'', x'''$; the lines MP, NQ, &c. z, z', &c. First, by the nature of the problem, we have $yy\dot{x} + y'y'\dot{x} + y''y''\dot{x} =$ a constant magnitude; whence, after having taken the differences, and observing that only y' and y'' are variable, we get $y'\dot{y}' = y''\dot{y}''$. Now we shall find, by known methods, that the attraction of the solid formed by the rotation of MN*mp*P about P*p* is $\overline{1-\frac{x}{z}} \times \dot{x} + \overline{1-\frac{x'}{z'}} \times \dot{x} + \overline{1-\frac{x''}{z''}} \times \dot{x}$, which should be a maximum: therefore, taking the difference, and observing that only z' and z'' are variable, and making it $= o$, we get $\frac{x'\dot{z}}{z' \times (z'-\dot{z}')} = \frac{x''\dot{z}''}{z'' \times (z''+z'')}$; but the right angled triangles ANQ, AOR, as the points Q and R are fixed, give $y'\dot{y}' = z'\dot{z}'$ and $y''\dot{y}'' = z''\dot{z}''$: Therefore $\frac{x'y'\dot{y}'}{z'z' \times (z'-\dot{z}')} = \frac{x''y''\dot{y}''}{z''z'' \times (z''+\dot{z}'')}$. And since $y'\dot{y}'$ is equal $y''\dot{y}''$ we shall have $\frac{x'}{z'z' \times (z'-\dot{z}')} = \frac{x''}{z''z'' \times (z''+\dot{z}'')} =$ a constant magnitude $\frac{1}{gg}$; therefore we shall have $z^3 = ggx$ for the equation of the curve required. The problem might be solved another far more simple way, by taking the attraction of any one point of the surface of the solid; which attraction is $x \div z^3$, and making it $=$ to a constant magnitude $1 \div gg$, we shall have $z^3 = ggx$, for the equation of the required curve, as above: For it is manifest, that if the attraction were less on any place of the surface than another, that point might be placed out of the solid, so as that it would attract more, and the solid would be no longer that of the greatest attraction, contrary to the hypothesis.

Question 47, *by Mr. Allen.*

To find the area of the curve, which, revolving about its axis, shall generate the solid of greatest attraction, supposing its force to act on a corpuscle placed on its surface; also, the content of the said solid, and the ratio of its attraction to that of a sphere of the same quantity of homogenious matter, taking the axis of the solid, equal the invariable quantity g. (see question 46).

Solution, *by the Proposer.*

Let AMBC (*Fig.* 12) represent the required solid. Put AB $= g$, AP $= x$, AM $= z$, and let $c = 3.1416$. Then for the area of the curve AMB. From the given equation $z^3 = ggx$, we have $x = z^3 \div g^2$, and therefore $(1 \div g^2) \times \surd(g^4 - z^4) \times z =$ PM. Moreover $\dot{x} = \frac{3z^2\dot{z}}{g^2}$ and consequently $\frac{3}{g^4}\surd(g^4 - z^4) \times z^3\dot{z}$ is the fluxion of the area, whose corrected fluent is $\frac{1}{2g^4} \times \overline{g^6 - \overline{g^4 - z^4}|^{\frac{3}{2}}}$, which when $z = g$, becomes $\frac{1}{2}g^2$ the area of the curve AMB required. *For the content of the required solid.* From what is given above, we have PM$^2 = 1 \div g^4 \times (g^4 - z^4) \times z^2$; therefore the fluxion of the solid will be $(3c \div g^6) \times g^4z^4\dot{z} - z^8\dot{z}$, whose fluent is $\frac{3c}{g^6} \times \left(\frac{g^4z^5}{5} - \frac{z^9}{9}\right)$, which when $z = g$ becomes $\frac{4g^3c}{15}$, for the content of the solid AMBC. *To find the force of attraction.* The force of a circle MPC will be, as $1 - x \div z$ (Simpson's Fluxions, Art. 375; or Emerson's Fluxions, 2d Ed. P. 367) or as $1 - x^{\frac{2}{3}} \div g^{\frac{2}{3}}$ by the equation of the curve, and therefore $\dot{x} - \frac{x^{\frac{2}{3}}\dot{x}}{g^{\frac{2}{3}}}$ is the fluxion of the force of the solid, whose fluent is $x - \frac{3x^{\frac{5}{3}}}{5g^{\frac{2}{3}}}$ which when $x = g$, becomes $2g \div 5$ for the attraction of the whole solid.

Let $a =$ the diameter of the sphere, ANDE $=$ to the solid AMBC, then will the fluxion of the force of the sphere be as $x - \frac{x\dot{x}}{\surd ax}$, and the fluent $x - \frac{2x^{\frac{3}{2}}}{3\surd a}$, and the force of the whole sphere, as $\frac{1}{3}a$. Moreover, the content of the sphere is $ca^3 \div 6$. Therefore $4g^3c \div 15 = ca^3 \div 6$, where $a = g \times \sqrt[3]{1.6}$. Whence the force of the solid to that of the sphere, is as $2g \div 5$ to $a \div 3$, or as $2g \div 5$ to $(g \div 3) \times \sqrt[3]{1.6}$, that is, as 4000 to 3899 nearly.

Question 48, *by Mr. Lowry, of Lambeth.*

Suppose a pedestal 100 yards high, erected on an horizontal plane, and at the top a statue 10 feet high is placed; it is required to find at what distance a person must stand from the base of the pedestal, to see the statue a maximum by geometry, only, the eye being elevated 5 feet above the plain.

Solution, *by Philomathematicus.*

From 100 yards = 300 feet take 5 feet the height of the observer's eye, and there remains DC (*Fig.* 13) = 295, and DB + 10 = AC = 305, the elevation of the highest part of the statue; then (per Eu. 6. 13.) a mean proportional between AC and DC set from C perpendicular to DC, gives BC required; thus, $\sqrt{(AC \times DC)} = BC$ = 299.958311 feet, and the subtending angle, ABD a maximum. Q. E. D.

Mr. Thomas Milner, of Catterick,

Sent calculations at full length of a solar and lunar eclipse, for the year 1794, which we are sorry we cannot with propriety insert; there being two of his in No. 2, of this Work already. The conclusions of the aforesaid eclipses are as follow:

Beginning of the solar eclipse is Jan. 31^{d} 10^{h} 59½^{m} morn.

Middle	11^{h}	46^{m}	54^{s}
End	0^{h}	34^{m}	18^{s}
Duration	1^{h}	34^{m}	48^{s}
Digits	2°	28′	23″

Beginning of the lunar eclipse.—Apparent time at Greenwich, is

February 14th, at night	8^{h}	8^{m}	26^{s}
Beginning of total darkness	9	14	4
Middle of the eclipse	10	5	5
End of total darkness	10	56	6
End of the eclipse	12	1	44
Duration of total darkness	1	42	6
Duration of the eclipse	3	53	18
Digits	20°	49′	17″

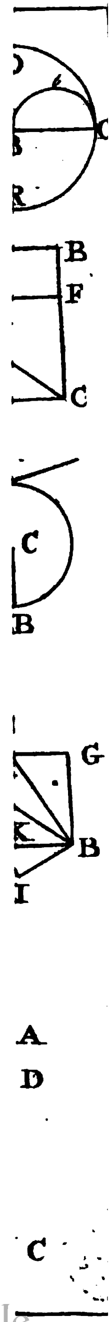
☞ We should be obliged to Mr. Milner to contract his solution to his question, by making references to Mayer' Tables for the requisites.

ARTICLE VIII.

Queries with their Solutions.

Query 1, *by Philotechnus.*

SHAWLS for ladies having of late years been brought from Asia, said to be made of Camels hair, and sold at great prices; we call upon your ingenious philosophic correspondents to say what particular quality they have to make them so valuable, and to point out a criterion to know those that are really made of Camel's hair from any others.


R
B
F
C
C
B
G
K
B
I
A
D
C

Solution, *by Mr. Charles Clarke, of Millbank-Row, Weſtminſter.*

The Camel being a native of Aſia, and is a patient, harmleſs, inoffenſive, and the moſt temperate of all animals; nature has pointed him out as a beaſt of burden, particularly for the ſandy deſarts of Arabia; he is accordingly made uſe of, to travel over thoſe vaſt and extenſive tracts, day and night, where he can meet with little or no ſhelter, to ſecure him from the great dews that fall, after the retreat of the ſun below the Horizon; therefore kind providence protects him, by cauſing the hair of this Animal to repulſe the water, the hair in conſequence is much ſought after, (and difficult to procure) for the purpoſe of making Shawls for the ladies of the Eaſt to wear after ſun ſet to protect them from the dews that fall having no rain in thoſe extenſive hot parts of the earth, but at particular periods, this particular quality makes them valuable, and the criterion to know them, is by their repelling Water, which no other ſort of Shawls will do.

Query 2 *by Philotechnus.*

Why is the eaſt wind hurtful to animal and vegetable life?

Solution *by Mr. Charles Clarke, of Millbank-Row, Weſtminſter.*

When we conſider that our earth being one of the planets, and making its way in its orbit round the ſun, as they do, by the power of the alwiſe Director of the univerſe; at the ſame time revolving round its own axis with the atmoſphere, in a regular and uniform order, from weſt to eaſt: Now, when the wind blows eaſt, being in a contrary direction to the motion of the atmoſphere, which moves with the earth in our parallel of latitude at the rate of more than 560 miles in one hour, therefore the atmoſphere is in ſome meaſure retarded, and kept back in its motion with the earth, ſo that a continual and quick change of cold, and more denſe wintery air, ſuddenly ruſhes upon us, ſo as I preſume to be prejudicial to animal and vegetable life, more particularly at the vernal equinox, becauſe we find that no ſuch effect ariſes by wind blowing from any other quarter.

The ſame *by the Rev. S. Williamſon, of Sheffield.*

If we attend to the various properties of air, and to the great improvements made by experiments on that fluid, we ſhall find that pure air is no where to be found. That which ſurrounds us is the moſt heterogenious body in nature. It is no other than an univerſal chaos, a compoſition of all kinds of bodies. All parts of the animal and mineral kingdoms muſt likewiſe be in the air. For the copious effluvia they emit by perſpiration whereby an animal in the courſe of its duration impregnates the air with many times the quantity of its own body. Any dead animal, when expoſed to the air, is in a certain time carried wholly off. And we know that all vegetables by putrifaction become volatile and evaporate into air.—Theſe conſiderations will lead us to an eaſy ſolution of the query before us. Eaſterly winds coming over a large continent before it comes to us cannot fail of being impregnated with many noxious qualities which are injurious to animal and vegetable life. On the contrary, a weſterly wind coming over an extenſive ocean is rendered more ſalubrious from the ſalt water, and is alſo leſs intermixed with putrified ſubſtance. In the united ſtates in America, an eaſterly wind is preferable to a weſterly one, for the health of the inhabitants and for the purpoſe of vegetation. The like obſervations have been made on other countries, which the climate uſually allotted to the anſwer of a query will not permit me more fully to enlarge upon.

N. B. We ſhould be much obliged to this ingenious gentleman to communicate his productions earlier, as we did not receive his letter till the work was gone to the preſs.

Query 3, *by Amanuenſis.*

Why do rocks and large ſtones ſometimes ſplit in a hard froſt?

Solution, *by the Propoſer.*

Rocks ſplit in a hard froſt from an obvious cauſe. All water congealed into ice ſwells, and takes up more room than when in thaw. Thus we ſee ice when broken, floats at the top of the water; a plain proof that it takes up more room than an equal quantity of water, otherwiſe it would ſink. Now let us ſuppoſe a ſmall quantity of water in the body of a rock or ſtone, and that this water becomes frozen; the conſequence will be, that ſwelling beyond its former bulk, it muſt either find or force room for itſelf, and therefore burſt the hardeſt flints that oppoſe its paſſage. I have known an iron cannon burſt in this manner.

Query 4, *by Dr. Sangrado.*

What is the reaſon that people in conſumptive habits raiſe their ſhoulders, and are never ſeen to have that elegant fall from the neck, which is obſervable in health.

Solution, *by the Propoſer.*

In people who are conſumptive, there is a great exertion of the breaſt required to dilated and expand itſelf with proper openneſs. Now the more the breaſt requires to be dilate, ſo much the more ſtrength is required from the muſcles that raiſe the ribs of the thorax, or breaſt. But when the ſhoulders fall low, the muſcles that raiſe the ribs do not pull ſo directly upward, but more obliquely, than when the ſhoulders are raiſed up towards the ears. Thus people in conſumptive habits are obliged to raiſe the ſhoulders in order to raiſe the ribs of the breaſt with greater eaſe, and thus to aſſiſt their difficult reſpiration.

Query 5, *by Philomathematicus.*

Why does the ſun extinguiſh a culinary fire, and yet not put out the flame of a farthing candle:

Solution, *by the Propoſer.*

The properties of fire are not yet throughly known: thus, for inſtance, in many of our mines in England; in ſome the miners are obliged to work by the light of a ſteel wheel ſtriking againſt a number of flints; for the flame of a candle would at once ſet the whole mine on fire. In other mines they are obliged to work by the light of a candle, and a ſingle ſpark from a flint and ſteel would ſet the mine into a blaze. Now with regard to the queſtion in hand, the blaze of a culinary fire is of ſuch a nature, that it requires a ſtrong body of air for its ſupport; but the warm ſun beaming in through a window rarifies the air, and takes away that fuel (if I may ſo call it) which feeds the fire. On the other hand, the flame of a candle is ſuch, that it requires the ſmalleſt and eveneſt quantity of air to keep it alive; therefore the rarefaction cauſed by the beams of the ſun can have no power upon it.

Query 6, *by Mr. John Franks, of York.*

What is the cause of those fine filaments, like cobwebs, that we see floating in the air, some of which have been found as high as York steeple?

Solution, *by the Proposer.*

It has long been a doubt with naturalists, whether these fine filaments were really the produce of spiders or not; however, at present, we may take it for granted, that they are; nor need we be surprised at their number, so as to cover the whole face of a meadow, when we consider that one single spider generally produces a thousand young ones, and these spin their web as soon as brought forth. The only difficulty that remains is, how they are found floating as high as York steeple. But this will vanish, if we consider that from their extreme fineness they become specifically lighter than air, and consequently float in it. The thread even of a silk-worm is near ten times thicker than one of these filaments in question; and yet this thread will be found to float in air, if thrown loosely forward, and will be seen to mount instead of sinking. In short, the thread of the spider when fixed firmly to blades of grass, stubble, &c. does not rise; but when by the wind or any other accident, it is let loose, it then rises; and if it be very long, and consequently have a large comparative surface, it will sometimes raise the young spider with it, which will therefore be supported by its own web. Thus all the difficulties, which have hitherto perplexed the learned upon this subject, are vanished.

Query 7, *by the Rev. Theophilus Grove, of Bristol.*

Travellers, who have visited the Empire of Peru, inform us, that it never rains at Lima; the reason of so remarkable a particular is required?

Solution, *by the Proposer.*

Various hypothesis have been framed for solving this famous question. But it is not my intention to repeat here what others have advanced, my design being to deliver my own thoughts on so intricate a subject. In order to which it may not be amiss to observe, that though no formal rain ever falls at Lima, yet there are wetting fogs, called there garuas, which continue the greatest part of the winter, but are never known in summer; and that the winds are always limited between the S. and S. E. no other wind being known at Lima. Experience abundantly informs us, that the wind is more violent in some regions of the atmosphere than in others: thus on the tops of high mountains a strong wind is felt, when very little can in the vallies below. Nor is this occasioned by the inequalities of the earth's surface, because the same thing is observable at sea; and consequently the earth's surface is not the place where it exerts its greatest force. It is also evident, that the vapours which exhale from the earth and sea, are not formed into drops of rain till they arrive at the region of the atmosphere, where their gravity becomes equal to that of the fluid which supports them.

These preliminary principles being premised, I think I may presume to assert, that the wind exerts its greatest force in a region of the atmosphere, at some distance from the earth's surface, but not in general higher than that where the rain is formed, or where the aqueous particles unite so as to form drops of a sensible gravity. And hence it follows, that in these countries, where the sun's rays, during the summer, are nearly perpendicular to the earth's surface, have the power of raising the vapours to a greater height than in winter. These vapours, on their approaching that part of the atmosphere where the wind exerts its greatest force, are hurried away before they can ascend to the height requisite for forming drops; and consequently no rain can be formed, for as the vapours issue from the earth, they are wafted along the lower regions of the air; and the wind blowing always from the south prevents their uniting; so that they are carried along in the lower parts of the atmosphere, till they are stopped by the mountains of the Andes, and there precipitate in astonishing torrents of rain. But during the winter, the rays of the sun acting in a more oblique direction, the vapours become less rarified, and the atmosphere considerably more condensed; and hence those wetting fogs, called garuas, which are almost continual at Lima during the winter, are formed. From what has been said it will follow, that in any country or climate, where the same wind always prevails, there can be no formal rain; for in order to form it, either the wind must entirely cease, or an opposite wind must arise, which, by checking the course of the vapours, brings them into contact with those already exhaled from the earth, and causes them to condense in proportion as they ascend by the action of the sun, till being rendered heavier than the air by which they were supported, they precipitate in drops of rain. Thus have I endeavoured to account for this wonderful phænomenon of nature; how well I have succeeded, is not for me to determine. Others may, doubtless, form different hypothesis which may answer the same intention. I have taken care to found what I have advanced on Philosophical principles; but as none can pretend to fathom the secrets of nature, so none can positively determine, in all cases, the principles on which she acts. The ascent of vapours, for instance, has long exercised the genius of philosophers, and produced innumerable hypothesis; though perhaps there is not one yet advanced but what is liable to very weighty objections; and consequently this common phænomenon of nature has hitherto eluded the utmost stretch of human sagacity. Many other instances might be brought to prove the small extent of the human understanding, with regard to the operations of nature; but what I have already observed will be sufficient for my purpose, and serve as an excuse for my advancing the above hypothesis, if it should not gain the approbation of philosophers.

Query 8, *by the Rev. Theophilus Grove.*

Which is the most satisfactory to an ingenuous mind, the relieving the wants of the needy, or the receiving a benefit from others, when it labours under the pressure of misfortunes?

Solution, *by the Proposer.*

When the mind is oppressed with misfortunes, it undoubtedly stands in need of support; and every one that administers the balm of comfort, may justly claim the title of friend; and though necessities may force us to receive a benefit, yet I cannot think, that an ingenuous mind receives so much satisfaction from the receipt of it, as it does from the bestowing it: for however great our necessities may be, the misery of our fellow-creatures excite in the mind generosity and compassion; and the secret delight the mind enjoys from relieving the wants of the needy, are sufficient indications of its satisfaction.

Let any one oppressed with misfortunes be asked, whether he feels most satisfaction in the receiving, or in the bestowing of a benefit? and, I will venture to say, he will declare on the side of the latter; for an ingenuous person cannot rest under the sense of an obligation. Every time he sees his benefactor, his thoughts rise in confusion at the remembrance of his bounty, and obliges him to be continually repeating his sense of the obligations received, which are heightened by that gratitude which overflows his mind. Whereas, it is a pleasure, even under the most oppressive misfortunes, to think that we have been instrumental to the happiness of others; and though their own misfortunes may some time or other oblige them to seek relief, yet they never repine at what they have bestowed; the reflection of having once dealt our bread to the hungry, yield us continual supplies of substantial pleasures.

Query 9, *by the Rev. Theophilus Grove.*

Which affords the mind the greatest degree of pleasure, the reception, or communication of knowledge?

Solution, *by the Proposer.*

The mind of man is endowed by the great author of its existence, with faculties capable of receiving the sublimest knowledge, in which every acquisition, or new improvement, affords it a very high and exquisite source of pleasure. And as the love and desire of knowledge are so strongly implanted in us by nature, it is no wonder that the attainment of it affords the mind so much delight and satisfaction. We think no labour nor trouble too great, provided we can arrive at the knowledge we desire: for when the mind is fully bent on the pursuit of any branch of science, whatever obstacles and difficulties may oppose it, instead of stopping its progress, they stimulate its ardour, and enhance the pleasure resulting from success. With what surprising and intense study, does the mathematician submit to the rugged and abstract laws of calculation? What dangers, what hardships and inconveniencies, does the curious traveller expose himself to, in order to gratify his boundless thirst of knowledge? The reception of it must therefore afford the mind a great degree of pleasure: for,

"'Tis a godlike attribute to know."

But why is knowledge so eagerly desired? Why do we spend so much time, pains and application in the prosecution of it? And why do we compass land and sea to discover it? Is it only for our own satisfaction, to bury it in our own breasts, and secretly enjoy the pleasure of it? No surely, man is a social being, and as such, must take delight in contributing whatever lies in his power to promote the benefit of human society. As we have naturally an ardent desire of knowledge, so we are naturally inclinable to communicate it to others, whatever pains or trouble the acquisition may have cost us. In fact, we find very few who are fond of having their knowledge concealed in a corner. Vanity, a desire of glory, and the hopes of gain, are all very powerful motives: and the pleasure resulting from the gratification of them, affects us in a very sensible manner. But not to insist upon these, the mind is endowed with more generous principles, which incline us to impart the useful knowledge we have acquired, merely for the pleasure we take in promoting the benefit and advantage of mankind. It certainly gives the highest pleasure imaginable to a truly benevolent spirit, to relieve the necessities, and advance the happiness of our fellow-creatures: and this benevolence is as much displayed in removing the wants and imperfections of the mind, as the misfortunes incident to the body. And because it is nobler to give, than to receive; to confer, than to accept a benefit, we may reasonably conclude, that the communication of knowledge affords the mind a far greater degree of pleasure than the reception of it.

Query 10, *by Dr. Sangrado.*

Children that have the rickets are always in-knee'd, or, as the usual expression is, knock-knee'd.—Why?

Solution, *by the Proposer.*

Because as the joints of the knees, from the nature of the disorder, begin to grow weak, the child is obliged to stride, in order to support his body, as we see wrestlers, who make the base of the column of their body as broad as they can, to stand more firmly. The child therefore standing with the legs spread, the weight will naturally fall upon the middle of the lower extremity, and thus bend it as we see at the joint; so that the knees in some very weak persons touch each other, while the feet are at a great distance.

Query 11, *by Dr. Sangrado.*

Men of weak constitutions generally after a full meal appear red in the face.—Why?

Solution, *by the Proposer.*

Men of weak constitutions are not so well able to digest their food, as those of a stronger habit of body; but their meat lies heavy on their stomach, as the expression is. Now, beneath the stomach lie two great blood-vessels, the aorta, and the vena cava: the vena cava sends its blood to the heart, which sends it to the head; and the aorta, which carries the blood from the head, being pressed by the incumbent stomach, does not receive all that comes from thence, but part of it flows into the face, and causes that redness sought for.

Query 12, *by Amanuensis.*

All bodies descending from a very high place, gain a swiftness of descent, in proportion to the height from which they fall. Hail descending from the clouds, which are sometimes a quarter of a mile high, does not, nevertheless, fall with such violence as we might reasonably expect. If it uniformly increased as it fell, it would come with a swiftness equal to a ball shot from a cannon; why then does it not strike us with equal force?

Solution, *by the Proposer.*

Falling bodies do really acquire a velocity uniformly accelerated, and if there were nothing to retard its fall, an hail-stone falling from a cloud might acquire the velocity of a bullet, before it reached the surface of the earth. But the truth is, it falls through a resisting body of air, which, the greater the velocity of the falling body, the greater is its resistance. Thus, when the hail-stone acquires a certain velocity, the resistance becomes strong enough to counter-act any new additions of velocity it might receive in its descent; so that it falls with a moderate degree of force, not very obnoxious to mankind.

Query 13, *by Mechanicus.*

It often happens, that if we take two horses, in every respect alike, both in strength and all other qualities; yet, if both are put to the draught, that horse which is most loaded, shall be capable of performing most work; so that, that horse will be capable of drawing the largest load, which carries the greatest weight.—How is this?

Solution, *by the Proposer.*

This happens to those horses which draw up weight from the shaft of a mine; the weight to be drawn up, being greatly below the draught, the horse has not power to fix himself firm enough to surmount it; but in this case, they generally place a moderate load on the horse, which fixes him firmer to the ground; and thus, though more loaded, he has the greater power over the weight to be raised from below.

Query 14, *by Charles Cadby.*

There is a place on the surface of the earth, where, if two men meet, the one shall find it so hot, that he will immediately throw off his cloaths through the heat, and the other shall find it so cold, that he will immediately put on those which the other has thrown off.—What place is this?

Solution, *by Thomas Cadby.*

This place is about the middle of the Andes, which are the highest mountains in the world, situated in South America. On the top of these mountains, the air seems temperate to the inhabitants of Quito, a country, almost upon their summit. At the bottom of the mountain, as it lies between the torrid zone, the climate is intensely hot. Now, as travellers go from the plain up the mountain, they find it colder and colder as they ascend; and, on the contrary, as the inhabitants from above come down the mountain, they find it hotter and hotter. Thus, when both meet at the middle of the ascent, those coming from above, are ready to throw off their cloaths from heat; those coming from below, are ready to put them on, through the increasing sensation of coldness.

ARTICLE IX.

THREE LETTERS,

Containing some Objections to the Editor's New THEORY on the ABERRATION of the FIXED STARS, *being all we have received in Support of* Dr. BRADLEY. *Though the Editor does not mean to be either Obstinate or Illiberal, yet he begs Leave to be allowed to make some Remarks, and some Objections to part of their Contents, in a future Number.*

LETTER I.

SIR,

I HAVE seen in one of your papers, an attempt to prove the *absurdity* of Dr. Bradley's Theory of the Aberration of Light, and something like a new Theory offered to the public. I have, therefore, taken the liberty to give you my thoughts upon the subject, in a free, but friendly manner.

In the first place, it appears to me that you are quite a stranger to Dr. Bradley's *own* Theory, which, I will venture to say, is so far from being *absurd*, that (even allowing him to be mistaken) it is a most simple, elegant, and conspicuous Theory. Secondly, I think your own Theory quite otherwise; for you have not made choice of any one particular ray, or particle of light, moving in a given direction, to illustrate your meaning; but have given a very obscure account of your own conceptions of this matter.

Lastly, in one of your papers, where you have been speaking of the velocity of light, you assert that its motion is retarded, when it enters a dense medium; and to determine the quantity of that retardation in the atmosphere, you quote a passage from Mr. Emerson; which quotation proves quite the contrary, for it proves that the motion of light is accelerated upon entering a dense medium: this is an unaccountable oversight, and I have taken notice of it here, because your Theory seems to be built on this false foundation of light being *retarded by*, *entangled in*, and *carried along with* the atmosphere.

I wish you to read Dr. Bradley's *own* Theory, and to make yourself better acquainted with the nature and operations of light, and by these means you will, perhaps, avoid those mistakes, which at present seem to endanger your reputation as an Author.

I am, Sir,

Your sincere well-wisher,

JOHN CARR.

Beverly, Yorkshire,
MAY 7th, 1793.

LETTER II.

SIR,

Fig. 1. IN the third Number of your SCIENTIFIC RECEPTACLE, you have proposed a query, relative to the cause of the Aberration of the fixed Stars ;— and in your fourth Number, have given an answer to it, which those who are your friends and understand the matter, must wish you had not done.

You seem to have misled yourself, by substituting the term *ray*, instead of *particle* (of light) which all the authors I have read on the subject use ; and to have formed an idea by it of something like a continued stream of light, constantly issuing from the star, and meeting your eye. Such an idea would, undoubtedly be just, if both the star and your eye were at rest ; but your eye being in motion, it is certain that only a single particle, in any *one* such ray, or stream of light, can meet your eye ; namely, that which strikes the eye as you are carried across such ray, by the motion of the earth ; and it will be a particle of *another* ray which strikes the eye the next moment, and so on, *ad-infinitum*.

The illustration which you have given of the matter, from Martin, is not greatly different from that which is given of it by every other author, who has written on the subject ; and is plain enough to me, as indeed it has been to every other person I have met with till now : but perhaps it may contribute toward rendering the thing clearer, if, instead of a particle of light coming from the star, we should suppose the star itself, considered as a point (which it appears to be) to fall toward the earth in the line SB, upon the line ABD, with a velocity equal to that which light moves with, at the same time that a tube, AC, is carried along the line AD, by the motion of the earth in its orbit, parallel to itself ; and let us suppose further, that the end C, of the tube, meets the line SB, at the instant that the star arrived at C, where it will enter the tube. Because, by Euclid 10. VI. AB : BC :: A*v* : C*v* :: *ab* : *vu* :: *bf* : *un* :: *f*B : *n*B :: the velocity with which the tube is carried along parallel to itself by the motion of the earth, is to the velocity with which the star descends along the line SB ; it follows, clearly, that when the end of the tube has moved from A to *a*, the star will have fallen from C to *v*, and therefore will still be in the axis of the tube, as it was when it first entered it, at C. And, for the same reason, when the end of the tube has got to *b*, the star will have descended from *v* to *u*, and consequently be still found in the axis of the tube, as at the first. Lastly, when the end of the tube arrives at B, it is manifest the star will be there also, just quitting the tube ; down which it has descended in the right line SB, notwithstanding the direction of the tube was inclined to that line under the angle ACB.

Let us now suppose AC to be the tube of Dr. Bradley's telescope, properly adjusted for observing *Draconis*, and SB the direction of that particle of light (issuing from the star) which impinged on the centre of his object glass as it was carried along by the annual motion of the earth in its orbit : is it not plain, from what has been said above, that this particle would continue to proceed along the axis of the telescope, in the direction SB, while the telescope was carried parallel to itself by the motion of the earth in the direction of the line AD, till it met the eye of the observer, at B ? In which case, the true altitude of the ſtar is evidently the angle SBD; but its apparent altitude, meaſured by the inſtrument, would be the angle •BD, which is the poſition of the teleſcope.

Before your hypotheſis for the cauſe of the aberration can be admitted, it is incumbent on you to ſhew, 1ſt. That the rays of light are retarded in paſſing through the atmoſphere of the earth; 2nd. That, if they are retarded in their paſſage through it, they are carried ſide-way with it; and, laſtly, After you have proved that both theſe circumſtances do take place, it behoves you to ſhew that the quantity of the aberration which ariſes from theſe cauſes, is agreeable to the quantity obſerved. All theſe things, I ſay, it would be incumbent on you to prove, before your hypotheſis could be admitted by any rational man, if the abſurdity of it were not palpable from this ſingle conſideration;—that if the cauſe lay wholly in the earth's atmoſphere, the aberrations of the ſun, moon, planets, and fixed ſtars, would all be the ſame under the ſame circumſtances: whereas we know, that the aberration of the fixed ſtars amounts to about 20″, while that of the moon is ſo ſmall as not to be obſervable; and that the aberrations of the planets vary with their diſtances and geocentric motions.

I am, Sir,

Your very humble ſervant,

ASTRONOMICUS.

LETTER III.

SIR,

Fig. 2. I HAVE lately been favored with a peruſal of two Numbers of your SCIENTIFIC RECEPTACLE, in the former of which you advance ſome objections to Dr. Bradley's Theory of the Aberration of Light; and in the latter, after informing the public that your ideas have been approved of by three correſpondents, you very candidly offer any of the Doctor's advocates a place in the Miſcellany before alluded to:—Relying on this promiſe, I have ventured to undertake his vindication.—In the firſt place, permit me to inform you and your readers, that the illuſtration you have extracted from Martin, is a very erroneous one: I do not charge the Editor of the Receptacle with falſe quotation, nor do I condemn the Author of the Philoſophia Britannica as a bad Geometrician, becauſe I am a ſtranger to the work: nothing therefore remains to be done, but to diſplace the demonſtration in queſtion by a better.—We know that light paſſes from the ſun to the earth in about eight minutes, in which time this planet moves on a medium through nearly 19 ſeconds of its orbit.—Now, in order to ſimplify the matter as much as poſſible, let this orbit, which is the path of the ſpectator's eye, be repreſented by the ſemi-circle DAE, then ſuppoſe OC to be part of a line joining a fixed ſtar, and the centre of the orbit O; make CA = AO, and let BA = an arc of 19 ſeconds. Then if the ſpectator's eye were fixed at B, it would ſee the ſtar in a right line BK ∥ to AC; on the contrary, ſuppoſing the ray BK intercepted, and the light in CA to

be deflected when it came to C with a velocity, which is to its velocity in CA, as BA to CA; then the spectator would see the star in the direction BC, making, with its true position, the angle KBC, which angle is equal 19″, because OA = AC, and BA being very small, may be considered as a right-line ⊥ to OC without sensible error; therefore ∠ KBC = ∠ BCA = ∠ AOB. If, instead of supposing the spectator fixed at B, we imagine, what is really the case, that he is carried from B to A, while a particle of light moves through CA, the quantity of Aberration will be the same; for between B and A an indefinite number of right-lines ∥ to AC may be drawn; of which, suppose *gb* to be one, meeting BC in *i*; also make R*g* = BA; then, since B*g* : BA = R*g* :: *gi* : AC = *gb*, it follows, that a particle of light at *b* when the spectator was at R, will be at *i* when he is at B, and at *g* when he is at *g*; and, therefore, all the particles of light that are in BC when he is at B, will successively fall on his eye in his passage from B to A, on which account the apparent direction of the ray will always be ∥ to BC, because it will be compounded of the motion of his eye in BA, and that of light ∥ to AC; therefore the quantity of the Aberration will be measured by the ∠ KBC in this case also.—*Corol. 1st.* If any point P be taken between EA or AD, and a line be drawn from it ∥ to AC, it will give the star's true place; and the Aberration at A will be to that at P, as radius to the sine of the ∠ POE; and therefore it is greatest at A, and nothing at D and E; consequently while the spectator moves from E to A the star is retrograde in respect to the annual motion of the Heavens, and drawing his passage from A to D direct. This may easily be demonstrated from the properties of the circle and the laws of optics. *Corol. 2d.* A quantity of Aberration is occasioned by the diurnal rotation of the earth, but is too small to be perceptible; for in the space of eight minutes, a point on the earth's surface moves through thirty-two minutes of a degree; and as small optic angles are nearly as the diameters they subtend, it is as radius : sine of 32′ :: the sun's parallax : the *maximum* of Aberration from this cause.

Scholium. I have here supposed the star, conformable to the illustration in the *Receptacle*, to be in the plane of the eye's path; but if we imagine it to have a sensible declination from the same, it forms a small ellipsis round its true place as a centre, the properties of which might be easily pointed out.—What I have already demonstrated shews plainly, that a cause of Aberration does actually exist such as Dr. Bradley assigned; what your Theory is I know not, and therefore can say nothing on the subject.

WESTMORLANDICUS.

X

ARTICLE X.

Mathematical Questions with their Solutions.

Question 49, *by Mr. John Lowry, of Houghton.*

GIVEN the sum of the sides, the line bisecting the vertical angle, and its prolongation 'till it meets the circumference of the circumscribing circle, to construct the triangle.

Solution, *by Mr. James Dennington.*

Geometrical Analysis. Suppose ABC (Fig. 3) the required triangle, O the centre of the inscribed circle, then LO a mean proportional to BL, IL is given; also, by a known theorem BL : IL :: AB + BC : AC, which is therefore given, but LC = LO is given; therefore no more is now required than to describe an isosceles triangle (ALC) all whose sides are given. *Otherwise.*

Construction. Draw PQ (Fig. 4) = bisecting line, QR = prolongation; make RS = a mean between PR, QR. Now with radius RS^2 = LO describe a circle, in which apply AC = a fourth proportional to PR, RS, and the given sum of sides; also apply LI to AC = given prolongation, and produce it till LB = PQ join AB, BC and the thing is done; for it is easily shewn that a circle will pass through the points ABCL, therefore BL bisects the vertical angle. Again, by Question 629, Ladies' Diary, LO : BL :: AC : AB + BC, but by construction RS (LO) : PR (BL) :: AC : sum of sides; hence AB + BC = given quantity. Q. E. D.

The same, *by Mr. John Ryley.*

Let CI be the given bisecting line, ID its prolongation; on DC (Fig. 5) describe a semi-circle, in which apply CE = half the sum of the given sides; take DO a mean proportional between DC and DI, and O will be the centre of the inscribed circle; draw OF ⊥ EC, with radius OF and center O describe the circle, through I draw AB to touch the circle in G; produce CE to meet BA in A, and draw CB to touch the circle in H, and ABC will be the triangle required. For it is evident that DC bisects the vertical angle, because it passes through the centre of the inscribed circle, and if DE be drawn it is known that AE is half the difference of the sides, and if EK be taken = AE, a circle whose radius is DO, and center D, will pass through the points A, K, O, B; therefore, because the angle ACB is bisected it is plain that KC = CB, and AB + CB = 2ED = the given sum of the sides.

The same, *by Mr. William Armstrong, Pupil in Mr. Howard's Academy, Carlisle.*

Suppose the thing done, ABC (Fig. 3, Plate 5) the triangle required; AD the given sum of the sides, MR the given bisecting line, and RN its prolongation; now it is well known that the rectangle AC.BC = MR.MN, therefore, if S be put for the side of a square = AC.BC the sides will be easily determined, by Pr. 22. VI. Bon. Geo. the rest is obvious.

Question 50, *by Mr. Thomas Whiting.*

At the point A (Fig. 6) stands a lamp on a post 10 feet high, at D stands a man six feet high, at the distance of 8 feet from the lamp-post, the man moves along DC: Required the area of the space passed over by the man's shadow, when he has gone 10 yards?

Solution, *by Mr. John Lowry.*

Take DE : DA :: 6 : 10 (Fig. 6) and FC : CA in the same ratio, join EF and DE FC is the space required. Calculation. 10 : 6 :: 8 : 4.8 = DE; $\sqrt{10^2 + 8^2} = 12.806$ = AC; and 10 : 6 :: 12.806 : 7.68 = FC; likewise $\sqrt{10^2 + 4.8^2} = 11.092$ EC, and the angle ECF = 115° 32′, hence the triangle ECF = 38.406, and the triangle CDE = 24. Therefore the whole space = 62.406 yards.

Question 51, *by Mr. Brown, a young Gentleman who has the Misfortune to be Blind. He was a Pupil in the Academy at Greenwich; and I have been told he possesses amazing Abilities in the Mathematics.*

In the triangle ABC there is given AB = 8, BC = 6, and AC = 10. Required the distance BE in the side CB continued, so that the arch GH being described with the radius EH, the parts AG, BH, cut off may be each = 3.

Solution, *by Mr. John Ryley.*

Let ABC (Fig. 7) be the given triangle, and take AG, BH each of the given lengths (3); join the points G, H, and bisect GH in F; draw FE perpendicular thereto, meeting CB produced in E; and BE is = the required length. Calculation. By similar triangles CA : AB :: CG : CI = 5.6; also, CA : CB :: CG : CI = 4.2. Hence HI = 1.2 GH = $\sqrt{32.8}$ and FH = $\sqrt{8.2}$. Now in the triangle IGH all the sides are given, to find the sine of the angle IGH = FEH = $\frac{1.2}{\sqrt{32.8}}$, and its cosine = $\frac{5.6}{\sqrt{32.8}}$; also in the triangle EFH all the angles are known, and side FH, to find the side EH = $13\frac{2}{3}$; therefore BE = $10\frac{2}{3}$ as required.

Question 52, *by Mr. O. Gregory.*

Given the radius of the base of a spheric segment = 20, and the angle formed by the said line, and a line drawn from the circumference of the base to the center of gravity = 12°; to find the content of the sphere of which this segment is a part.

Solution, *by Mr. William Armstrong.*

Let O represent the center of gravity (Fig. 8) then will OBQ be the given angle. Put BV = AV = a, AQ = x, and BQ = 20 = c, then, per Trig. $s. \angle$ BOQ : BQ :: $s. \angle$ OBQ : OQ = 4.251, which call r, and (47. E. 1.) QV = $\sqrt{a^2 - c^2} = a - x$, hence $x = a - \sqrt{a^2 - c^2}$, and AO = $a - r - \sqrt{a^2 - c^2}$, but by Art. 178 Simpson's Fluxions, the distance AO is = $\frac{x \times (4a - 3x)}{6a - 4x} = a - r - \sqrt{a^2 - c^2}$ and by substitution $\frac{(a - \sqrt{a^2 - ^2}) \times (a + 3\sqrt{a^2 - c^2})}{2a + 4\sqrt{(a^2 - c^2)}} = a - r - \sqrt{a^2 - c^2}$; this reduced becomes $a^2 + \frac{ac^2}{3r} = \frac{c^4}{12r^2} + \frac{4c^2}{3}$ and by completing the square we obtain a equal $\sqrt{\frac{c^4}{9r^2} + \frac{4c^2}{3}} - \frac{c^2}{6r} = 23.218$ ∴ the content is $(23.218)^3 \times .5236 = 6553.5049$.

Question 53, *by Mr. Richard Nicholson, Master of Kirkbyoverblow School.*

From the point of intersection P of two given circles O and C, to draw geometrically a right line, PQ cutting the least circle in R, and terminating in the periphery of the greatest, so that PR may be = QR.

Solution, *by Mr. John Salter, of Bilston.*

Construction. From P (Fig. 9) the point of intersection of the two circles to the center C of the greater, draw PC, on which describe the semi-circle PRC cutting the less in R, from P through R, draw PQ and the thing is done. For if RC be joined, the angle PRC being in a semi-circle will be a right one ∴ RC is perpendicular to PR, and by passing through the center C, consequently bisects PQ ∴ PR = QR. Q. E. D.

The same, *by Mr. John Ryley.*

Let O (Fig. 9) and C, be the centers of the given circles, and P the point of intersection; draw PC and bisect it in A; join AO, and draw CR parallel thereto; through R draw PQ, and the thing is done. For by Eu. III. and 3d. OB is perpendicular, and also bisects it: Likewise, as CR is parallel to AO, it is perpendicular to PQ and also bisects it; therefore PR = QR. Otherwise.—Upon PC the radius of the greater circle describe a semi-circle, and through the point of intersection R draw PQ and the thing is done. For the angle PBC in the semi-circle is a right angle, therefore CR is perpendicular to, PQ, and also bisects by the Prop. above quoted.

The same, *by Mr. John Lowry.*

Generally when PR (Fig. 10) is to PQ in any given ratio. Imagine PRQ drawn as required; and draw PA, RA, QA, then because the angles ARP, AQP are constant, the ratio of RQ to RA will be given; and the ratio of PR to RQ is given per question, ∴ the ratio of PR to RA is given. Whence this Construction. Divide PA in D in the given ratio of PR to RA; make the perpendiculars PE, DC = DP, DA respectively, through EC draw ECG meeting PA produced in G, with radius GD and center G describe the arch DR cutting the less circle in R, through R draw PRQ and the thing is done. This construction is manifest from Prob. 13, Page 220, Simpson's Geometry.

The same, *by Mr. William Armstrong.*

Analysis. Mr. Simpson has demonstrated in Prop. 27 of his Geometry, that if O and C the centers of any two circles be joined, and a semi-circle ODC (Fig. 11) formed upon OC, and OD taken any given length, then RQ made by drawing PRQ through P ∥ to OD, will be equal to 2OD. From O let fall OF upon PQ, then, since O is the center of the circle PR; PF = FR = OD, and if we join DR then ED is a parallelogram, and ∴ DR will pass through C. Again, join O, P, and C, P, then since PF = FR the line CP will be bisected in I: hence this Construction. Bisect PC in I and from O thro' OI draw OIF, and from P draw PERQ at right-angles to OF, so will PQ be the line required.

Question 54, *by Mr. Richard Nicholson, of Kirkbyoverblow.*

It is required to draw geometrically a right line, the length of which is a minimum, to cut off from a given triangle, another triangle having a given magnitude.

Solution, *by Mr. W. Armstrong, at Mr. Howard's school.*

Construction. Make VG = the given side VF (Fig. 12) then by Bon. Geometry 16. VI. △ VGF : GV^2 :: △ VAB : AV^2, hence, through A draw AB ∥ to GF and AB will be the shortest line possible.

Demonstration. Let any other line CD be drawn to cut off the given area AVB, then from P the point of intersection AB, CD lay off the triangle PQL = ACP having the angle L = C, and which because PBD = CPA will make the triangle BLO = QOD. (O being the point where LQ cuts BD) Now by reason of the similar and equal triangles ACP, PLQ we have AP = PQ; CP = PL and ∴ AP + PB is less than AP + PL = CP + PQ, therefore AP + PB must be less than CP + PD = CD. Q. E. D.

Question 55, *by Mr. W. Armstrong.*

In a right-angled triangle there is given the sum of the sides about the right-angle, and the side of the inscribed square to construct the triangle.

Solution, *by Mr. John Salter, of Bilston.*

Construction. Constitute the square (Fig. 13) PQSB = the given one; and produce BP to F so that SB + BF = the given sum of the sides, divide FP in A, so that FA ×

$AP = PB^2$, from A through Q draw AQC cutting BS produced in C, then will ACB be the triangle required. For by similar triangles, as $AP : PQ (= PB) :: PB : CS \therefore AP \times CS = PB^2$ consequently $AF = CS$, and $FB + BS = AB + BC$. W. W. R.

The same, *answered by the Proposer.*

Analysis. Suppose ABC (Fig. 13) the required triangle, and BPQS the given inscribed square, then by similar triangles $BC : AB :: CS : SQ = BS$, and by composition $BC + AB : AB :: CS + SQ = BC : BS$, therefore $BS \times (BC + AB) = AB \times BC$. Whence this *Construction.* On AD = the given sum of the sides describe the semi-circle AED, and at D erect the ⊥ DR = a mean proportional between the sum of the sides, and the side of the inscribed square, draw RE parallel to AD, and let fall the perpendicular EB, make BD = BC and join A, C, then is ABC the required triangle. For $AB.BC = AB.BD = BE^2$ (per circle) $= SB \times (AB.BC)$ by Construction. Note Because RE cuts the circle in two points E and *e* the problem is capable of two solutions. But when RE neither cuts nor touches the circle, the problem is impossible.

Question 56, *by Mr. James Dennington, of London.*

Given the difference of the angles at the base, and the perpendicular falling from the angle at the base of the given inscribed square on the line bisecting the vertical angle, to construct the triangle.

Solution, *by Mr. W. Armstrong, Pupil in Mr. Howard's School.*

Analysis. Suppose the thing done, and ABC the triangle required; the rest as per Fig. (14) demit the ⊥ CP, then will the ∠ LCP = ½ the given diff. (by Dr. Hutton's Math. Mis. page 271) but the ∠'s at P and E are right, and the ∠ L common to the 2 △'s CLP, RLE ∴ ∠ERL = ∠ LCP, from which the position of LC and the ∠'s MI, NI will be given, but $MI : NI :: MC : CN$ (and by Prop. 3. VI. Bon. Geom.) $MC : CN :: AM : BN$, now a given ratio; hence if from M the ∠ AMF be laid off = the given diff. ∠ MFW = BAC + AMF = ABC, and also MF = NB, from which all the ∠s will be known; hence the following

Construction. Having drawn MN and laid off the ∠s M and N equal A and B at the distance MN draw the parallel AB meeting CM and CN produced in A and B, and the thing is done.

The same, *by the Proposer.*

Construction. On the base of the given square MR (Fig. 14) take the ∠ LRE = ½ the given one, take RE = given line, and through E draw the indefinite ⊥ LC cutting MN in I, then by Simpson's Algebra, page 336, $MC : NC :: MI : NI$; through M and N draw CA, CB meeting RL produced in A and B, so shall ABC be the triangle required.

Demonstration. Let fall the perpendicular CP, then the triangles CPL, REL having ∠ P = ∠ E, and ∠ L common, will have the angles LRE, PCL also equal; hence ∠ A

— ∠ C = 2∠ LRE = the given ∠ by construction. Again, because MC : MI :: NC : NI, by Eu. 3. VI. CL bisects the vertical angle. The rest is obvious.

Question 57, *by Mr. Thomas Hewitt, of Spitalfields.*

Required a general rule for making right-angled triangles, by having two numbers a and b given.

Solution, *by Mr. William Armstrong.*

Mr. Bonnycastle determines at Page 243 of his Algebra, that $2ab$; $a^2 - b^2$ and $a^2 + b^2$ are the perp. base, and hyp. of a right-angled triangle. Mr. Lowry refers to page 238 of Emerson's Algebra, for the same thing.

Question 58, *by Mr. Olinthus Gregory.*

Yaxley church, in N. lat. 52° 30′ stands on the summit of an hill which is inclined to the horizon 3° 30′. On the morning of the longest day, the sun shining brightly when at due East, I found the length of the steeple's shadow, on the slant side of the hill, to be 103 yards. Now exercise yourselves ye young students, and find the steeple's height.

Solution, *by the Proposer.*

First, as the sine of the latitude 52° 30′ : radius :: sine of the declination 23° 28′ : sine 30° 7′ 42″ the true altitude of the sun's center: With proper allowances for the sun's semi-diameter, parallax, and refraction, we get 30° 24′ 34″ apparent altitude of the sun's upper limb. Then in Figure 9, Plate 2, we have the ∠ BCD = 30° 24′ 34″ from which take the angle ACD = 30° 30′, remains 26° 54′ 34″ = angle ACB. Therefore as cosine of DCB, or sine of B = 59° 35′ 26 : AC = 103, :: sine ACB = 26° 54′ 34″ : AB = 54.05175 yards, height of Yaxley-Church steeple.

Question 59, *by Mr. William Burdon, Acaster Malbis.*

In a four sided field, ABCD, given AB = 820 links, BC = 600, AD = 760, the angle BCD = 57° 30′, and the angle AB makes with DC = 20° 16′, to determine the area.

Solution, *by Mr. John Salter, of Bilston.*

Let ABCD (Fig. 15) represent the field, draw CE parallel to AB, then there is given per Question, AB, BC, AD, the angle BCD = 57° 30′ and the angle ECD = 20° 16′, therefore the angle BCE = 77° 46′ and angle ABC = 180° — 77° 46′ = 102° 14′. Whence per Trig. as 820 + 600 : 820 — 600 :: tang. angle 38° 53′ (half the sum of the angles ACB, BAC) : tang. 7° 7′ (half the difference of the angles ACB and BAC) therefore ACB = 46° and BAC = 31° 46′, again, as sine 31° 46′ : 600 :: sine 102° 14′ : 1113.8 links = AC, also, as 760 : sine 11° 30′ :: 1113.8 : 163° (ADC) hence CAD = 5° 30′, and consequently the area of ABCD = 1113.8 × 410 × sine 31° 46′ + 1113.8 × 380 × sine 5° 30′ = 280975 square links. W. W. R.

Note. This question admits of another answer, *i. e.* when the angle ADC = 180° —

163° = 17°, and in that case the area will exceed the other by $\frac{AD^2}{2} \times$ sine of 17°.

Question 60, *by Mr. Davison, being his 97th in the Algebra.*

The Equations, as expressed in the Question.		
	1	$\frac{z^2 + 2zm}{y - 2m} = y$
	2	$\frac{xy}{x + m^2} = z$
	3	$um + vn + mn - m^2 = vm + un$
	4	$uw + 2vy = vw + 2uy$
	5	$\frac{uwn + wzn}{u^2 - z^2} = \frac{uym - yzm}{(u - z)^2}$
	6	$wyzn - vymn = vwn^2$
	7	$w^2 - x^2 - 2xm^2 = m^4$
	8	$\frac{u^2 - um}{m^2 n^2} = u - m$

A general Solution, *by Mr. Thomas Keith, of London.*

By red. com. square and ext. 1st eq.	9	$z + m = y - m$; $y = z + 2m$, & $2y = 2z + 4m$.
Second reduced - - - - - - - - - -	10	$xy = zx + zm^2$; $z = \frac{xy}{x + m^2}$.
4 transposed - - - - - - - - - - -	11	$uw - vw = 2uy - 2vy$; $w = \frac{2uy - 2vy}{u - v} = 2y$.
7th transposed and extracted - - -	12	$w = x + m^2$
10th, 11th, and 12th, compared	13	$z = \frac{xy}{x + m^2} = \frac{xy}{w} = \frac{xy}{2y} = \frac{x}{2}$, $2z = x$.
For 2z eq. 9th write x and compare it with the 12th.	14	$x + m^2 = x + 4m$ ∴ $m = 4$
5th × $(u^2 - z^2)$ and 2d part ÷ $(u - z)^2$ - - - - - - - - -	15	$uwn + wzn = 4uy + 4yz$.
15th for 4y write 2w - - - - -	16	$uwn + wzn = 2uw + 2wz$.
16th ÷ w and transposed - - - -	17	$un + zn = 2u + 2z$ ∴ $n = \frac{2u + 2z}{u + z} = 2$.
8th ÷ (u − m) - - - - - - - - -	18	$\frac{u}{m^2 n^2} = 1$ ∴ $u = m^2 n^2 = 64$.
3d transposed and ÷ (m − n) -	19	$v = \frac{um + mn - un - m^2}{m - n} = u - m = 60$.
In 6th eq. for w, write 2y - - -	20	$y = \frac{2vn + vm}{2z} = z + 2m$ by equation 9th
20th × 2z and sq. completed, &c.	21	$z = \sqrt{(m^2 + \frac{1}{2}vm + vn)} - m = 12$.
By 13th, 12th, and 11th, we get	22	$x = 24$, $w = 40$, and $y = 20$.

Note. This Question is not properly limitted, for from the eleventh step it evidently appears that it may be $= \frac{vw - 2vy}{w - 2y} = v$, and by writing u for v, or v for u in the third equation, we find $m = n$; and hence the 8 equations will be reduced to 6; but we must remark that it will not then be $= 2y$ for that fraction will vanish. The 7th equation may likewise have two roots, &c. In the above solution, more attention has been paid to perspicuity than brevity.

The same, *by Mr. John Ryley.*

Let s, t, u, v, w, x, y, and z represent the required numbers respectively; then per question (see Page 152 of Davison's Algebra) $\frac{x^2 + 2xy}{w - 2y} = w$; $\frac{vw}{v + y^2} = x$; $sy + tz + yz - y^2 - sz = ty$; $su + 2tw = tu + 2sw$; $\frac{suz + uxz}{s^2 - x^2} = \frac{swy - wxy}{(s - x)^2}$; $uwxz - tuz^2 = twyz$; $u^2 - v^2 - 2vy^2 = y^4$, and $\frac{s^2 - sy}{y^2 z^2} = s - y$. Now by multiplication the first equation becomes $x^2 + 2xy = w^2 - 2wy$, and if y^2 be added to each side thereof, and the square root extracted, it becomes $x + y = w - y$; therefore $x = w - 2y$: moreover it is evident from the seventh equation, that $u = y^2 + v$; consequently the second equation may be reduced to $ux = vw$: Also, the fourth equation by transposition becomes $su - tu = 2sw - 2tw$; or $u \times (s - t) = 2w \times (s - t)$; $\therefore u = 2w$; and if each denominator in the fifth equation be divided by $s - x$ it becomes $\frac{uz \times (s + x)}{s + x} = \frac{wy \times (s - x)}{s - x}$; hence $uz = wy$; but by the above $u = 2w$; consequently $y = 2z$. Now if $2z$ be substituted in the third equation for y, its equal, and the equation reduced, we shall obtain $t = s - 2z$; and after the same manner the seventh equation is reduced to $vw = 4tz$, and the eighth $s = 4z^4$: Hence the original equations are transformed to the following ones, viz. $x = w - 2y$; $vw = ux$; $t = s - 2z$; $u = 2w$; $y = 2z$; $ux = 4tz$; $u = y^2 + v$, and $s = 4z^4$. Now from the first and second of those equations $x = w - 4z = \frac{1}{2}v$, and from the seventh $v = 2w - y^2 = 2w - 4z^2 = 2w - 8z$; or $4z^2 = 8z$; hence $z = 2$: Therefore the rest are easily found as follows, viz. $s = 64$, $t = 60$, $u = 40$, $v = 24$, $w = 20$, $x = 12$, and $y = 4$:

Question 61, *by Mr. O. Gregory.*

Suppose a perfectly elastic ball should fall from the top down a plane, whose length is 60 feet, and inclination to the horizon 40°, on a firm plane, parallel to the horizon, it would then rebound, and fall again on that plane. Query, how far from the inclined plane will it fall; also, the time the ball has been in motion, when it alights after reflection.

Solution, *by Mr. John Ryley.*

Because the length and inclination of the plane are given, the height by Trig. is found to be 38.567256 feet; therefore the velocity with which the ball quits the plane will be expressed by $2\sqrt{(16\frac{1}{12} \times 38.567256)} = 49.812$ feet per second. Also, the time of descent through CB (Fig. 16) $= 2CB \div 49.812 = 1.5485$ seconds; and BC : CA :: 1.5485 : 2.409 seconds the time of descent through AC. Now because the angle of incidence is = to the angle of reflection, the angle EAD and velocity of projection are given; therefore by the Theory of projectiles, radius : sine of twice the angle EAD :: $49.812^2 \div 32\frac{1}{6}$ to the amplitude $AE = 75.965$ feet; also radius : sine angle EAD :: $49.812 \div 16\frac{1}{12}$: 1.9908 seconds the time of flight; which added to 2.409 seconds the time of descent down the plane, gives 4.3998 seconds; the whole time the ball was in motion.

Question 62, *by Mr. John Lowry.*

Given the two sides of a spherical triangle to construct it, when the ∠ at the vertex is equal to the difference of the angles at the base.

Solution, *by the Proposer.*

Construction. Bisect one of the given sides, AB in D, (Fig. 17) and about D as a pole, with the distance AD or BD, describe the lesser circle ACB, and with distance BC = the other given side, and center B, describe the lesser circle C*c* cutting the former small circle in C and *c*, draw the great circles BC, AC, then is ABC or AB*c* the triangle required.

Demonstration. Since BD = DC = DA; the angle B = ACD, therefore the angle A = angle C = angle B, and AB, BC are = the given sides by construction. Q. E. D.

Corol. When C*c* cuts the circle, there will be two triangles answering the Question; but when it only touches, then BC = BA, and the Question is impossible.

Question 63, *by Mr. John Lowry, of Houghton.*

There is a piece of land in the form of a parabola, whose base and abscissa are a and b (16 and 11) chains respectively. The land on one side of the abscissa is worth d (20) shillings, and on the other c (14) shillings per acre. Query, the direction of an hedge, made from the corner of the base on the side, worth d shillings; so that the field may be divided into two parts of equal value.

Solution, *by Mr. William Armstrong, Pupil in Mr. Howard's School, Carlisle.*

Let AQ (Fig. 18) represent the hedge, draw DQ ∥ to BP, and RQ ∥ to AC. Put PC = a, PD = QR = x, QD = y, RS = z, p = parameter, and b, c, d, as per question. Here PS = $y - x$, and per conics, $p : a + x :: a - x : \frac{a^2 - x^2}{p}$ equal y, and per sim. Δs $a : y - z :: x : z$, and $z = \frac{xy}{a+x} = \frac{a^2x - x^3}{p \times (a+x)}$. But $\frac{1}{2}xy$ = area QBS,

$\frac{2}{3}x \times (b-y) =$ Do. BRQ and $\frac{2}{3}ab - \frac{1}{2}a \times (y-z) =$ Do. ABS. Whence, per question $\frac{1}{2}cxz + \frac{2}{3}cx \times (b-y) + \frac{2}{3}abd - \frac{1}{2}ad \times (y-z) = \frac{2}{3}ab \times \frac{c+d}{2}$, which reduced is $x^4 - 4ax^3 + \overline{\frac{3a^2d}{c} + 4bp - a^2} \times x^2 + \overline{\frac{2abdp}{c} + 2abp - 4a^3} \times x = 2a^2bp - \frac{2a^2bdp}{c} + \frac{3a^4d}{c}$, or in numbers, $x^4 + 32x^3 + \frac{3264}{7}x^2 + \frac{3072}{7}x = \frac{98304}{7}$; hence $x = 4.39842$ and $y = 7.67476$ extremely near.

Question 64, *by Mr. John Lowry.*

Given $\left\{\begin{array}{l} 64z^2 - y^4 + 136y^2 - z^3y = 4624 \\ 176z^2 - z^4 + 64y^2 - y^3z = 7744 \end{array}\right\}$ To find the value of x and y.

Solution, *by Mr. W. Armstrong.*

Put $mn = y$, and $n = z$, then the equations become $64n^2 - m^4n^4 + 136m^2n^2 - n^4m = 1624$, and $176n^2 - n^4 + 64m^2n^2 - m^3n^4 = 7744$, hence from 484 times the first, take 289 times the second; and from m^2 times the first, take the second, and we get $484.64 - 484m^4n^2 + 484.136m^2 - 484n^2m = 289.176 - 289n^2 + 289.64m^2 - 289m^3n^2$, and $n^4 - 176n^2 + 7744 = m^6n^4 - 136m^4n^2 - 4624m^2$, or $n^2 - 88 = m^3n^2 - 68m$ and $n = \sqrt{\frac{88-68m}{1-m^3}}$, this substituted in the other & reduced is $m^5 + \frac{15564}{3604}m^4 - \frac{5605}{1802}m^3 - \frac{5015}{901}m^2 + \frac{15561}{3604}m = \frac{5544}{14416}$, hence $m = \frac{3}{4}$, and $n = 8$. $\therefore y = 6$, and $z = 8$.

Question 65, *by Euclid.*

From a given point, to draw geometrically a right line equal to a given finite right line.

Solution, *by Mr. Thomas Leybourn.*

Let A (Fig. 1, Plate 5) be the given point, BC the given right line, it is required to draw a right line from the point A that shall be equal to BC. Join the points AB, and upon BA describe the equilateral triangle BAD. (Eu. 1. 1.) From the point B at the distance BC describe the circle CEF, and produce DB to meet the circumference in F. From the point D, at the distance DF, describe the circle HFG, and produce DA to meet the circumference in G, so shall AG be equal to BC as required.

Note. This is Euclid's 2 Prop. Book 1. It was reproposed on account of its having been done wrong in Dr. Simpson's and some other modern books. They cut off a line instead of drawing a line.

Momentum's Question, *answered experimentally, by Mr. Peter Nicholson.*

When the resistance acts perpendicular to the side.

ABC (Fig. 4) is a wedge, or an isoceles triangle; AB being the base, and AC and

BC the two equal sides, the wedge being laid upon a level board GHIK, with two rollers under it, which are shown at L and M, each roller parallel to the side or base AB of the wedge: Now if the angle ACB is bisected by the right-line CQ, and any point D being taken in CQ, and tie three threads to a pin fixed at the point D, making one thread pass over a pully at R in the direction QC, another string from D over a pully at E in the direction DE ⊥ to the side BC of the wedge, the third string from D over a pully at F; in the direction DF perpendicular to the other side AC; then if weights S, T, and U, are hung to the threads DES, DFT, and DRU, the weights S, T, and U, being respectively in the same proportion to each other as the opposite sides BC, AC, and AB, which their lines of direction are perpendicular to: the wedge in this position will be found by this experiment to remain in equilibrio. Consequently this proves that the wedge will remain in equilibrio, when the power is to the resistance against either side as the base is to one of its sides; therefore the power is to the resistance against both sides, as the base is to the sum of the two sides, which agrees with all the writers on this subject that I have yet seen.

When the resistance acts parallel to the base.

ABC (Fig. 5) is a wedge made of two rectangular pieces, hinged together at B, so that the sides AB and BC may be made to form any angle with each other, a circular piece DME is fixed to the lower side AB, and runs freely in a mortice cut through the upper piece BC at M, with a wedge or screw at M, so that it may be fixed to any angle; this being laid upon a rectangular board, HIKL, with two rollers, which are shewn under it at F and G, PO is a square piece perpendicular to the board HIKL; ON is another piece, ‖ to AB, movable round a pin at O, put through a mortice in the perpendicular piece PO; under the end N is a roller to keep the end at N from rubbing on the top of the wedge BC; then a piece TU being put across the side BC directly over the roller at N, and put a thread round the joint at B, to go over the pullies at R and S; then if two equal weights are hung at W and X, also two other equal weights at Y and Z, from the pullies at R and S; then the weights Y and Z will be to the weights W and X, when the wedge is in equilibrio; as a perpendicular BC to VB, as this experiment shews; that is as the base is to the perpendicular in the half wedge.

Plate. 5.

ARTICLE XI.

Queries with their Solutions.

Query 15, *by Dr. Sangrado.*

WHETHER doth the moon at full and change really affect lunatic people; and if it doth, how, and in what manner doth it work this strange alteration in them?

Solution, *by the Proposer.*

Before we attempt a solution of this question, it may not be unnecessary to premise, that the ancients did not apply the term lunatic in so general a sense as the moderns; we make use of it to express almost every disease of the mind; the ancients applied it only to those people who were subject to epilepsies, because they observed the moon's influence more particularly to affect that disorder. The influence of the moon and other planets on the human frame has been long since observed. Hippocrates, who was a cotemporary with Democritus, so long ago as the 80th Olympiad, about the year of the world 3490, or 457 years before Christ, recommends (Epist. ad Thessalum) to his son Thessalus the study of numbers and Geometry; because, says he, *De aere, aquis et locis*, the rising and setting of the stars, &c. have a great effect upon distempers. Hippocrates, before giving this advice to his son, had certainly remarked the influence the planets had upon diseases; for he was a man by no means given to theoretical speculation, as we may see in his book De Arte, de Decora, de Perceptionibus, where he speaks expressly against it; but wrote entirely from practice and observation, unmixed with the subtle philosophy of Aristotle, which was so much in vogue about 110 years afterwards, when that philosopher began to flourish, and which has so much perplexed and rendered obscure the writings of Galen, and many other of the ancient physicians. Galen, who lived 144 years after Christ, expressly says, *De diebus criticis*, the moon governs the periods of epileptic cases. After him we do not find it so particularly mentioned for many ages; nor is this to be wondered at, for they neglected philosophy, and a knowledge of nature, therefore could not assign a cause for it; and it was customary with them to deny the existence of a thing they could not account for. Even in the time of Galen, this absurd custom prevailed, as Galen himself testifieth. To enter into the particulars of what every author has said, who has touched on the subject of the moon's influence, would render this paper tedious to your readers, and swell it to a much larger bulk than your book would well admit of; I shall only mention the names of those who, in their own writings, or the writings of others, have given cases, or made observations of the effect of the moon's influence; neither shall I take the trouble to consult chronology to set them down in the order in which they lived, but promiscuously as they happen to occur to my memory, as Aristotle, J. Bartholine, Carolus Piso, Kirchringius, Rammazini, Ballorius, Van Helmont, Sanctorius, Baccius, Baglivi, Groenvelt, Diemerbroeck, Tyson, Sir John Floyer, Pitcairn, and Dr. Musgrave, in the Philosophical Transactions, No. 272. All these authors do not relate cases of the moon's influence on those particular people called lunatics only, but of its influence in many other diseases. Now after the testimony of so many learned men, most of them esteemed for their accurate observations on different diseases. I think we can no longer doubt of the certainty of the moon's influence on the human species; and Horus Apollo asserts, that it has a particular effect upon many of the brute species also. It now remains to be shewn, that the moon's influence is greater at full and change than at some other periods of it, and to explain in what manner it acts upon the body.

To account properly for the moon's influence upon the human body, we have occasion for three powerful assistants, anatomy, physiology, and philosophy. The first, from which we want the most assistance, affords us the least, as it is defective in that particular part which the moon's influence most affects; I mean the brain and nervous system. Physiology will assist us but little more, as it is also defective in the same part: though so far it informs us, that man is compounded of two distinct beings, united one to the other, namely, the mind and the body; which, however different in their nature, do yet appear, from undoubted observation, to be so linked one to the other, that certain thoughts of the mind are ever united with determinate changes, or conditions of the body. Philosophy will lend us considerably more assistance, it will enable us to prove that there must be a great alteration in things around us at every new and full moon, and at some other periods of it; which if anatomy and physiology did but sufficiently help us to apply to the human frame, we should be able to account for the effect of the moon upon lunatics, with some degree of certainty. But as this is not the case, I will endeavour to prove, that the body must be much affected at those seasons also; and leave futurity to account how this change immediately affects the mind, when they have carried their researches farther into the arcana of nature, and are become more enlightened by better observations and experiments than the preceding times have furnished us with. To do this, it will be necessary to say something of the air, or atmosphere, in which we live: because it must be by this medium that the moon, or any other planet, can have connection with us. The atmosphere then is an elastic fluid, one part of which gravitates upon another, whose pressure, or force, is communicated every way; though elasticity is not an immutable property of air, for we find it may be changed from an elastic to a fixed state by fumes of sulphur; but yet it is a property absolutely necessary for the preservation of life; for we find that burning sulphur, or charcoal, in a close room will cause the death of any person imprudent enough to remain there. We have great reason to think too, that the lungs reduce it from an elastic to a fixed state in respiration, seeing that no animal can live long in a place where there is not a continual admiſſion of freſh air. Though ſome have aſſerted, that this is owing to expiration from the lungs, and the tranſpiration from the other parts of the body, which renders the air injurious to life, and unfit for reſpiration; others have ſaid it is owing to the air's being conſumed in reſpiration; others again, that it is owing to a vivifying ſpirit in air neceſſary for the preſervation of life; which being continually ſecreted by the lungs, and at laſt conſumed, puts a period to the life of the animal; but that there is no ſuch vivifying ſpirit in air, (ſee Hale's Statical Eſſays, Vol. I.) and I believe the death of the animal ſo ſhut up will be found entirely owing to the elaſticity of the air being deſtroyed. Otto Guerick, a German, by inventing an inſtrument (the air pump) about the middle of the laſt century, ſince much improved by Mr. Boyle, has taught us, that there is air contained in the moſt denſe and compact ſubſtances, whether animal, vegetable or mineral; and that this included air, by leſſening the preſſure of the atmoſphere (the action of the air-pump) will expand itſelf, and a quantity of it get out: for air occupies ſpaces which are in a reciprocal ratio of the compreſſing weight: from hence we may infer, that whatever will leſſen the elaſticity, or denſity of the atmoſphere, muſt alſo leſſen the quantity of air, or cauſe it to expand itſelf in all bodies, whether animal or other, and no perſon can conceive that a quantity of air can make its eſcape from an animal body, or expand itſelf in that body, without cauſing ſome conſiderable alteration. Now that there are things which leſſen the gravity of the atmoſphere is beyond contradiction, and that very conſiderable too; for the difference of the preſſure of the atmoſphere upon the ſurface of the human body at different times here in England, computed in a ratio of the weight of the quickſilver, in the torricellian tube, or barometer, is 3062 pounds. That the ſun and moon may be reckoned amongſt thoſe things which cauſe a diminution of gravity in the atmoſphere, will appear more than propable from the following conſideration. It is proved (ſee Dr. Halley's Theory of the Tides) that at particular periods, the ſun and moon conſiderably attract the waters of the ſea, even ſometimes $10+\frac{1}{2}$ perpendicular height; by which means it cauſes in the tides that continual circulation calculated to prevent the waters of the ſea from corrupting, which all ſtagnated waters are ſo liable to. The attractive powers of theſe two bodies are not equal; that of the moon exceeds that of the ſun, as $4+\frac{1}{2}:1$; hence the effect of the moon's influence is more apparent than that of the ſun; and they vary according to their greater or leſs diſtance from our earth; but their greateſt effect upon the waters is when they both conſpire in their attraction, which is every new and full moon; from theſe circumſtances ariſe all the variety of tides. Now ſeeing that air is a fluid as well as water, ſubject alſo to putrefaction from ſtagnation, and differing not materially from it but in being leſs denſe, and more elaſtic, is it not reaſonable to conclude that the atmoſphere has its regular tides as well as the ſea, but in a much greater degree in a ratio, as the denſity of water is to that of air, which is as 800 to 1. This ſuppoſition is greatly corroborated by the obſervation of the whole world; for change of weather is always expected at the new and full moon; and the vernal and autumnal equinoxes, have always been obſerved to be the moſt windy ſeaſons of the year. This being the caſe at certain periods of the moon, but more particularly at the full and new, for the reaſon mentioned above, the preſſure of the atmosphere will be lessened, consequently give leave to part of the elastic aura contained in the blood vessels or other parts of animals, to make its exit out of, or expand itself in them; which must cause some considerable intestine motion, by which intestine motion, supposing the lunatic to have the predisposing cause of lunacy already, his paroxisms may be brought about. Another cause that may contribute towards bringing on the fit may arise from the lungs, for we know from undoubted observation that they have at all times a considerable effect upon the blood; we know also that the lungs require air of a certain determinate gravity to expand them; and that if this gravity be considerably lessened, the animal will die. The truth of this we plainly perceive in going from low situations up to the tops of high mountains; though we cannot immediately say what effect this levity of air, at those periods before-mentioned, has upon the human frame; yet does it not seem probable that it contributes more or less to the bringing on of the paroxisms of lunatic people. When we consider again, as before observed, that there is elastic air contained in all vegetables and minerals, as well as animals, which must also at those periods breed intestine motion, and a quantity escape out of those vegetable and mineral substances, carrying with it particles of the same substances, it may not be too chimerical to conceive, that some one species of particles or other among so many, may, by being drawn into the body, stimulate the predisposing cause to lunacy, and by that means assist in bringing on the fit. I shall not attempt at present to carry this enquiry farther, as I am afraid I have taken up too much of your time. From all that has been said, it appears how necessary a knowledge of the other parts of philosopy, as well as the animal oeconomy, is for acquiring a good knowledge of physic. This is a subject I could wish to attend to, as I am confident it would lead to the discovery of the causes of many disorders unknown; and till we are better acquainted with those causes, we shall be but meanly qualified to cure, or even to alleviate those disorders on which the life and happiness of man so much depend.

Query 16, *by Philo.*

It is well known, from repeated observations, that in hot weather when the sun has shined for several days successively, the effect of the burning glass is much weaker than when the sun shines immediately after a shower. Required the reason of this phenomenon.

Solution, *by the Proposer.*

From experience it is well known, that heat exhales from the earth a prodigious number of sulphureous homogeneous particles, which by their gravity float in the atmosphere, absorb and prevent the incorporating rays from falling parallel, or with such great coalescence upon the mirror. Whence immediately after a shower, the rain precipitating the sulphureous particles purges the air of its absorbing matter, so that the numerous converging rays fall parallel upon the mirror, and are driven against the combustible body, with an incredible, superlative, inflammable force.

ARTICLE XII.

Mathematical Questions and Solutions.

Question 67, by *Mr. William Davis.*

A PERSON has a meadow in form of a parabola, containing an acre of ground, and its greatest ordinate is in ratio to its axis as 3 : 5. Required the length of a rope (one end of which is fixed in the vertex) whereby a horse is to be fixed so as to eat half an acre of grass; supposing the rope to be fixed to the horse's head?

Solution *by Mr. John Ryley, of Leeds.*

Let AVB (Fig. 1.) be the given parabolic field, and VDGE that part of it wherein the horse is to eat; then by the question $5 : 3 :: VC : \frac{3}{5} VC = CB$; therefore $\frac{6}{5} VC = AB$ the double ordinate, and $\frac{2}{3} VC \times \frac{6}{5} VC = \frac{4}{5} VC^2 = 4840$ square yards: hence $VC = \sqrt{6050} = 55\sqrt{2}$, and $AB = 66\sqrt{2}$.

Now put 2420 yards, the quantity to be cut off, $= a$, $AB = 2b$, $VC = c$, $GF = x$, and $FE = y$; then by the property of the circle $FV = \frac{y^2 - x^2}{2x}$; and by the property of the parabola $\frac{y^2 - x^2}{2x} : y^2 :: c : b^2$; by multiplying means and extremes $cy^2 = \frac{y^2 - x^2}{2x} \times b^2$; therefore $y^2 = \frac{b^2 x}{b^2 - 2cx}$. Moreover by Rule 4th, Page 104, Dr. Hutton's Mensuration, first Edition, the area of the segment DGE will be $\overline{2y + \frac{4}{3}\sqrt{x^2 + y^2}} \times \frac{2}{5} x$: and if the above-found value of y^2 be substituted in the expression for FV it becomes $\frac{cx^2}{b^2 - 2cx}$; therefore the area of the parabola VDC will be expressed by $\frac{4bcx^3}{3\sqrt{b^2 - 2cx} \times \overline{b^2 - 2cx}}$; and the area of the whole curvilinear figure VDGE, when y is expunged, becomes $\frac{4bcx^3}{3\sqrt{b^2 - 2cx} \times \overline{b^2 - 2cx}} + \frac{8bx^2}{10\sqrt{b^2 - 2cx}} +$

$\frac{16}{30} x \sqrt{\frac{2b^2x^2 - 2cx^3}{b - 2cx}} = a$. Now if this equation be multiplied by $15\sqrt{b^2 - 2cx}$ it will be transformed to $\frac{20bcx^3}{b^2 - 2cx} + 12bx^2 + 8x^2\sqrt{2b^2 - 2cx} = 15a\sqrt{b^2 - 2cx}$ which being put into numbers and solved, gives $x = 12.43$: hence by the property of the circle VG is found $= 54.465$ yards, the required length of the rope.

Question 68, *by a Friend to the Publication.*

Suppose a globe whose diameter is twelve inches, be suspended with its axis parallel to the horizon, and a thread of the dimensions of that mentioned in Question 23, Page 52, No. 2. to be wrapped round in the same manner; to determine. &c.

Solution, *by Mr. William Davis.*

Because the area's of segments are as their heights, this question may be solved in a similar manner to Question 23, No. 2. Thus $12 \times 3.1416 \times 12 = 452.3904 =$ the area of the globe; this divided by $.01 = 45239.04 =$ the length of the string; hence by Euclid's 47th and 1st. 45239.0015 inches is the distance gone over by the person.

Question 69, *by Mr. John Salter, of Bilston.*

Given $\overline{x+y} \times \sqrt[3]{z} = 1775$.
And $xyz = 28336$.
Also $\overline{x^3y^2z + x^2y^3} \times \sqrt[3]{z^4} = 178149848800$

Where x represents the year, y the month, and z the day of the month wherein I was born.

Solution, *by Mr. Salter, the Proposer.*

Divide the third equation by the product of the first and second, and you will have $xy = 3542$, and by the second $xy = 28336 \div z$. these values of xy being equated, z is found $= 8$, now z being known the first and second equations become $x + 2y = 1775$, and $xy = 3542$, whence by subtracting 8 times the latter from the square of the former, and extracting the root we get $x - 2y = 1767$, hence by addition and subtraction x is found $= 1771$ and $y = 2$, consequently I was born February 8th, 1771.

Question 70, *by Mr. James Stevenson.*

Given the solidity of a right hexagonal prism $= 20000$ inches, to determine its dimensions when its surface is a minimum.

Solution, *by Mr. Jonathan Mabbott, of Oldham.*

Put $a = 20000$, $b = 2.5980762$; and let x represent the length of the prism, and y = each side of its end, then per question $by^2x = a$, and $2by^2 + bxy =$ a minimum.—

From the first equation $x = a \div by^2$, which being substituted in the minimum, and $2by^2 + \frac{6a}{by} =$ a minimum, this in fluxions is $4by\dot{y} - \frac{6ab\dot{y}}{b^2y^2} = o$, this reduced is $4b^2y^3 = 6a$, hence $y =$ the cube root of $6a \div 4b^2 = 16.29$, and thence $x = 29.21$. W. W. R.

Question 71, *by Mr. Olinthus Gregory.*

Suppose Georgium Sidus revolves round the sun in a circular orbit, which is 19 times farther from the sun than the earth is, and that the earth goes through her orbit (which we here suppose circular) in 365.2565 days; now if the projectile motion of Georgium Sidus were destroyed, how long would it be falling to the sun—gravity alone acting?

Solution, *by Mr. Olinthus Gilbert Gregory.*

Thorp, in his Commentary on Newton's Principia, has (from prop. 36. book 1.) proved that the periodical time in a circle whose radius is SC, is to the time of the rectilinear descent (in the circumstances mentioned in the question) from C to S, as $4 \times \sqrt{2}$ to 1. (See Vol I. page 193.) Hence by Keplar's rule, as $1^3 : (365.2625)^2 :: 19^3 : 915075039.72404275$ the square root of which is 30250.2072 days the periodical time of Georgium Sidus: and consequently as $4 \times \sqrt{2}$ (or 5.65685424) : 1 :: 30250.2072 : 5347.5316 days, or nearly 5347 days, 12 hours, 45 minutes, the answer.

Question 72, *by Mr. Thomas Leybourn.*

How many forms of a geometrical progression may be made with 5 threes?

Solution, by *Mr. Stevenson, of Heath.*

$\frac{3}{3 \times 3^{3^{3}}} = \frac{1}{19683}$, $= \frac{3}{(3 \times 3 \times 3)^3} = \frac{1}{6561}$, $\frac{3 \times 3}{3^{3^{3}}} = \frac{1}{2187}$, $\frac{3^3}{3^{3^{3}}} = \frac{1}{729}$, $\frac{3}{3^3 \times 3^3} = \frac{1}{243}$, $\frac{3}{3 \times 3 \times 3^3} = \frac{1}{81}$, $\frac{3}{3 \times 3 \times 3 \times 3} = \frac{1}{27}$, $\frac{3 \times 3}{3 \times 3^3} = \frac{1}{9}$, $\frac{3 \times 3}{3 \times 3 \times 3} = \frac{1}{3}$, $\frac{3^3}{3 \times 3 \times 3} = 1$, $\frac{3 \times 3 \times 3}{3 \times 3} = 3$, $\frac{3 \times 3^3}{3 \times 3} = 9$, $\frac{3 \times 3 \times 3 \times 3}{3} = 27$, $\frac{3 \times 3 \times 3^3}{3} = 81$, $3 \times 3 \times 3 \times 3 \times 3 = 243$, $3^3 \times 3 \times 3 \times 3 = 729$, $3 \times 3^3 \times 3^3 = 2187$, $\frac{(3 \times 3 \times 3)^3}{3} = 6561$, $\frac{3 \times 3^{3^{3}}}{3} = 19683$, $3 \times (3 \times 3 \times 3)^3 = 59049$, $3 \times 3 \times 3^{3^{3}} = 177147$, $3^3 \times 3^{3^{3}} = 531441$, and $3 \times (3^3 \times 3)^3 = 1594323$: are 23 different forms, producing 23 terms in geometric progression; the common ratio being 3.

Question 73, *by Mr. John Lowry.*

Given the difference between each side and the adjacent segment of the base made by the perpendicular, and the difference between the perpendicular and radius of the inscribed circle to construct the triangle.

Solution, by *the Proposer.*

Construction. Let D and D′ be the given differences of the sides and their adjacent segments, take O*b* (Fig. 2) $= \frac{D-D'}{2}$ and perp. thereto erect *b*C = the given difference of the ⊥ and radius of the inscribed circle, join OC and upon OC describe the semicircle CD*b*O and apply CD $= \frac{D-D'}{2}$, with radius OD and center O describe the circle DPQ draw OQ ∥ C*b* and through Q draw the tangent AQB, produce CD to meet it in B and draw the tangent C*d*A to meet it in A, then is ABC the required △.

Demonstration. Because CD = C*d* $= \frac{D-D'}{2}$ and O*b* = QP $= \frac{D-D'}{2}$ ∴ CD + QP = D, and CD − PQ = D′; and since A*d* = AQ AC − AP will be = CD − PQ = D′, and BC − BP = CD + QP = D. And PC − QO = PC − P*b* = *b*C the given difference. Q. E. D.

Question 74, *by Mr. John Lowry.*

Given the vertical angle, one side, and the difference of the angles at the base of a spherical triangle to construct it.

Solution, *by the Proposer.*

Draw the great circles (Fig 3.) AD DC containing the given vertical angle, and having DC = the given side, make the ∠ D C P = the given difference of the angles at the base. About P as a pole with the distance PC describe the lesser circle ACB cutting DA in A, through the points A, C describe the great circle AC and ACD is the triangle required.

Demonstration. Because AP = PC ∴ ∠ A = ACP and ACD − A = DCP = the given difference by construction, CD = the given side, and ∠ CDA = the given vertical angle. Q. E. D.

Question 75, *by Mr. John Lowry.*

Given the vertical angle, and the side of the inscribed square, to construct the triangle, when the rectangle of the sides is to the square of the base in a given ratio.

Solution, *by the Proposer.*

Analysis. Suppose the thing done and ABC (Fig. 4.) the required triangle. From A upon CB demit the perpendicular AP and produce it to R so that AC may be to AR in the given ratio of AC. CB to AB^2. Then AC : AP :: AC. CB (Eu. 6. 1.) but AP . CB = CQ . AB ∴ AC : AP :: AC . CB : CQ . AB and AP : CQ . AB :: AR : AB^2, or AP : AR :: CQ : AB and since the vertical angle is given the ratio of AC to AP is given, and the ratio of AC to AR is given, therefore the ratio of AP to AR is given, and consequently the ratio of CQ to AB or EF to C*q*, is given, whence this

Construction. Upon EF = the side of the given inscribed square describe a segment of a circle capable of containing the given vertical angle, erect the ⊥ *db* and thereon take I*b* : EF :: AP : AR, draw IC ∥ EF meeting the circle in C, on EF describe the square DEFG, through the points EF draw CEA, CFB meeting DG produced both ways in A and B and the thing is done.

The same, *by Mr. Richard Nicholson, of Kirkbyoverblow.*

Construction. Make the angle EDF (Fig. 5.) = the given vertical ∠ C, and ED (the side of the given inscribed square) : DF :: the square of the base : rectangle of the sides, upon ED describe an arch to contain the ∠ C, and from a point where it cuts FG (*a* ∥ ED drawn from the point F) and through E, D, draw CA, CB meeting the line AB (*a* ∥, and at the distance ED from ED) in A and B, then ABC is the triangle required.

Demonstration. Join EF, then the △ EDF = ECD, consequently per construction EC . CD = ED . DF, but ED : DF :: ED^2 : ED. FD :: ED^2 : EC . CD :: AB^2 : AC . CB (per construction, and similar triangles) the given ratio, ED = DH = the side of the inscribed square and ACB the given vertical ∠ per construction. Q. E. D.

Limitation. When the ∠ EDC is greater than a right angle, or when the arch ECD does not touch or cut the line FG the problem is impossible.

Question 76, *by Mr. John Lowry.*

Given the base, the difference of the sides, and the radius of the inscribed circle of a spherical triangle to construct it.

Solution, *by the Proposer.*

Construction. Let AB (Fig. 6) be the given base, and D = the difference of the sides, take $QA = \frac{AB + D}{2}$ and at Q erect the ⊥ QO = the given radius and about O as a pole with distance OQ describe the lesser circle QPD, from the points A, B draw the great circles APC, BDC to touch the lesser circle in P and D and meet in C then is ABC the required triangle.

Demonstration. By the nature of tangents PC = CD, AP = AQ, and DB = QB ∴ AC − CB = AQ − QB, but AQ = $\frac{AB - D}{2}$ per Const. ∴ QB = $\frac{AB - D}{2}$ and AQ − QB = D. And AB is the given base and OQ = the given radius. Q. E. D.

Question 77, *by Mr. John Lowry.*

Given the line bisecting the vertical angle, the ⊥, and the radius of the inscribed circle of a spherical triangle to construct it.

Solution, *by the Proposer.*

Construction. Upon the given ⊥ BP (Fig. 7.) construct the right-angled triangle PBQ having the side BQ = the given bisecting line: again construct the right-angled △ QOD the ∠ Q being given and the side OD = the given radius; about O as a pole, with distance OD describe the lesser circle D*mn*, from the point B draw the great circles B*m*A, B*n*C meeting PQ produced in A and C and touching the lesser circle in *m* and *n*, then ABC is the required triangle. This I presume is too manifest to need a particular demonstration in this place.

Question 78, *by Mr. John Lowry.*

Given the radius of the circumscribing circle of a spherical triangle, and the segments of the base made by the ⊥ to construct it.

Solution, *by the Proposer.*

Construction. Let BP, PC (Fig 8.) be the given segments of the base, bisect BC in Q, and upon BQ describe the right-angled triangle BQO having the side BO = the given radius, and about O as a pole with distance BO describe the lesser circle BAC, at P erect the ⊥ great circle PA cutting the lesser circle in A; lastly, describe the arches AB, AC, and ABC is the required triangle. This construction is obvious from P. 53, page 194, Emerson's Trigonometry.

Question 79, *by Mr. O. G. Gregory.*

Having the axis of a sphere; or the fixed and revolving axes of a spheroid; or the altitude and diameter of the base of a paraboloid; or the dimensions of the base, and the height of any prism or pyramid, to determine the content of either of them in ale and wine gallons, malt bushels, and cubic feet, at once setting the slide on the gauging rule.

Solution, *by the Proposer.*

To the {diameter / side} of the base, on the line D, set the length on the slide, (put in an inverted manner) and against the respective gauge points on D, you will find the contents in ale and wine gallons, &c. on the inverted slide.

Our next business is to find the gauge points; in order to which we must proceed as follows:

$$\left.\begin{array}{l} \sqrt{\dfrac{282}{\cdot 7854}} = 18\cdot 95 \text{ for ale-gallons} \\ \sqrt{\dfrac{231}{\cdot 7854}} = 17\cdot 15 \text{ for wine gallons} \\ \sqrt{\dfrac{2150.4}{\cdot 7854}} = 52\cdot 32 \text{ for malt bushels} \\ \sqrt{\dfrac{1728}{\cdot 7854}} = 46\cdot 9 \text{ for cubic feet} \end{array}\right\} \text{Gauge points for cylinders.}$$

And by dividing 282, 231, &c. by the respective factors in a table of regular polygons, and extracting the square roots of the quotients; we shall have gauge points for prisms whose bases are any regular polygons, as in the following table:

Prism whose base is A.	Gauge Points for Ale Gallons.	Gauge Points for Wine Gallons.	Gauge Points for Malt Bushels.	Gauge Points for Cubic Feet.
Trigon	25.52	23.097	70.47	63.18
Tetragon or square	16.79 +	15.185	46.37	41.57
Pentagon	12.8	11.59	35.35	31.7
Hexagon	10.42	9.43	28.77	25.79
Heptagon	8.8	7.97 +	24.33	21.8
Octagon	7.641	6.92	21.1	18.92
Nonagon	6.755	6.11	18.65	16.72
Decagon	6.05	5.48	16.71	14.99 +
Enneagon	5.48	4.97	15.15	13.58
Duodecagon	5.02	4.54	13.86	12.42

It is entirely unnecessary to find gauge points for the sphere; for, as the sphere is $\frac{2}{3}$ of the circumscribing cylinder, we have only to set $\frac{2}{3}$ of the axis on the inverted slide, to the axis on D; and the same gauge points will answer as in the cylinder. In like manner, because pyramids and cones are $\frac{1}{3}$ of prisms of the same base and altitude, we shall have to set $\frac{1}{3}$ of the altitude on the inverted slide to $\left\{\begin{array}{l}\text{a side}\\\text{the diameter}\end{array}\right\}$ of the base on D; and against the gauge points for prisms of similar bases on D, will be the contents on the inverted slide. The content of a spheroid may be found by setting $\frac{2}{3}$ of the fixed axe on the slide inverted against the revolving axe on D, and using the gauge points for the cylinder. The content of a paraboloid may be found by setting ½ the altitude, on the slide inverted against the diameter of the base on D, and using the gauge points for the cylinder.

Example 1. The diameter of a cylinder is 40, and the length 60 inches. Required the content in ale gallons, &c.?

	on D.		on Slide.	
Set 60 on the slide inverted to 40 on D, and against	18.95	are	267.37	ale gallons
	17.15	—	326.4	wine gallons
	52.32	—	35.06	malt bushels
	46.9	—	43.63	cubic feet

Example 2. Each side of the base of a square prism is 40, and the length 60. Required the content in ale gallons, &c.?

	on D.		on Slide	
Against 40 on D set 60 on the slide inverted, and against	16.79	are	340.4	ale gallons
	15.185	—	415.6	wine gallons
	46.37	—	44.6	malt bushels
	41.57	—	55.55	cubic feet.

Example 3. The axis of a sphere is 60 inches. What's the content in ale and wine gallons, &c.?

	on D.		on Slide.	
Against 60 on D set ⅔ of 60, or 40, on the slide inverted, and against	18.95	are	401.06	ale gallons
	17.15	—	489.6	wine gallons
	52.32	—	52.6	malt bushels
	46.9	—	65.45	cubic feet.

Example 4. Each side of the base of an hexagonal pyramid is 10 inches, and the altitude is 60. Quere the content in A. G. &c.?

	on D.		on Slide.	
Against 10 on D set ⅓ of 60 (viz. 20) on the slide inverted, and against	10.42	are	18.43	ale gallons
	9.43	—	22.5	wine gallons
	28.77	—	2.42	malt bushels
	25.79	—	3.00	cubic feet.

Question 80, *by Mr. O. G. Gregory.*

There is a circle, the sum of the areas of whose circumscribed and inscribed squares is 121.5. Required the area of the circle, the dimensions and area of its greatest inscribed right-angled triangle, also the dimensions and area of the inscribed duodecagon?

Solution, *by the Proposer.*

By corollary to Prop. 7. Book 5, Martin's Euclid "The square circumscribed about a circle, is double the square inscribed in the same circle." Hence $\sqrt{\frac{121.5\times 2}{3}}$ equal $\sqrt{81} = 9$, side of the circumscribed square, which also is evidently the diameter of the circle, and therefore its area is $9^2 \times \cdot 785398 = 63.617238$. The hypothenuse of the greatest inscribed right-angled triangle is $= 9$, the circle's diameter; and fluxional writers shew that the greatest right-angled triangle to a given hypothenuse is when the base is $=$ to the $\perp$, therefore in the case before us, it is obvious that the area of the inscribed right-angled triangle will be half the inscribed square, viz. $\frac{121.5}{3} \div 2 = \frac{40.5}{2} = 20.25$; and its base and perpendicular will each be equal to $\sqrt{(40.5)} = 6.36396$. The side of the inscribed duodecagon (by Table 2, p. 144, Robertson's Mensuration, Ed. 2) is $\frac{\cdot 5176381 \times 9}{2} = 2.3293714$; and its area is $\frac{9}{2} \times \frac{9}{2} \times 3 = 60.75$.

Question 81, *by Mr. W. Armstrong, Pupil in Mr. Howard's School, Carlisle.*

A gentleman dying, left by will a field in form of a parabola, between his son and daughter, the land was divided into two equal parts by a hedge drawn $\parallel$ to the base, on account of their different qualities, viz. the part next the vertex was worth 20 and the other 8s. per acre, but he wished to have it divided between them by a hedge drawn $\parallel$ to the abscissa, and that the value of the son's part might be double that of the daughter's; now supposing the abscissa to be 18, and that a line drawn from the corner of the base to the focus measure 20 chains, at what distance from the abscissa must the line of division be drawn?

Solution, by *the Proposer.*

Let F (Fig. 9) $=$ focus, draw GQ $\parallel$ to BD and let GO or QD represent the required distance. Put $a = 18 = $ BD, $b = 20 = $ AF, $m = 20$ and $n = 8$, also put $x =$ IR, $y = $ GQ and $z = $ QD $=$ GO, and call p the parameter, then, per Conics, $b - a = $ BF and DF $= 2a - b$ $\therefore$ (47. e. 1) AD $= \sqrt{b^2 - \overline{2a-b}|^2}$ which call c; also $c^2 : a :: x^2 : \frac{ax^2}{c^2} = $ BR $\therefore$ 2IR $\times \frac{2}{3}$ BR $= \frac{4ax^3}{3c^2} = \frac{2ac}{3}$ per question, or $x = c\sqrt[3]{\frac{1}{2}}$ whence BR $= a\sqrt[\frac{2}{3}]{\frac{1}{2}}$ and DR $= a - a\sqrt[\frac{2}{3}]{\frac{1}{2}}$ which call s, again, per Conics, $p : c + z :: c - z : y = \frac{c^2 - z^2}{p}$ $\therefore$ BO $= a - y$ whence $\frac{2z}{3} \cdot \overline{a-y} = $ area GOB and $yz = $ DO.DQGO hence $\frac{2ac}{3} - \overline{yz \quad \frac{2az - 2yz}{3}} = $ DO.AGQ, also GS $= y - s$, $\therefore$ $yz - sz = $ DO.RSGO whence $\frac{ac}{3} - \overline{yz - sz + \frac{2az - 2yz}{3}} = $ DO.IGS and $\frac{2ac}{3} - \overline{yz + \frac{2az - 2yz}{3}} = \frac{ac}{3} + yz - sz + \frac{2az}{3} - \frac{2yz}{3} = \frac{ac}{3} - sz = $ DO.AISQ $\therefore$

the value of the daughter's part is $\frac{2acm + 2acn}{9} = \frac{acn}{3} - nsx + \frac{acm}{3}$ − [illegible] + [illegible] $- \frac{2amx}{3} + \frac{2mpx}{3}$ this reduced is $x^3 - cx^2 + 3psx - \frac{3npsx}{m} - 2apx = \frac{acp}{3} - \frac{acpn}{3m}$ or in numbers $x^3 - 336{\cdot}025x = -806{\cdot}4$ and $x = 2{\cdot}443$ the diſtance required.

The ſame, by *Mr. Nicholſon, of Kirkbyoverblow.*

The line drawn from the focus to the corner of the baſe will be = the diſtance of the baſe and directrix. Conſequently $20 - 18 = 2$ chains = the diſtance of the vertex from the focus: $\sqrt{(18 \times 8)} = 12$ chains = half the baſe, and $24 \times 18 \times \frac{2}{3} = 288 =$ the area, ∴ the area of the two equal parts are each 144 ſquare chains. Alſo $288 : 12^3 :: 144 : 864$, the cube root of which is 9.52 chains = half the line which divides the field into two equal parts and it croſſes the abſciſs $9.52^2 \div 8 = 11.33$ chains from the vertex, and $18 - 11.33 = 6.67$ chains from the baſe. Now put $x =$ the required line of diviſion from the abſciſſa to which it will be an ordinate, then will $x^2 \div 8$ be = the abſciſſa to that ordinate x, and half the area of that parabola will be $x^2 \div 8 \times 2 \div 3 \times x = x^3 \div 12$, and the area of the parallelogram formed by the diſtance of the required line of diviſion from the abſciſſa and the line which divides the parabola into two equal parts will be $(11.33 - \frac{1}{8} x^2) \times x = 11.33x - \frac{1}{8} x^3$, and the area of the parallelogram formed by the ſaid diſtance and the diſtance between the baſe and the other ſaid line is $6.67x$, hence we get this equation:

$\overline{72 - \frac{1}{12} x^3 - \overline{11.33x} + \frac{1}{8} x^3} \times 20 + \overline{72 - 6.67x} \times 8 \times 2 = \overline{72 + \frac{1}{12} x^3 + 11.33x - \frac{1}{8} x^3} \times 20 + \overline{72 - 6.67x} \times 8$, which put into numbers and reduced we get $2016 + 2.5x^3 = 839.88x$, hence $x = 2$ chains, 44 links the diſtance required.

Queſtion 82, by *Mr. W. Armſtrong.*

Given the difference of the ſegments of the baſe made by a perpendicular, the difference of the ſquares of the two ſides, and the difference of the angles at the baſe to conſtruct the triangle.

Solution, by *the Propoſer.*

Conſtruction. Having drawn (Fig. 10) AD = the difference of the ſegments, and made S = the ſide of a ſquare = the difference of the ſquares given, take AB a third proportional to AD and S; biſect DB in B at which erect the perpendicular CP, then on AD deſcribe a ſegment of a circle, to contain an angle = to the given difference where this interſects the perpendicular gives the vertex of the triangle.

Demonſtration. Since DB is biſected by the Δ CDB is iſoſceles and ∴ ∠ CDB = ∠ CBD but ∠ ACD = CDB − CAD = ∠ CBD − ∠ CAD = the given difference. Let AE be taken = the difference of the ſides, then (by II. 13, Bon.Geo.) the difference of the squares of AC and CB is $=$ AE. $\overline{AC+CB}$ and by the Cor. II. 16, AD.AB $=$ AE. $\overline{AC+CB}$ that is AD : $\sqrt{AE.\overline{AC+CB}}$ :: $\sqrt{AE.\overline{AC+CB}}$: AB. Q. E. D.

Note. When the circle cuts the perpendicular in two points Cc the problem is capable of two solutions.

The same, by *Mr. Richard Nicholson, of Kirkbyoverblow.*

Geom. Anal. Suppose ABC the Δ, BD the $\perp$, the base AC; (Fig. 11) then by a known theorem $AB^2 - BC^2 = AD^2 - DC^2 = AC.(AD - DC) =$ a given rectangle; but AD $-$ DC is given; therefore AC is given, which bisect in E; then ED ($\frac{1}{2}$ the difference of the segments of the base) $+$ AE $=$ AD, and AE $-$ ED $=$ DC, the segments of the base are given. Now take DH $=$ DC, and let BG bisect the $\angle$ ABC, then DHB (DCB) $-$ DAB $=$ ABH $=$ 2GBD, hence this

Construction. Take AC $=$ the given difference of the squares of the sides divided by the difference of the segments of the base, which bisect in E, take ED $=$ half the last-mentioned difference, make DH $=$ DC, then upon AH the difference of the segments of the base, describe a circle to contain the given difference of the angles at the base; then from the point of intersection of which and the $\perp$ BD, join AB, BC, so will ABC be the Δ, which is evident from the above Analysis.

Limitation. When the circle cuts the $\perp$ in more than one point, the problem admits of two solutions, when it neither cuts or touches the $\perp$ the problem is impossible.

Question 83, *by Mr. Archibald Gilchrist, Bombardier, Royal Artillery.*

Given the chord AB, and the versed sines of the two segments of circles AEB, and AGB to draw a line FE, so that FG, GE, may obtain a given ratio.

Solution, by *Mr. John Lowry.*

This problem admits of different cases according as the given segments are greater or less than semi-circles.

Case 1. When they are both greater. (Fig. 12) Let O and C be the centers of the given circles, and *m* to *n* the given ratio of FG to GE. Through the centre draw the line QOCIL bisecting AB in Q and cutting the circles in I and L, draw QR $\parallel$ AB and produce it to *b*, so that $Cb^2 = CL^2 - OI^2$, draw *bs* $\parallel$ QIL, and with radius *bf* $=$ CQ describe the arch *rf*, take *bg* : *bf* :: *m* : *m*+*n*, draw QGE to meet the arch in *f*, through *b* draw *fbd* meeting QL in *d*, draw *d*G $\parallel$ AB to cut the circle in G, and lastly, through G draw EGF $\parallel$ LIQ and the thing is done.

Demonstration. Draw CP $\parallel$ AB and draw the radii OG, CE then by reason of the $\parallel$ lines OR $=$ CP, hence $PE^2 - RG^2 = CE^2 - OG^2 = CL^2 - OI^2 = Ob^2$ by Construction. But O*d* $=$ RG and $bd^2 - dO^2 = Ob^2$ $\therefore$ $PE^2 - RG^2 = bd^2 - RG^2$ or PE $=$ *bd*, and by Construction *bf* $=$ CQ $=$ PE, $\therefore$ *df* $=$ EF and *d*Q $=$ GF, and by sim. Δs and Construction $dQ(=GF) : df(=BF) :: bg : bf :: m+n : m$, or, $FG : EG :: m : n$.

Case. 2. When they are both less. (Fig. 13) Take $Cb^2 = OI^2 - CL^2$, draw $bs \parallel LIQ$, with radius $bf = QO$ describe the arch fr, take $bg : bf :: m+n : m$, draw Qfg to cut the arch in f, through f draw bfd meeting QIL in d, draw $dE \parallel AB$ cutting the circle in E, and lastly, draw $EGF \parallel LIQ$ and the thing is done.

Demonstration. By reasoning as before $RG^2 - PE^2 = Cb^2 = db^2 - dc^2$, but $dC = PE \therefore bd = RG$, and by Construction $bf = QO = RE \therefore df = GF$ and $dQ = EF$, and per Construction and similar Δs $dQ(=EF) : df(=CE) :: bQ : bf :: m+n : m$, or $GE : GF :: n : m$.

Case. 3. When one is greater and the other less. The construction and demonstration of this Case is the very same as Case 2, making use of Fig. 14, instead of 13.

Cor. In this last Case, OI may be $=$ CL and then this Construction fails; but in that case it may be easily constructed from what is done above.

Question 84, by *J. P. O. Sullivan, Esq.*

From the following equations be pleased to try
If you can find x equal four times y.

$$x + y + x^3 \div y = 144, \text{ and } x^2 + y^2 + x^5 \div y^2 = 4608.$$

Solution, by *Mr. Jonathan Mabbott.*

Put $a = 144$, $b = 4608$, and let x be to y as 1 to m. Then $y = mx$, substitute this value of y in the room of its equal in the given equations, and they will become $x + mx + \frac{x^2}{m} = a$, and $x^2 + m^2x^2 + \frac{x^3}{m^2} = b$; from the first equation $x = \sqrt{am + \left(\frac{m^2+m}{2}\right)^2} - \frac{m^2+m}{2}$: which wrote in the second $\overline{m^4+m^2} \times \left(\sqrt{am + \left(\frac{m^2+m}{2}\right)^2} - \frac{m^2+m}{2}\right)^2 + \left(\sqrt{am + \left(\frac{m^2+m}{2}\right)^2} - \frac{m^2+m}{2}\right)^3 = bm^2$, from which equation $m = 4$, therefore $y = 4x$, which was required.

Question 85, *by Mr. Thomas Leybourn.*

Required the area of a curve whose equation is $(1 - x^{60}).y^{n+1}x + (x^{38} - x^{30} - x^8 - 1).y^n x^{23} + (1 - x^{60}).y = (1 + x^{30} + x^8 - x^{38}).x^{22}$.

Solution, by *the Proposer.*

The given equation being multiplied out, transposed, divided by $y^n x + 1$ and then reduced, we obtain $y = \frac{x^{32}}{1 - x^{30}} + \frac{x^{40}}{1 + x^{30}}$ and the fluxion of the area is $y\dot{x} = \frac{x^{12}\dot{x}}{1 - x^{30}} + \frac{x^{40}\dot{x}}{1 + x^{30}}$; but $\frac{x^{32}\dot{x}}{1 - x^{30}} = \frac{x^2\dot{x}^2}{1 - x^{30}} - x^2\dot{x}$; and $\frac{x^{40}\dot{x}}{1 + x^{30}} = x^{10}\dot{x} - \frac{\lambda^{10}\dot{x}}{1 + x^{30}}$, (Fig. 15) $\therefore y\dot{x} = \frac{x^2\dot{x}}{1 - x^{30}} - \frac{x^{10}\dot{x}}{1 + x^{30}} + x^{10}\dot{x} - x^2\dot{x}$. Now to find the fluent of the first term, the fluxion corresponding is that in Problem 5, page 371, Simpson's Fluxions, where, by comparing the forms $\frac{x^{m-1}\dot{x}}{r^n - x^n} \Big| \frac{x^2\dot{x}}{1 - x^{30}}$ we have $m = 3$, $n = 30$, $r = 1$, $x = x$, $b =$ cosine of $0° = 1$, $c =$ cosine $\frac{360°}{30} = 12° = .9781476$, $d =$ cosine $24° = .9135455$, $e =$ cosine $36° = .8090170$, $f =$ cosine $48° = .6691306$, $g =$ cosine $60° = .5$, $h =$ cosine $72° = .3090170$, $i =$ cosine $84° = .1045285$, $k =$ cosine $96° = -.1045285$, $l =$ cosine $108° = -.3090170$, $m =$ cosine $120° = -.5$, $n =$ cosine $132° = -.6691306$, $o =$ cosine $144° = -.8090170$, $p =$ cosine $156° = -.9135455$, $q = 168° = -.9781476$, $r =$ cosine $180° = -1$, $s =$ cosine $192° = -.9781476$, $t =$ cosine $204° = -.9135455$, $u =$ cosine $216° = -.8090170$, $v =$ cosine $228° = -.6691306$, $w =$ cosine $240° = -.5$, $z =$ cosine $252° = -.3090170$, B = cosine $264° = -.1045285$, C = cosine $276° = .1045285$, D = cosine $288° = .3090170$, E cosine $300° = .5$, F = cosine $312° = .6691306$, G = cosine $324° = .8090170$, H = cosine $336° = .9135455$, I = cosine $348° = .9781476$; $R = m \div n \times 360 = 36°$. Sine R = .5877853, cosine R = .8090170; sine 2 R = .9510565, cosine 2 R = 3090170; sine 3 R = .9510565, cosine 3 R = −.3090170, sine 4 R = .5877853, cosine 4 R = −.8090170; sine 5 R = 0, cosine 5 R = −1; sine 6 R = −.5877853, cosine 6 R = .8090170; sine 7 R = −.9510565, cosine 7 R = −.3090170; sine 8 R = −.9510565, cosine 8 R = .3090170; sine 9 R = −.5877853, cosine 9 R = .8090170; sine 10 R = 0, cosine 10 R = 1; sine 11 R = .5877853, cosine 11 R = .8090170; sine 12 R = .9510565, cosine 12 R = .3090170; sine 13 R = .9510565, cosine 13 R = −.3090170; sine 14 R = .5877853, cosine 14 R = −.8090170; sine 15 R = 0, cosine 15 R = −1; sine 16 R = −.5877853, cosine 16 R — .8090170; sine 17 R = −.9510565, cosine 17 R = −.3090170; sine 18 R = −.9510565, cosine 18 R = .3090170; sine 19 R = −.5877853. cosine 19 R = .8090170; sine 20 R = 0, cosine 20 R = 1; sine 21 R = .5877853, cosine 21 R = .8090170; sine 22 R = .9510565, cosine 22 R = .3090170; sine 23 R = .9510565, cosine 23 R = −.3090170; sine 24 R = −.5877853, cosine 24 R = −.8090170; sine 25 R = 0, cosine 25 R = −1; sine 26 R = —

.5877853, cosine 26 R $= -.8090170$; sine 27 R $= -.9510565$, cosine 27 R $= -.3090170$; sine 28 R $= -.9510565$, cosine 28 R $= .3090170$; sine 29 R $= -.5877853$, cosine 29 R $= .8090170$; —, $Q = 0$, $\overset{\mathrm{I}}{Q} = 12^\circ$, $\overset{\mathrm{II}}{Q} = 24^\circ$, $\overset{\mathrm{III}}{Q} = 36^\circ$, $\overset{\mathrm{IV}}{Q} = 48^\circ$, $\overset{\mathrm{V}}{Q} = 60^\circ$, $\overset{\mathrm{VI}}{Q} = 72^\circ$, $\overset{\mathrm{VII}}{Q} = 84^\circ$, $\overset{\mathrm{VIII}}{Q} = 96^\circ$, $\overset{\mathrm{IX}}{Q} = 108^\circ$, $\overset{\mathrm{X}}{Q} = 120^\circ$, $\overset{\mathrm{XI}}{Q} = 132^\circ$, $\overset{\mathrm{XII}}{Q} = 144^\circ$, $\overset{\mathrm{XIII}}{Q} = 156^\circ$, $\overset{\mathrm{XIV}}{Q} = 168^\circ$, $\overset{\mathrm{XV}}{Q} = 180^\circ$, $\overset{\mathrm{XVI}}{Q} = 192^\circ$, $\overset{\mathrm{XVII}}{Q} = 204^\circ$, $\overset{\mathrm{XVIII}}{Q} = 216^\circ$, $\overset{\mathrm{XIX}}{Q} = 228^\circ$, $\overset{\mathrm{XX}}{Q} = 240^\circ$, $\overset{\mathrm{XXI}}{Q} = 252^\circ$, $\overset{\mathrm{XXII}}{Q} = 264^\circ$, $\overset{\mathrm{XXIII}}{Q} = 276^\circ$, $\overset{\mathrm{XXIV}}{Q} = 288^\circ$, $\overset{\mathrm{XXV}}{Q} = 300^\circ$, $\overset{\mathrm{XXVI}}{Q} = 312^\circ$, $\overset{\mathrm{XXVII}}{Q} = 324^\circ$, $\overset{\mathrm{XXVIII}}{Q} = 336$, $\overset{\mathrm{XXIX}}{Q} = 348^\circ$.

Hence $M =$ H. L. $\sqrt{1 - 26x + x^2} =$ H. L. $\sqrt{1 - 2x + x^2} =$ H. L. $\overline{1 - x}$; $\overset{\mathrm{I}}{M} =$ H. L. $\sqrt{1 - 2cx + x^2} =$ H. L. $\sqrt{1 - 1.9562952x + x^2}$; $\overset{\mathrm{II}}{M} =$ H. L. $\sqrt{1 - 2dx + x^2} =$ H. L $\sqrt{1 - 1.8270910x + x^2}$; $\overset{\mathrm{III}}{M} =$ H. L. $\sqrt{1 - 2ex + x^2} =$ H. L. $\sqrt{1 - 1.6180340x + x^2}$; $\overset{\mathrm{IV}}{M} =$ H. L. $\sqrt{1 - 2fx + x^2} =$ H.L. $\sqrt{1 - 1.3382612x + x^2}$; $\overset{\mathrm{V}}{M} =$ H. L. $\sqrt{1 - 2gx + x^2} =$ H. L. $\sqrt{1 - x + x^2}$; $\overset{\mathrm{VI}}{M} =$ H. L. $\sqrt{1 - 2hx + x^2} =$ H. L. $\sqrt{1 - .7080340x + x^2}$; $\overset{\mathrm{VII}}{M} =$ H. L. $\sqrt{1 - 2ix + x^2} =$ H. L. $\sqrt{1 - .2090570x + x^2}$; $\overset{\mathrm{VIII}}{M} =$ H. L. $\sqrt{1 - 2kx + x^2} =$ H.L. $\sqrt{1 + .2090570x + x^2}$ $\overset{\mathrm{IX}}{M} =$ H. L. $\sqrt{1 - 2lx + x^2} =$ H. L. $\sqrt{1 + .7080340x + x^2}$; $\overset{\mathrm{X}}{M} =$ H.L. $\sqrt{1 - 2mx + x^2} =$ H. L. $\sqrt{1 + x + x^2}$; $\overset{\mathrm{XI}}{M} =$ H. L. $\sqrt{1 - 2nx + x^2} =$ H.L. $\sqrt{1 + 1.3382612x + x^2}$; $\overset{\mathrm{XII}}{M} =$ H. L. $\sqrt{1 + 2xo + x} =$ H. L. $\sqrt{1 + 1.6180340x + x^2}$; $\overset{\mathrm{XIII}}{M} =$ H. L. $\sqrt{1 - 2px + x^2} =$ H. L. $\sqrt{1 + 1.8270910x + x^2}$; $\overset{\mathrm{XIV}}{M} =$ H.L. $\sqrt{1 + 1.9562952x + x^2} =$ H.L.($\sqrt{1 - 2qx + x^2}$); $\overset{\mathrm{XV}}{M} =$ H. L. $\sqrt{1 - 2rx + x^2} =$ H. L. $\sqrt{1 + 2x + x^2} =$ H. I. $\overline{1 + x}$; $\overset{\mathrm{XVI}}{M} =$ H. L. $\sqrt{1 - 2sx + x^2} =$ H. L. $\sqrt{1 + 1.9562952x + x^2}$; $\overset{\mathrm{XVII}}{M} =$ H. L. $\sqrt{1 - 2tx + x^2} =$ H. L. $\sqrt{1 + 1.8270910x + x^2}$; $\overset{\mathrm{XVIII}}{M} =$ H. L. $\sqrt{1 - 2ux + x^2} =$ H. L. $\sqrt{1 + 1.6180340x + x^2}$; $\overset{\mathrm{XIX}}{M} =$ H. L. $\sqrt{1 - 2vx + x^2} =$ H. L. $\sqrt{1 + 1.3382612x + x^2}$; $\overset{\mathrm{XX}}{M} =$ H. L. $\sqrt{1 - 2wx + x^2} =$ H. L. $\sqrt{1 + x + x^2}$; $\overset{\mathrm{XXI}}{M} =$ H. L. $\sqrt{1 + .7080340x + x^2}$, $=$ H. L. $\sqrt{1 - 2zx + x^2}$; $\overset{\mathrm{XXII}}{M} =$ H. L. $\sqrt{1 - 2Bx + x^2} =$ H. L. $\sqrt{1 + .2090570x + x}$; $\overset{\mathrm{XXIII}}{M} =$ H. L. $\sqrt{1 - 2Cx + x^2} =$

H. L. $\sqrt{1-.2090570x+x^2}$; $\overset{xxiv}{M}$ = H. L. $\sqrt{1-2Dx+x^2}$ = H. L. $\sqrt{1-.7080340x+x^2}$; $\overset{xxv}{M}$ = H. L. $\sqrt{1-2Ex+x^2}$ = H. L. $\sqrt{1-x+x^2}$; $\overset{xxvi}{M}$ = H. L. $\sqrt{1-2Fx+x^2}$ = H. L. $\sqrt{1-1.3382612x+x^2}$; $\overset{xxvii}{M}$ = H. L. $\sqrt{1-2Gx+x^2}$ = H. L. $\sqrt{1-1.6180340x+x^2}$; $\overset{xxviii}{M}$ = H. L. $\sqrt{1-2Hx+x^2}$ = H. L. $\sqrt{1-1.8270910x+x^2}$; $\overset{xxix}{M}$ = H. L. $\sqrt{1-2Ix+x^2}$ = H. L. $\sqrt{1-1.9562952x+x^2}$;

N = cir. arc. rad. 1 and ſine ($x \times$ ſine Q ÷ $\sqrt{1-2bx+x^2}$) = o; ($\overset{i}{N}$ = cir. arc. rad. 1 and ſine (x. ſine $\overset{i}{Q}$ ÷ $\sqrt{1-2cx+x^2}$ = $.2079117x$ ÷ $\sqrt{1-1.9562952x+x^2}$; $\overset{ii}{N}$ = cir. arc. rad. 1 and ſine (x. ſine $\overset{ii}{Q}$ ÷ $\sqrt{1-2dx+x^2}$ = $.4067366x$ ÷ $\sqrt{1-1.8270910x+x^2}$;) $\overset{iii}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{iii}{Q}$ ÷ $\sqrt{1-2ex+x^2}$ = $.5877853x$ ÷ $\sqrt{1-1.6180340x+x^2}$;) $\overset{iv}{N}$ = cir. arc. rad. 1 and ſine (x. ſine $\overset{iv}{Q}$ ÷ $\sqrt{1-2fx+x^2}$ = $.7431448x$ ÷ $\sqrt{1-1.3382012x+x^2}$;) $\overset{v}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{v}{Q}$ ÷ $\sqrt{1-2gx+x^2}$ = $.8660254x$ ÷ $\sqrt{1-x+x^2}$;) $\overset{vi}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{vi}{Q}$ ÷ $\sqrt{1-2hx+x^2}$ = $.9510565x$ ÷ $\sqrt{1-.7080340x+x^2}$;) $\overset{vii}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{vii}{Q}$ ÷ $\sqrt{1-2ix+x^2}$ = $.9945219x$ ÷ $\sqrt{1-2090570x+x^2}$) $\overset{viii}{N}$ = cir. arc. rad. 1 and ſine (x. ſine $\overset{viii}{Q}$ ÷ $\sqrt{1-2kx+x^2}$ = $.9945219x$ ÷ $\sqrt{1+.2090570x+x^2}$;) $\overset{ix}{N}$ = .cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{ix}{Q}$ ÷ $\sqrt{1-2lx+x^2}$ = $.9510565x$ ÷ $\sqrt{1+.7080340x+x^2}$;) $\overset{x}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{x}{Q}$ ÷ $\sqrt{1-2mx+x^2}$ = $.8660254x$ ÷ $\sqrt{1+x+x^2}$;) $\overset{xi}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xi}{Q}$ ÷ $\sqrt{1-2nx+x^2}$ = $.7431448x$ ÷ $\sqrt{1+1.3382612x+x^2}$) $\overset{xii}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xii}{Q}$ ÷ $\sqrt{1-2ox+x^2}$ = $.5877853x$ ÷ $\sqrt{1+1.6180340x+x^2}$;) $\overset{xiii}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xiii}{Q}$ ÷ $\sqrt{1-2px+x^2}$ = $.4067366x$ ÷ $\sqrt{1+1.8270910x+x^2}$;) $\overset{xiv}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xiv}{Q}$ ÷ $\sqrt{1-2qx+x^2}$ = $.2079117x+x^2$ ÷ $\sqrt{1+1.9562952x+x^2}$;) $\overset{xv}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xv}{Q}$ ÷ $\sqrt{1-2rx+x^2}$ = o;) $\overset{xvi}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xvi}{Q}$ ÷ $\sqrt{1-\text{[illegible]}x+x^2}$ = $-.2079117x$ ÷ $\sqrt{1+1.9562952x+x^2}$;) $\overset{xvii}{N}$ = cir. arc. rad. 1 and ſine ($x \times$ ſine $\overset{xvii}{Q}$ ÷ $\sqrt{1-2tx+x^2}$ = $-.406766x$ ÷

$\sqrt{1+1.8270910x+x^2};)\ \overset{xviii}{N} =$ cir. arc. rad. 1 and sine $(x \times \overset{xviii}{Q} \div \sqrt{1-2ux+x^2}$ $= -.5877853\,x \div \sqrt{1+1.6180340x+x^2};)\ \overset{xix}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xix}{Q} \div \sqrt{1-2vx+x^2} = -.7431448\,x \div \sqrt{1+1.3382612x+x^2};)\ \overset{xx}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xx}{Q} \div \sqrt{1-2ux+x^2} = -.8660254x \div \sqrt{1+x+x^2})$ $\overset{xxi}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxi}{Q} \div \sqrt{1-2zx+x^2} = -.9510565\,x \div$ $\sqrt{1+.7080340\,x+x^2};)\ \overset{xxii}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxii}{Q} \div \sqrt{1-2Bx+x^2}$ $= -.9945219\,x \div \sqrt{1+.2090570x+x^2};)\ \overset{xxiii}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxiii}{Q} \div \sqrt{1-2Cx+x^2} = -.9945219\,x \div \sqrt{1-.2090570\,x+x^2};)\ \overset{xxiv}{Q} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxiv}{Q} \div \sqrt{1-2xD+x^2} = -.9510565\,x \div$ $\sqrt{1-.7080340\,x+x^2};)\ \overset{xxv}{N} =$ cir. arc. rad, 1 and sine $(x \times$ sine $\overset{xxv}{Q} \div \sqrt{1-2Ex+x^2}$ $= -.8660254\,x \div \sqrt{1-x+x^2};)\ \overset{xxvi}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxvi}{Q}$ $\div \sqrt{1-2Fx+x^2} = -.7431448\,x \div \sqrt{1-1.3382612x+x^2};)\ \overset{xxvii}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxvii}{Q} \div \sqrt{1-2Gx+x^2} = -.5877853\,x \div$ $\sqrt{1-1.6180340x+x^2};)\ \overset{xxviii}{N} =$ cir. arc. rad. 1 and sine $(x \times$ sine $\overset{xxviii}{Q} \div$ $\sqrt{1-2Hx+x^2} = -.4067366x \div \sqrt{1-1.8270910\,x+x^2};)\ \overset{xxix}{N} =$ cir. arc. and rad. 1 and sine $(x \times$ sine $\overset{xxix}{Q} \div \sqrt{1-2Ix+x^2} = -.2079117\,x \div$ $\sqrt{1-1.9562952\,x+x^2}.)$ Hence the fluent $\frac{rmn-n}{n}$ drawn into: $-M+$ sine R $\times \overset{I}{N} -$ cosine R $\times \overset{I}{M} +$ sine 2 R $\times \overset{II}{N} -$ cosine 2 R $\times \overset{II}{M} +$ sine 3 R $\times \overset{III}{N} -$ cosine 3 R $\times \overset{III}{M} +$ sine 4 R $\times \overset{IV}{N} -$ cosine 4 R $\times \overset{IV}{M} +$ sine 5 R $\times \overset{V}{N} -$ cosine 5 R $\times \overset{V}{M} +$ sine 6 R $\times \overset{VI}{N} -$ cosine 6 R $\times \overset{VI}{M} +$ sine 7 R $\times \overset{VII}{N} -$ cosine 7 R $\times \overset{VII}{M} +$ sine 8 R $\times \overset{VIII}{N} -$ cosine 8 R $\times \overset{VIII}{M} +$ sine 9 R $\times \overset{IX}{N} -$ cosine 9 R $\times \overset{IX}{M} +$ sine 10 R $\times \overset{X}{N} -$ cosine 10 R $\times \overset{X}{M} +$ sine 11 R $\times \overset{XI}{N} -$ cosine 11 R $\times \overset{XI}{M} +$ sine 12 R $\times \overset{XII}{N} -$ cosine 12 R $\times \overset{XII}{M} +$ sine 13 R $\times \overset{XIII}{N} -$ cosine 13 R $\times \overset{XIII}{M} +$ sine 14 R $+ \overset{XIV}{N} -$ cosine 14 R $\times \overset{XIV}{M} +$ sine 15 R $\times \overset{XV}{N} -$ cosine 15 R $\times \overset{XV}{M} +$ sine 16 R $\times \overset{XVI}{N} -$ cosine 16 R $\times \overset{XVI}{M} +$ sine 17 R $\times \overset{XVII}{N} -$ cosine 17 R $\times \overset{XVII}{M}$

$+$ sine 18 R $\times N^{xviii}$ — cosine 18 R $\times M^{xviii}$ + sine 19 R $\times N^{xix}$ — cosine 19 R $\times M^{xix}$ + sine 20 R $\times N^{xx}$ — cosine 20 R $\times M^{xx}$ + sine 21 R $\times N^{xxi}$ — cosine 21 R $\times M^{xxi}$ + sine 22 R $\times N^{xxii}$ — cosine 22 R $\times M^{xxii}$ + sine 23 R $\times N^{xxiii}$ — cosine 23 R $\times M^{xxiii}$ + sine 24 R $\times N^{xxiv}$ — cosine 24 R $\times M^{xxiv}$ + sine 25 R $\times N^{xxv}$ — cosine 25 R $\times M^{xxv}$ + sine 26 R $\times N^{xxvi}$ — cosine 26 R $\times M^{xxvi}$ + sine 27 R $\times N^{xxvii}$ — cosine 27 R $\times M^{xxvii}$ + sine 28 R $\times N^{xxviii}$ — cosine 28 R $\times M^{xxviii}$ + sine 29 R $\times N^{xxix}$ — cosine 29 R $\times M^{xxix}$ by substituting the above value becomes $\frac{1}{30} \times$: into the following quantities :

$-$ hyp. log. of $\overline{1 - x}$

.5877853 × arch to rad. 1 and sine $.2079117\,x \div \sqrt{1 - 1.9562952\,x + x^2}$ $-$ .8090170 × hyp. log. of $\sqrt{1 - 1.9562952\,x + x^2}$

.9510565 × ... ... $.4067366\,x \div \sqrt{1 - 1.8270910\,x + x^2}$ $-$ .3090170 × hyp. log. of $\sqrt{1 - 1.8270910\,x + x^2}$

.9510565 × ... ... $.5877853\,x \div \sqrt{1 - 1.6180340\,x + x^2}$ $+$ .3090170 × hyp. log. of $\sqrt{1 - 1.6180340\,x + x^2}$

.5877853 × ... ... $.7431448\,x \div \sqrt{1 - 1.3382612\,x + x^2}$ $+$ .8090170 × hyp. log. of $\sqrt{1 - 1.3382612\,x + x^2}$

$+$ hyp. log. of $\sqrt{1 - x + x^2}$

$-$ .5877853 × ... ... $.9510565\,x \div \sqrt{1 - .7080340\,x + x^2}$ $+$ .8090170 × hyp. log. of $\sqrt{1 - .7080340\,x + x^2}$

$-$ .9510565 × ... ... $.9945219\,x \div \sqrt{1 - .2090570\,x + x^2}$ $+$ .3090170 × hyp. log. of $\sqrt{1 - .2090570\,x + x^2}$

$-$ .9510565 × ... ... $.9945219\,x \div \sqrt{1 + .2090570\,x + x^2}$ $-$ .3090170 × hyp. log. of $\sqrt{1 + .2090570\,x + x^2}$

$-$ .5877853 × ... ... $.9510565\,x \div \sqrt{1 + .7080340\,x + x^2}$ $-$ .8090170 × hyp. log. of $\sqrt{1 + .7080340\,x + x^2}$

$-$ hyp. log. of $\sqrt{1 + x + x^2}$

.5877853 × ... ... $.7431448\,x \div \sqrt{1 + 1.3382612\,x + x^2}$ $-$ .8090170 × hyp. log. of $\sqrt{1 + 1.3382612\,x + x^2}$

.9510565 × ... ... $.5877853\,x \div \sqrt{1 + 1.6180340\,x + x^2}$ $-$ .3090170 × hyp. log. of $\sqrt{1 + 1.6180340\,x + x^2}$

.9510565 × ... ... $.4067366\,x \div \sqrt{1 + 1.8270910\,x + x^2}$ $+$ .3090170 × hyp. log. of $\sqrt{1 + 1.8270910\,x + x^2}$

.5877853 × ... ... $.2079117\,x \div \sqrt{1 + 1.9562952\,x + x^2}$ $+$ .8090170 × hyp. log. of $\sqrt{1 + 1.9562952\,x + x^2}$

$+$ × hyp. log. of $\sqrt{1 + x}$

.5877853 × ... ... $.2079117\,x \div \sqrt{1 + 1\,9562952\,x + x^2}$ $+$ .8090170 × hyp. log. of $\sqrt{1 + 1.9562952\,x + x^2}$

.9510565 × ... ... $.4067366\,x \div \sqrt{1 + 8\,1270910\,x + x^2}$ $+$ .3090170 × hyp. log. of $\sqrt{1 + 1.8270910\,x + x^2}$

$+ .9510565 \times$ arch rad. 1 and sine $.5877853\,x \div \sqrt{1+1.6180340\,x+x^2} - .3090170 \times$ hyp. log. of $\sqrt{1+1.6180340\,x+x^2}$

$.5877853 \times$ ... ... $.7431448\,x \div \sqrt{1+1.3382612\,x+x^2} - .8090170 \times$ hyp. log. of $\sqrt{1+1.3382612\,x+x^2}$

$-$ hyp. log. of $\sqrt{1+x+x^2}$

$-.5877853 \times$ ... ... $.9510565\,x \div \sqrt{1+.7080340\,x+x^2} - .8090170 \times$ hyp. log. of $\sqrt{1+.7080340\,x+x^2}$

$-.9510565 \times$ ... ... $.9945219\,x \div \sqrt{1+.2090570\,x+x^2} - .3090170 \times$ hyp. log. of $\sqrt{1+.2090570\,x+x^2}$

$-.9510565 \times$ ... ... $.9945212\,x \div \sqrt{1-.2090570\,x+x^2} + .3090170 \times$ hyp. log. of $\sqrt{1-.2090570\,x+x^2}$

$-.5877853 \times$ ... ... $.9510565\,x \div \sqrt{1-.7080340\,x+x^2} + .8090170 \times$ hyp. log. of $\sqrt{1-.7080340\,x+x^2}$

$+$ hyp. log. of $\sqrt{1-x+x^2}$

$.5877853 \times$ ... ... $.7431448\,x \div \sqrt{1-1.3382612\,x+x^2} + .8090170 \times$ hyp. log. of $\sqrt{1-1.3382612\,x+x^2}$

$.9510565 \times$ ... ... $.5877853\,x \div \sqrt{1-1.6180340\,x+x^2} + .3090170 \times$ hyp. log. of $\sqrt{1-1.6180340\,x+x^2}$

$.9510565 \times$ ... ... $.4067366\,x \div \sqrt{1-1.8270910\,x+x^2} - .3090170 \times$ hyp. log. of $\sqrt{1-1.8270910\,x+x^2}$

$.5377853 \times$ ... ... $.2079117\,x \div \sqrt{1-1.9562952\,x+x^2} - .8090170 \times$ hyp. log. of $\sqrt{1-1.9562952\,x+x^2}$

but the above expression is evidently $=$ to $\frac{1}{10}$ drawn into

hyp. log. of $\overline{1-x}$ + hyp. log. $\overline{1+x}$

$.5877853 \times$ 2 cir. arc. rad. 1. sine $.2079117\,x \div \sqrt{1-1.9562952\,x+x^2} - .8090170 \times$ hyp. log. of $\overline{1-1.9562952\,x+x^2}$

$.9510565$ ... ... $.4067366\,x \div \sqrt{1-1.8270910\,x+x^2} - .3090170$ hyp. log. of $\overline{1-1.8270910\,x+x^2}$

$.9510565$ ... ... $.5877853\,x \div \sqrt{1-1.6180340\,x+x^2} + .3090170$ hyp. log. of $\overline{1-1.6180340\,x+x^2}$

$.5877853$ ... ... $.7431448\,x \div \sqrt{1-1.3382612\,x+x^2} + .8090170$ hyp. log. of $\overline{1-1.3382612\,x+x^2}$

$+$ hyp. log. of $\overline{1-x+x^2}$

$-.5877853$ ... ... $.9510565\,x \div \sqrt{1-.7080340\,x+x^2} + .8090170$ hyp. log. of $\overline{1-.7080340\,x+x^2}$

$-.9510565$ ... ... $.9945219\,x \div \sqrt{1-.2090570\,x+x^2} + .3090170$ hyp. log. of $\overline{1-.2090570\,x+x^2}$

$-.9510565$ ... ... $.9945219\,x \div \sqrt{1+.2090570\,x+x^2} - .3090170$ hyp. log. of $\overline{1+.2090570\,x+x^2}$

$-.5877853$ ... ... $.9510565\,x \div \sqrt{1+.7080340\,x+x^2} - .8090170$ hyp. log. of $\overline{1+.7080340\,x+x^2}$

$-$ hyp. log. of $\overline{1+x+x^2}$

$.5877853$ ... ... $.7431448\,x \div \sqrt{1+1.3382612\,x+x^2} - .8090170$ hyp. log. of $\overline{1+1.3382612\,x+x^2}$

$.9510565 \times 2$ cir. arc. rad. 1. sine $.5877853\, x \div \sqrt{1+1.6180340\, x + x^2} - .3090170 \times$ hyp. log. of $\overline{1+1.6180340\, x + x^2}$
$.9510565$ ... ... $.4067366\, x \div \sqrt{1+1.8270910\, x + x^2} + .3090170$ hyp. log. of $\overline{1+1.8270910\, x + x^2}$
$.5877853$ ... ... $.2079117\, x \div \sqrt{1+1.9562952\, x + x^2} + .8090170$ hyp. log. of $\overline{1+1.9562952\, x + x^2}$
which will be the fluent of the given expreſſion, according to Mr. Simpſon's method. But I will put the calculation down as obtained from Landen's Lucubrations; where the terms to be compared are $\frac{x^{m-1}\dot{x}}{x^n - x^{2n}}$ and $\frac{y^2 \dot{y}}{1-y^{30}}$, from whence $a = 1$, $m = 3$, $n = 30$, and therefore $c =$ tangent $\frac{180^\circ}{30} = 6^\circ = .1051042$, $d =$ tangent $\frac{2 \times 180^\circ}{30} = 12^\circ = .2125566$, $e =$ tangent $\frac{3 \times 180^\circ}{30} = 18^\circ = .3249197$, $f =$ tangent $\frac{4 \times 180^\circ}{30} = 24^\circ = .4452287$, g tangent $\frac{5 \times 180^\circ}{30} = 30^\circ = .5573503$, $h =$ tangent $\frac{6 \times 180^\circ}{30} = 36^\circ = .7265425$, $i =$ tangent $= \frac{7 \times 180^\circ}{30} = .9004040$, $k =$ tangent $\frac{8 \times 180^\circ}{30} = 48^\circ = 1.1106125$ $l =$ tangent $\frac{9 \times 180^\circ}{30} = 54^\circ = 1.3763819$, $n =$ tangent $\frac{10 \times 180^\circ}{30} = 60^\circ = 1.7320508$, $p =$ tangent $\frac{11 \times 180^\circ}{30} = 66^\circ = 2.2460368$, $q = \frac{12 \times 180^\circ}{30} = 72^\circ = 3.0776835$, $r =$ tangent $\frac{13 \times 180^\circ}{30} = 78^\circ = 4.7046301$, $s =$ tangent $= \frac{14 \times 180^\circ}{30} = 84^\circ = 9.5143645$; $\underset{\text{I}}{\text{A}} =$ cir. arc. rad. 1. tangent c, $\underset{\text{II}}{\text{A}} =$ cir. arc. rad. 1. tangent d, $\underset{\text{III}}{\text{A}} =$ cir. arc. rad. 1. tangent e, $\underset{\text{IV}}{\text{A}} =$ cir. arc. rad. 1. tangent f, $\underset{\text{V}}{\text{A}} =$ cir. arc. rad. tangent g, $\underset{\text{VI}}{\text{A}} =$ cir. arc. rad. 1. tangent h, $\underset{\text{VII}}{\text{A}} =$ cir. arc. rad. 1. tangent i, $\underset{\text{VIII}}{\text{A}} =$ cir. arc. rad. 1. tangent k, $\underset{\text{IX}}{\text{A}} =$ cir. arc. rad. 1. tangent l, $\underset{\text{X}}{\text{A}} =$ cir. arc. rad. 1. tangent n, $\underset{\text{XI}}{\text{A}} =$ cir. arc. rad. 1. tangent p, $\underset{\text{XII}}{\text{A}} =$ cir. arc. rad. 1. tangent q, $\underset{\text{XIII}}{\text{A}}$ cir. arc. rad. 1. tangent r, $\underset{\text{XIV}}{\text{A}} =$ cir. arc. rad. 1. tangent s; $\overset{\text{I}}{\text{M}} = 2 \times$ sine $6\,\underset{\text{I}}{\text{A}} = 2 \times$ sine $36^\circ = 2 \times .5877853$, $\overset{\text{II}}{\text{M}} = 2 \times$ sine $6\,\underset{\text{II}}{\text{A}} = 2 \times$ sine $72^\circ = 2 \times .9510565$, $\overset{\text{III}}{\text{M}} = 2 \times$ sine $6\,\underset{\text{III}}{\text{A}} = 2 \times$ sine $108^\circ = 2 \times .9510565$, $\overset{\text{IV}}{\text{M}} = 2 \times$ sine $6\,\underset{\text{IV}}{\text{A}} = 2 \times$ sine $144^\circ = 2 \times .5877853$, $\overset{\text{V}}{\text{M}} = 2 \times$ sine $6\,\underset{\text{V}}{\text{A}} = 2 \times$ sine $180^\circ = 0$, $\overset{\text{VI}}{\text{M}} = 2 \times$ sine $6\,\underset{\text{VI}}{\text{A}} = 2 \times$ sine $216^\circ = -2 \times$

.5877853, $\overset{vii}{M} = 2 \times$ sine $6 \underset{vii}{A} = 2 \times$ sine $252^\circ = -2 \times .9510565$, $\overset{viii}{M} = 2 \times$ sine $6 \underset{viii}{A} = 2 \times$ sine $288^\circ = -2 \times .9510565$, $\overset{ix}{M} = 2 \times$ sine $6 \underset{ix}{A} = 2 \times$ sine $324^\circ = -2 \times 5877853$, $\overset{x}{M} = 2 \times$ sine $6 \underset{x}{A} = 2 \times$ sine $360^\circ = 0$, $\overset{xi}{M} = 2 \times$ sine $6 \underset{xi}{A} = 2 \times$ sine $396^\circ = 2 \times .5877853$, $\overset{xii}{M} = 2 \times$ sine $6 \underset{xii}{A} = 2 \times$ sine $432^\circ = 2 \times .9510565$ $\overset{xiii}{M} = 2 \times$ sine $6 \underset{xiii}{A} = 2 \times$ sine $468^\circ = 2 \times .9510565$, $\overset{xiv}{M} = 2 \times$ sine $6 \underset{xiv}{A} = 2 \times$ sine $504^\circ = 2 \times .5877853$; $\overset{i}{N} = 2 \times$ cosine $6 \underset{i}{A} = 2 \times$ cosine $36^\circ = 2 \times .8090170$ $\overset{ii}{N} = 2 \times$ cosine $6 \underset{ii}{A} = 2 \times$ cosine $72^\circ = 2 \times .3090170$, $\overset{iii}{N} = 2 \times$ cosine $6 \underset{iii}{A} = 2 \times$ cosine $108^\circ = -2 \times .3090170$, $\overset{iv}{N} = 2 \times$ cosine $6 \underset{iv}{A} = 2 \times$ cosine $144^\circ = -2 \times .8090170$, $\overset{v}{N} = 2 \times$ cosine $6 \underset{v}{A} = 2 \times$ cosine $180^\circ = -2$, $\overset{vi}{N} = 2 \times$ cosine $6 \underset{vi}{A} = 2 \times$ cosine $216^\circ = -2 \times .8090170$, $\overset{vii}{N} = 2 \times$ cosine $6 \underset{vii}{A} = 2 \times$ cosine $252^\circ = -2 \times .3090170$, $\overset{viii}{N} = 2 \times$ cosine $6 \underset{viii}{A} = 2 \times$ cosine $288 = 2 \times .3090170$, $\overset{ix}{N} = 2 \times$ cosine $6 \underset{ix}{A} = 2 \times$ cosine $324^\circ = 2 \times .8090170$ $\overset{x}{N} = 2 \times$ cosine $6 \underset{x}{A} = 2 \times$ cosine $360 = 2$, $\overset{xi}{N} = 2 \times$ cosine $6 \underset{xi}{A} = 2 \times$ cosine $396^\circ = 2 \times .8090170$, $\overset{xii}{N} = 2 \times$ cosine $6 \underset{xii}{A} = 2 \times$ cosine $430 = 2 \times .3090170$, $\overset{xiii}{N} = 2 \times$ cosine $6 \underset{xiii}{A} = 2 \times$ cosine $468 = 2 \times .3090170$, $\overset{xiv}{N} = 2 \times$ cosine $6 \underset{xiv}{A} = 2 \times$ cosine $504^\circ = 2 \times .8090170$, and $x = x$; and the fluent becomes $\frac{1}{30}$ drawn into $-$ hyp. log. DP $\div$ CP $2 \times .5877853 \times$ arc $(\overset{i}{CPD}) - 2 \times .8090170 \times$ hyp. log. $\overset{i}{DP} \div \overset{i}{CP}$; $2 \times .9510565 \times$ arc $(\overset{ii}{CPD}) - 2 \times .3090170 \times$ hyp. log. $\overset{ii}{DP} \div \overset{ii}{CP}$; $2 \times .9510565 \times$ arc $(\overset{iii}{CPD}) + 2 \times .3090170 \times$ hyp. log. $\overset{iii}{DP} \div \overset{iii}{CP}$; $2 \times .5877853 \times$ arc $(\overset{iv}{CPD}) + 2 \times .8090170 \times$ hyp.

log. $D\overset{\text{IV}}{P} \div C\overset{\text{IV}}{P}$; $+\ 2 \times$ hyp. log. $D\overset{\text{V}}{P} \div C\overset{\text{V}}{P}$; $-\ 2 \times .5877853 \times$ arc $(C\overset{\text{VI}}{P}D) + 2 \times .8090170 \times$ hyp. log. $D\overset{\text{VI}}{P} \div C\overset{\text{VI}}{P}$; $-\ 2 \times .9510565 \times$ arc $(C\overset{\text{VII}}{P}D) + 2 \times .3090170 \times$ hyp. log. $D\overset{\text{VII}}{P} \div C\overset{\text{VII}}{P}$; $-\ 2 \times .9510565 \times$ arc $(C\overset{\text{VIII}}{P}D) - 2 \times .3090170 \times$ hyp. log. $D\overset{\text{VIII}}{P} \div C\overset{\text{VIII}}{P}$; $-\ 2 \times .5877853 \times$ arc $(C\overset{\text{IX}}{P}D) - 2 \times .8390170 \times$ hyp. log. $D\overset{\text{IX}}{P} \div C\overset{\text{IX}}{P}$; $-\ 2 \times$ hyp. log. $D\overset{\text{X}}{P} \div C\overset{\text{X}}{P}$; $2 \times .5877853 \times$ arc $(C\overset{\text{XI}}{P}D) - 2 \times .8090170 \times$ hyp. log. $D\overset{\text{XI}}{P} \div C\overset{\text{XI}}{P}$; $2 \times .9510565 \times$ arc $(C\overset{\text{XII}}{P}D) - 2 \times .3090170 \times$ hyp. log. $D\overset{\text{XII}}{P} \div C\overset{\text{XII}}{P}$; $2 \times .9510565 \times$ arc $(C\overset{\text{XIII}}{P}D) + 2 \times .3090170 \times$ hyp. log. $D\overset{\text{XIII}}{P} \div C\overset{\text{XIII}}{P}$; $2 \times .5877853 \times$ arc $(C\overset{\text{XIV}}{P}D) + 2 \times .8090170 \times$ hyp. log. $D\overset{\text{XIV}}{P} \div C\overset{\text{XIV}}{P}$.

Now to find the measures of the angles, $(C\overset{\text{I}}{P}D)$ $(C\overset{\text{II}}{P}D)$, &c. to $(C\overset{\text{XIV}}{P}D)$ and of the fractions $(DP \div CP)$, $(D\overset{\text{I}}{P} \div C\overset{\text{I}}{P})$, $(D\overset{\text{II}}{P} \div C\overset{\text{II}}{P})$, &c. to $(D\overset{\text{XIV}}{P} \div C\overset{\text{XIV}}{P})$. We must observe that $P\overset{\text{I}}{a} = \overset{\text{I}}{a}\overset{\text{I}}{A} = \overset{\text{II}}{A}\overset{\text{II}}{a} = \overset{\text{II}}{a}\overset{\text{II}}{A}$, &c. $= {}^{1}/_{n} \times PO = \frac{1}{30} \times 90^\circ = 3^\circ$, $P\overset{\text{I}}{A} = 6^\circ$, $P\overset{\text{II}}{A} = 12$, $P\overset{\text{III}}{A} = 18^\circ$, $P\overset{\text{IV}}{A} = 24^\circ$, $P\overset{\text{V}}{A} = 30^\circ$, $P\overset{\text{VI}}{A} = 36^\circ$, $P\overset{\text{VII}}{A} = 42^\circ$, $P\overset{\text{VIII}}{A} = 48^\circ$, $P\overset{\text{IX}}{A} = 54^\circ$, $P\overset{\text{X}}{A} = 60^\circ$, $P\overset{\text{XI}}{A} = 66^\circ$, $P\overset{\text{XII}}{A} = 72^\circ$, $P\overset{\text{XIII}}{A} = 78^\circ$, $P\overset{\text{XIV}}{A} = 84^\circ$; and $P\overset{\text{I}}{P} =$ tangent $6^\circ = c$, $P\overset{\text{II}}{P} =$ tangent $12^\circ = d$, $P\overset{\text{III}}{P} =$ tangent $18^\circ = e$, $P\overset{\text{IV}}{P} =$ tangent $24^\circ = f$, $P\overset{\text{V}}{P} =$ tangent $30^\circ = g$, $P\overset{\text{VI}}{P} =$ tangent $36^\circ = h$, $P\overset{\text{VII}}{P} =$ tangent $42^\circ = i$, $P\overset{\text{VIII}}{P} =$ tangent $48^\circ = k$, $P\overset{\text{IX}}{P} =$ tangent $54^\circ = l$, $P\overset{\text{X}}{P} =$ tangent $60^\circ = n$, $P\overset{\text{XI}}{P} =$ tangent $66^\circ = p$, $P\overset{\text{XII}}{P} =$ tangent $72^\circ = q$, $P\overset{\text{XIII}}{P} =$ tangent $78^\circ = r$, $P\overset{\text{XIV}}{P} =$ tangent $84^\circ = s$, CP being 1.

And from the theorem we have $BP : CD :: a + x : x$, or $a + x : x :: BP : CD$, that is $1 + x : x :: 2 : \frac{2x}{1+x} = CD$, $DP = PC - DC = 1 - \frac{2x}{1+x} = \frac{1 + x - 2x}{1 + x} = \frac{1 - x}{1 + x}$. Hence $\frac{DP}{CP} = \frac{1 - x}{1 + x}$, and the hyp. log. $\frac{DP}{CP} =$ hyp. log. $\overline{1 - x}$ −

hyp. log. $\overline{1+x}$; $D\overset{I}{P} = \sqrt{\overline{P\overset{I}{p}}|^2 + \overline{PD}|^2} = \sqrt{c^2+1} \times \sqrt{\dfrac{1 + \dfrac{2c^2-2}{c^2+1}x + x^2}{1+x}} = \dfrac{\sqrt{c^2+1}.\sqrt{1-1.9562952\,x+x^2}}{1+x}$ and $C\overset{I}{P} = \sqrt{\overline{P\overset{I}{p}}|^2 + \overline{PC}|^2} = \sqrt{c^2+1}$, and $(D\overset{I}{P} \div C\overset{I}{P}) = \sqrt{1 + \dfrac{2c^2-2}{c^2+1}x + x^2} \div \overline{1+x} = \sqrt{1-1.9562952\,x+x^2} \div \overline{1+x}$. Consequently the hyp. log. $(D\overset{I}{P} \div C\overset{I}{P})$ = hyp. log. $\sqrt{1-1.9562952\,x+x^2}$ − hyp. log. $\overline{1+x}$. Whence by the very same methods we easily find the hyp. log. of $(D\overset{II}{P} \div C\overset{II}{P})$ = hyp. log. of $\sqrt{1-1.8270910\,x+x^2}$ − hyp log. $\overline{1+x}$, hyp. log. $(D\overset{III}{P} \div C\overset{III}{P})$ = hyp. log. $\sqrt{1-1.6180340\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{IV}{P} \div C\overset{IV}{P})$ = hyp. log. $\sqrt{1-1.3382612\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{V}{P} \div C\overset{V}{P})$ = hyp. log. $\sqrt{1-x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{VI}{P} \div C\overset{VI}{P})$ = hyp. log. $\sqrt{1-.7080340\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{VII}{P} \div C\overset{VII}{P})$ = hyp. log. $\sqrt{1-.2090570\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{VIII}{P} \div C\overset{VIII}{P})$ = hyp. log. $\sqrt{1+2090570\,x+x^2}$ − hyp. log. $\overline{1+x}$ hyp. log. $(D\overset{IX}{P} \div D\overset{IX}{P})$ = hyp. log. $\sqrt{1+.7080340\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{X}{P} \div C\overset{X}{P})$ = hyp. log. $\sqrt{1+x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{XI}{P} \div C\overset{XI}{P})$ = hyp. log. $\sqrt{1+1.3382612\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{XII}{P} \div C\overset{XII}{P})$ = hyp. log. $\sqrt{1+1.6180340\,x+x^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{XIII}{P} \div C\overset{XIII}{P})$ = h. l. $\sqrt{1+1.8270910x+y^2}$ − hyp. log. $\overline{1+x}$; hyp. log. $(D\overset{XIV}{P} \div C\overset{XIV}{P})$ = hyp. log. $\sqrt{1+1.9562952\,x+x^2}$ − hyp. log. $\overline{1+x}$. Again $D\overset{I}{P}$: CD :: sine DCP : sine $C\overset{I}{P}D$ that is $(\sqrt{c^2} \times 1 \ \sqrt{1 + \dfrac{2c^2-2}{c^2+1}x + x^2} \div \overline{1+x}) : \dfrac{2x}{1+x} :: c : (.2079117\,x \div \sqrt{1-1\,9562952\,x+x^2}$, and in the very same way will the sines of the arches $CP\overset{IV}{D}$, CPD, &c. be found, *viz.*

Sine $C\overset{II}{P}D = .4067366\,\dot{x} \div \sqrt{1-1.8270910\,x+x^2}$; sine $C\overset{III}{P}D = .5877853\,\dot{x} \div \overline{1-1.6180340\,x+x^2}$;

Sine $C\overset{IV}{P}D = .7431448\,\dot{x} \div \sqrt{1+1.3382612\,x+x^2}$; sine $C\overset{VI}{P}D = .9510565\,\dot{x} \div \sqrt{1+.7080340\,x+x^2}$;

Sine $C\overset{VII}{P}D = .9945219\,\dot{x} \div \sqrt{1-.2090570\,x+x^2}$; sine $C\overset{VIII}{P}D = .9945219\,\dot{x} \div \sqrt{1+.2090570\,x+x^2}$;

Sine $C\overset{IX}{P}D = .9510565\,\dot{x} \div \sqrt{1+.7080340\,x+x^2}$; sine $C\overset{XI}{P}D = .7431448\,\dot{x} \div \sqrt{1+1.3382612\,x+x^2}$;

Sine $C\overset{XII}{P}D = .5877853\,\dot{x} \div \sqrt{1+1.6180340\,x+x^2}$; sine $C\overset{XIII}{P}D = .4067366\,\dot{x} \div \sqrt{1+1.8270910\,x+x^2}$;

Sine $C\overset{XIV}{P}D = .2079117\,\dot{x} \div \sqrt{1+1.9562952\,x+x^2}$.

Whence by collecting the terms, the fluent will be $= \frac{1}{30}$ drawn into $-$ hyp. log. $\overline{1-y}$ $+$ hyp. log. $\overline{1+y}$

$2\times.5877853 \times$ c. a. r. 1. fi. $.2079117\,x \div \sqrt{1-1.9562952\,x+x^2} - 2\times.8090170\times(\text{h.l.}\sqrt{1-1.9562952\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.9510565 \times$ ... ... $.4067366\,x \div \sqrt{1-1.8270910\,x+x^2} - 2\times.3090170\times(\text{h.l.}\sqrt{1-1.8270910\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.9510565 \times$ ... ... $.5877853\,x \div \sqrt{1-1.6180340\,x+x^2} + 2\times.3090170\times(\text{h.l.}\sqrt{1-1.6180340\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.5877853 \times$ ... ... $.7431448\,x \div \sqrt{1+1.3382612\,x+x^2} + 2\times.8090170\times(\text{h.l.}\sqrt{1-1.3382612\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$+2 \times(\text{h.l.}\sqrt{1-x+x^2} - \text{h.l.}\,\overline{1+x})$

$-2\times.5877853 \times$ ... ... $.9510565\,x \div \sqrt{1-.7080340\,x+x^2} + 2\times.8090170\times(\text{h.l.}\sqrt{1-.7080340\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$-2\times.9510565 \times$ ... ... $.9945219\,x \div \sqrt{1-.2090570\,x+x^2} + 2\times.3090170\times(\text{h.l.}\sqrt{1-.2090570\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$-2\times.9510565 \times$ ... ... $.9945219\,x \div \sqrt{1+.2090570\,x+x^2} - 2\times.3090170\times(\text{h.l.}\sqrt{1+.2090570\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$-2\times.5877853 \times$ ... ... $.9510565\,x \div \sqrt{1+.7080340\,x+x^2} - 2\times.8090170\times(\text{h.l.}\sqrt{1+.7080340\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$-2\times(\text{h.l.}\sqrt{1+x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.5877853 \times$ ... ... $.7431448\,x \div \sqrt{1+1.3382612\,x+x^2} - 2\times.8090170\times(\text{h.l.}\sqrt{1+1.3382612\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.9510565 \times$ ... ... $.5877853\,x \div \sqrt{1+1.6180340\,x+x^2} - 2\times.3090170\times(\text{h.l.}\sqrt{1+1.6180340\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.9510565 \times$ ... ... $.4067366\,x \div \sqrt{1+1.8270910\,x+x^2} + 2\times.3090170\times(\text{h.l.}\sqrt{1+1.8270910\,x+x^2} - \text{h.l.}\,\overline{1+x})$

$2\times.5877853 \times$ ... ... $.2079117\,x \div \sqrt{1+1.9562952\,x+x^2} + 2\times8090170\times(\text{h.l.}\sqrt{1+1.9562952\,x+x^2} - \text{h.l.}\,\overline{1+x})$

In this last expression for the fluent it is easily seen that $-$ hyp. log. $\overline{1+y}$ when drawn into the different multiplicators, there will b found 7 terms affected the sign $+$, and 7 terms with the sign $-$, the first line remaining as above $-$, and the fourteen terms will b destroyed; the 7 terms with the sign $+$, cancelling those 7 terms with the sign $-$; I mean only that part of each line after the firs which is affected with the quantity hyp. log. $\overline{1+y}$.

Whence it is evident that the fluent will be expressed by $\frac{1}{30}$ drawn into........... .... $-$ hyp. log. $\overline{1-x}$ $+$ hyp. log. $\overline{1+x}$

$.5877853 \times$ 2 cir. arc. rad. 1 sine $.2079117\,x \div \sqrt{1-1.9562952\,x+x^2} - .8090170 \times$ hyp. log. of $\sqrt{1-1.9562952\,x+x}$

$.9510565 \times$ ... . $.4067366\,x \div \sqrt{1-1.8270910\,x+x^2} - .3090170 \times$ hyp. log. of $\sqrt{1-1.8270910\,x+x}$

$.9510565 \times$ ... ... $.5877853\,x \div \sqrt{1-1.6180340\,x+x^2} + .3090170 \times$ hyp. log. of $\overline{1-1.6180340\,x+x}$

$.5877853 \times$ ... ... $.7431448\,x \div \sqrt{1-1.3382612\,x+x^2} + .8090170 \times$ hyp. log. of $\overline{1-1.3382612\,x+x}$

$+$ hyp. log. of $\overline{1-x+x^2}$

$-.5877853 \times$ ... ... $.9510565\,x \div \sqrt{1-.7080340\,x+x^2} + .8090170 \times$ hyp. log. of $\overline{1-.7080340\,x+x}$

$-.9510565 \times$ ... ... $.9945219\,x \div \sqrt{1-.2090570\,x+x^2} + .3090170 \times$ hyp. log. of $\overline{1-.2090570\,x+x}$

$-.9510565 \times$ ... ... $.9945219\,x \div \sqrt{1+.2090570\,x+x^2} - .3090170 \times$ hyp. log. of $\overline{1+.2090570\,x+x}$

$-.5877853 \times$ ... ... $.9510565\,x \div \sqrt{1+.7080340\,x+x^2} - .8090170 \times$ hyp. log. of $\overline{1+.7080340\,x+x}$

$-$ hyp. log. of $\overline{1+y+y^2}$

$.5877853 \times$ ... ... $.7431448\,x \div \sqrt{1+1.3382612\,x+x^2} - .8090170 \times$ hyp. log. of $\overline{1+1.3382612\,x+x}$

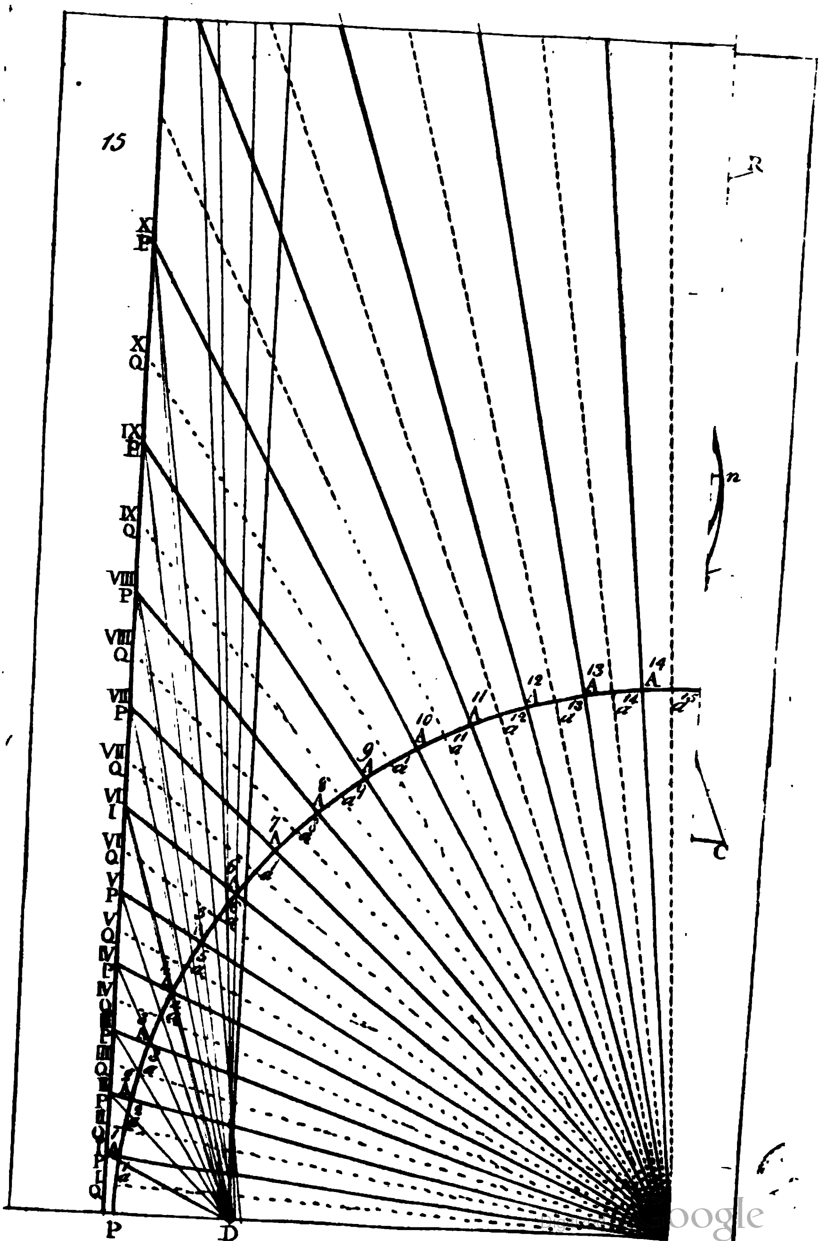
$.9510565 \times$ ... . $.5877853\,x \div \sqrt{1+1.6180340\,x+x^2} - .3090170 \times$ hyp. log. of $\overline{1+1.6180340\,x+x}$

$.9510565 \times$ ... ... $.4067366\,x \div \sqrt{1+1.8270910\,x+x^2} + .3090170 \times$ hyp. log. of $\overline{1+1.8270910\,x+x}$

$.5877853 \times$ ... ... $.2079117\,x \div \sqrt{1+1.9562952\,x+x^2} + .8090170 \times$ hyp. log. of $\overline{1+1.9562952\,x+x}$

which is the very same expression for the fluent as that obtained from Mr. Simpson's method. The very same conclusion may be draw out from Emerson's 8th form, but I am tired of the processes. Only you must observe, in finding the fluent by Mr. Landen's method I have, by mistake, supposed the given fluxion $\frac{y^2\dot{y}}{1-y^{30}}$ instead of $\frac{x^2\dot{x}}{1-x^{30}}$, however that is easily rectified, and you will see that I hav made every y into an x.

Having shewn how to find the fluent of the first term, the fluent of the second term may be found after the same way, from Probler 4, Page 366, Simpson's Fluxions, or by form 7th, Emerson's Fluxions, or by Page 98, Landen's Lucubrations, and the fluents of th 3d and 4th terms is easily had by common methods, and therefore the area itself may be expressed by $\int\frac{x^2\dot{x}}{1-y^{30}} - \int\frac{x^{1'}\dot{x}}{1+x^{10}} + \int x'$ $\dot{x} - \int x^2\dot{x}$, all which may be found from what is done above.


15
R
n
C
P
D
X
P
X
Q
IX
P
IX
Q
VIII
P
VIII
Q
VII
P
VII
Q
A
a

Question 86, *by L. L. D. F. R. S.*

Suppose a comet containing the same quantity of matter as the moon, to pass between the moon and earth, equally distant from the two bodies, to find how high it will raise the water in the ocean?

Solution, *by Mr. John Bickford, of Westminster.*

Let a = the earth's radius = 3985 miles, e = its density = 9, d = the density of the comet which is supposed to be = the moon's = 11, s = the sine of the comet's apparent semi-diameter, as seen from the earth being 31′ 14″. Then by Mr. Emerson's Fluxions, page 423, we have $\frac{3ds^3a}{e}$ = 57.85 feet, the height of the tide required.

From hence we may learn, that if a comet approaches the earth in its orbit round the sun, at no less a distance than 120000 miles, and at the same time be no bigger than the moon, yet the waters will be so attracted by it, as to deluge the greatest part of the habitable parts of our globe. But it is exceeding probable, that most of the comets are as large or larger than the earth, which is 50 times as large as the moon; what then must be the consequence of the approach of such a comet, but entire destruction to the present order of things on the surface of our globe?

Question 87, *by Mr. Thomas Whiting.*

There is a clock which keeps true time at London, its pendulum is 6 inches long.— Now admit this clock to ascend perpendicularly upwards, with an uniform velocity of one mile per day; how much will it gain or lose in 400 days; and how many times will it vibrate in a minute when arrived at that height?

Solution, *by Mr. W. Armstrong, Pupil to Mr. Howard.*

Let A (Fig. 1) represent the surface, and C the center of the earth, and D the place to which the clock is to ascend. Put radius CA = r = 3967, CD = a = 4367 miles, CP = x, n = 6 inches, t = 400 days = 34560000 seconds, and m = 39.13, then, per mechanics $\sqrt{m} : \sqrt{n} :: 1 : \sqrt{\frac{n}{m}}$ = .3915 = the time of vibrating at A. Again the time of vibration of the same pendulum at any other place is by Prob. 196, Emerson's Algebra, as the distance from the earth's center $\therefore r : \sqrt{\frac{n}{m}} :: x : \frac{x}{r}\sqrt{\frac{n}{m}}$ = the time of vibrating at P. Let Pp = x' represent any small indefinite distance, then by uniform motion $a - r : t :: x' : \frac{tx'}{a-r}$ = the time of describing the distance Pp; now since the effect of gravity may be supposed to remain constant, during the time of describing this increment, and in uniform motion, the number of vibrations being as the time, we have $\frac{t\dot{x}}{a-r} \times \frac{x}{r} \times \sqrt{\frac{n}{m}}$ = the fluxion of the number of vibrations, and its fluent $= \frac{x^2}{2} \times \frac{t}{ar - r^2} \times \sqrt{\frac{n}{m}}$ which needs no correction, and when $x = a$ it becomes $\frac{a^2t}{2ar - 2r^2} \times \sqrt{\frac{n}{m}} = 81305457.251985$ = the number of vibrations made, but t divided by $\sqrt{\frac{n}{m}} = 88273563.22$ is the number it would have made at the surface, therefore their difference = 6968105.958 is the number lost, and is = 31 days, 13 hours, 46 minutes, 53½ seconds. Now to find the number of vibrations made when arrived at D in one minute $r : \sqrt{\frac{n}{m}} :: a : \frac{a}{r} \times \sqrt{\frac{n}{m}} = .43097$ = the time of vibrating, ∴ $\frac{a}{r} \times \sqrt{\frac{n}{m}}$: 1 vibration, :: 60 seconds : 139.2205 the number required.

Question 88, *by Mr. T. Whiting.*

There is an ellipsis whose diameters are 15 and 10 feet respectively, in the center of which stands a lamp 10 feet from the ground, and suppose a man 10 feet high to walk once round the circumference of the ellipse. Required the area of the space passed over by his shadow, also the circumference of the shadow's extremity; and nature of the curve?

Solution, *by Mr. W. Armstrong.*

Since the ordinate of the required curve is always in the constant ratio (to that of the given curve) of the height of the lamp to the difference between it and the man's height, it necessarily follows, that it will be a curve of the same order with that of the given one, and in the present case an ellipsis, whose required properties are well known.

Question 89, *by Mr. James Stevenson, of Heath.*

Given $x^{\frac{2m}{n}}y^{\frac{m}{n}} - x^{\frac{m}{n}}y^{\frac{2m}{n}} = a$, and $x^{\frac{3m}{n}} - y^{\frac{3m}{n}} = b$; to find x and y. And to give an example when $a = 120000$, $b = 387000$, $m = 100$, and $n = 99$?

Solution, *by the Proposer.*

By subtracting 3 times the 1st equation from the 2d, and taking the cube root of the remainder, we get a 3d equation, *viz.* $x^{\frac{m}{n}} - y^{\frac{m}{n}} = \overline{b - 3a}|^{\frac{1}{3}}$; then by dividing the

1st equation by the 3d, we get $\overline{xy}|^{\frac{m}{n}} = \frac{a}{\overline{b-3a}|^{\frac{1}{3}}}$; and by adding 4 times this equation to the square of the 3d, and taking the square root of the sum, we obtain $x^{\frac{m}{n}} + y^{\frac{m}{n}} = \sqrt{\overline{b-3a}|^{\frac{2}{3}} + \frac{4a}{\overline{b-3a}|^{\frac{1}{3}}}}$; also by adding and subtracting the 3d equation to and from the last, we have $2x^{\frac{m}{n}}$ and $2y^{\frac{m}{n}} = \sqrt{\overline{b-3a}|^{\frac{2}{3}} + \frac{4a}{\overline{b-3a}|^{\frac{1}{3}}}} \pm \overline{b-3a}|^{\frac{1}{3}}$; $\therefore$ x and $y =$

$$\overline{\frac{\sqrt{\overline{b-3a}|^{\frac{2}{3}} + \frac{4a}{\overline{b-3a}|^{\frac{1}{3}}}} \pm \overline{b-3a}|^{\frac{1}{3}}}{2}}\Bigg|^{\frac{n}{m}} = 76.57009, \text{ and } 48.08176, \text{ in this case.}$$

Question 90, *by Mr. James Stevenson.*

Given the solidity of a right dodecagonal pyramid, $= 36000$ inches, to determine the dimensions when the surface is a minimum.

Solution, *by Mr. Jonathan Mabbott.*

Put $a = 36000$, $b = 1.9318516$, $c = 11.1961524$, and let x represent the $\perp$ height of the pyramid, and $y =$ each side of its end, or base. Then per question $cy^2 x = 3a$, and $12y \times \frac{1}{4}\sqrt{b^2y^2 + x^2} =$ a minimum, hence from the first equation $x = 3a \div cy^2$, which wrote instead of its equal in the minimum, and $6y\sqrt{b^2y^2 + \frac{9aa}{c^2y^4}}$ is a minimum, or $b^2y^4 + \frac{9aa}{c^2y^2}$ is also a minimum, this in fluxions is $4b^2y^3\dot{y} - \frac{18aac^2y\dot{y}}{c^4y^4} = o$, this reduced is $2by^3 = ac\sqrt{18}$, hence $y =$ the cube root of $ac\sqrt{18} \div 2b = 59.3$ and thence $x = 27.4$ as required.

Question 91, *by Mr. O. G. Gregory.*

Given $\sqrt[4]{x^3y + x^3} \times \sqrt[3]{x^2y + 2xy^2 - y^3} = x$, and $x^2 + 2xy = b^2 - y^2$, to find x and y, when $b^2 =$ the number of inches in a cubic foot.

Solution, *by Mr. Jonathan Mabbott.*

By proper reduction the first equation becomes $x^2 - 4xy + y^2 = o$, and the latter is $x^2 + 2xy + y^2 = b^2$. Take their sum and difference, and $x^2 - xy + y^2 = \frac{1}{2}b^2$, and $xy = \frac{1}{6}b^2$, take the latter of these from the former, and $x^2 - 2xy + y^2 = \frac{1}{3}b^2$, hence taking the square root of this, and second equation we have $x - y = b\sqrt{\frac{1}{3}}$, and $x + y = b$, ∴ by addition and subtraction $x = \frac{b + b\sqrt{\frac{1}{3}}}{2} = 32.7846$, and $y = \frac{b - b\sqrt{\frac{1}{3}}}{2} = 8.7846$ as required.

Question 92, *by Mr. Stevenson, of Heath.*

Given the area, and two opposite angles of a trapezium ABCD, whose sides AB, BC, CD, and DA are to each other as four known quantities a, b, c, and d, respectively; to find the sides and diagonals.

Solution, *by Mr. John Ryley.*

Let ABCD (Fig. 2) be the trapezium, ABC, ADC the given angles whose sines are here expressed by m and n. Put AB $= ax$, BC $= bx$, CD $= cx$, DA $= dx$, and the given area $= s$. Then per Trig. $\frac{ax \times bx}{2} \times m + \frac{cx \times dx}{2} \times n = s$; hence $abmx^2 + cdnx^2 = 2s$, and $x = \sqrt{\frac{2s}{amb + cdn}}$. Therefore AB $= a\sqrt{\frac{2s}{abm + cdn}}$; CD $= b\sqrt{\frac{2s}{abm + cdn}}$; CD $= c\sqrt{\frac{2s}{abm + cdn}}$; and DA $= d\sqrt{\frac{2s}{abm + cdn}}$.— Moreover, by Case V. page 124, Emerson's Trigonometry, AC $= \sqrt{\frac{2a^2s + 2b^2s + 4abs\sqrt{1 - m^2}}{abm + cdn}}$. Now, as all the sides and diagonal AC are known the angle BCD, per Trig. will become known, and thence BD will be found, after the same manner as we have found AC.

Question 93, *by Mr. R. Nicholson, of Kirkbyoverblow.*

Surveying a field in form of a parabola, the base of which was 12 chains in a diameter, and from its vertex I measured 284 links at the outside of the field, and then found if a tangent were drawn from thence to the hedge, it would just cut the base when produced 160 links from the corner of the field. Required a geometrical construction of the field, when the diameter is the axis, and the base an ordinate to it; also, the area without algebra, or fluxions.

Solution, *by Mr. John Ryley.*

By the property of the parabola DV $=$ VG, therefore VG is given; also EL^2 $=$ AE $\times$ EB (Fig. 3) which is given, consequently EL is given $= 80\sqrt{34} = 466.48$, and also, AL $= 80\sqrt{34} - 160 = 306.48$. Moreover EA : AL :: AL : 2FG; ∴ FG is given $= 760 - 80\sqrt{34} = 293.52$: And by similar triangles DG : GF :: DC

: CE; therefore GF : DG :: CE : CD = 1470, hence VC = 1186, and the area = 9a. 1r. 38p.

Question 94, *by Mr. W. Davis.*

There is an island, whose area is 70686 square feet, round which is a semi-circular concave ditch, whose radius is 6 feet; in the center of the island stands a may-pole 300 feet high; it so happened that a high wind broke the pole in such a manner, that the top struck the ditch 30° degrees beyond the middle. How high is the piece left standing?

Solution, *by Mr. John Ryley.*

Let AB (Fig. 4) be the given circular island, and B*nb* a section of the ditch, OCD the broken may-pole; then per Trig. rad. 1 : OD :: sine ∠ *r*D*o* : *or* = FO = $3\sqrt{3}$: also radius 1 : *o*D :: sine ∠ D*or* : *r*D = 3. But OB = $\frac{1}{2}\sqrt{\frac{70686}{.7854}}$ = 150; therefore FD = 159. Now put CO = *x*, and CD will be = 300 − *x*; therefore by Euclid's 47. 1. $(x + 3\sqrt{3})^2 + 159^2 = (300 - x)^2$; or $6x\sqrt{3} + 27 + 25281 = 90000 - 600x$; hence we have $x = \frac{10782}{100+\sqrt{3}}$ = 105.9843 the height of the part left standing.

Question 95, *by Mr. Jonathan Mabbott, of Oldham, Lancashire.*

Sterling, at page 27 of his Methodus Differentialis, says, that the aggregate of the thirteen initial terms of the series $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} + \frac{1}{9.10} + \frac{1}{11.12} + \frac{1}{13.14} + \frac{1}{15.16} + \frac{1}{17.18} + \frac{1}{19.20} + \frac{1}{21.22} + \frac{1}{23.24} + \frac{1}{25.26}$, is = to .674285961. Required the investigation?

Solution, *by the Proposer.*

If each numerator be multiplied into all the denominators but its own, for a new numerator, we shall have the following 13 new numerators, *viz.*

2016457305633028177920000000
336076217605504696320000000
134430487042201878528000000
72016332344036720640000000
44810162347400626176000000
30552383418682245120000000
22158871490472837120000000
16803810880275234816000000
13179459513941360640000000

$$\begin{array}{r} 10612933187542253568000000 \\ 8729252405337784320000000 \\ 7306004730554449920000000 \\ 6204484017332394393600000 \\ \hline \end{array}$$

Their sum is $27193377046163106594816000000$

And the product of the denominators, or common denominator is 403291461126605635584000000. Whence by dividing the sum of the numerators by the denominator, the quotient is $.674285961\frac{32783648840186763778}{403291461126605635584}$, which was to be investigated.

Question 96, *by Mr. Thomas Leybourn.*

Required the sum of the series $\frac{1}{2.6.8} + \frac{1}{3.8.10} + \frac{1}{4.10.12} +$, &c. by Mr. Sterling's method?

Solution, *by Mr. Jonathan Mabbott.*

The general term of this series is $\frac{1}{\overline{z+1}.\overline{2z+4}.\overline{2z+6}}$ in which 1, 2, 3, 4, &c. must be respectively wrote for z, and $\frac{1}{\overline{z+1}.\overline{2z+4}.\overline{2z+6}}$, must be reduced to a summable form, and which, by division, becomes $\frac{1}{4z^3} - \frac{6}{4z^4} + \frac{25}{4z^5} - \frac{90}{4z^6} + \frac{201}{4z^7} -$ &c. hence (Vide Sterling's Methodus Differentialis, page 11, 12) $A = o$, $B = 1$, $C = -6$, $D = 25$, $E = -90$, $F = 201$, &c. and hence $a = o$, $b = 1$, $c = -3$, d, e, f, &c. $= o$, and therefore $\frac{1}{\overline{z+1}.\overline{2z+4}.\overline{2z+6}}$, reduced to a summable form is $\frac{1}{4z.\overline{z+1}.\overline{z+2}} - \frac{3}{4z.\overline{z+1}.\overline{z+2}.\overline{z+3}}$. Whence (by Cor. 1, to Prop. 2, page 23) the sum will be $\frac{1}{8z.\overline{z+1}} - \frac{1}{4z.\overline{z+1}.\overline{z+2}}$, or $\frac{1}{8z.\overline{z+1}.\overline{z+2}}$, which when $z = 1$, becomes $\frac{1}{8.2.3} = \frac{1}{48}$, the sum required.

Question 97, *by Mr. Thomas Leybourn.*

Required the sum of the series $\frac{1}{2.6.4.5} + \frac{1}{4.8.5.6} + \frac{1}{6.10.6.7} +$ &c. by Mr. Sterling's method.

Solution, *by Mr. Jonathan Mabbott.*

The expression defining the terms of the given fines is $\frac{1}{2z.2z+4.z+3.z+4}$, in which $z = 1, 2, 3$, &c. and $\frac{1}{2z.2z+4.z+3.z+4}$ reduced to a summable form, (as in the preceding solution) becomes $\frac{1}{4z.z+1.z+2.z+3} - \frac{3}{4z.z+1.z+2.z+3.z+4}$. Whence (by Cor. 1, to Prop. 2, page 23) the sum will be $\frac{1}{12z.z+1.z+2} - \frac{3}{16z.z+1.z+2.z+3}$, which joined together is $\frac{4z+3}{48z.z+1.z+2.z+3}$. Now, for z substitute unity, and we have $\frac{7}{48.\ 2.\ 3.\ 4} = \frac{7}{1152}$, the sum required.

Question 98, *by Mr. Thomas Mabbott.*

Required the sum of the series $\frac{1}{1.2.6} + \frac{3}{1.3.8} + \frac{5}{3.4.10} +$ &c. by Mr. Sterling's method.

Solution, *by Mr. Jonathan Mabbott.*

Every term of the given series is assigned by $\frac{2z-1}{z.z+1.2z+4}$, in which $z = 1, 2, 3, 4$, &c. and in order to reduce this expression to a summable form, it must be reduced into an infinite series, and will then become $\frac{2}{2z^2} - \frac{7}{2z^3} + \frac{17}{2z^4} - \frac{37}{2z^5} +$ &c. compare this series with the general one, (Vide Sterling's Methodus Differentialis, page 11 and 12) and A $= 2$. B $= -7$, C $= 17$, D $= -37$, &c. these values being substituted in the general one, of a due form *(ibid)*, we have $a = 2$, $b = -5$, c, d, e, &c. $= 0$, consequently the series breaks off, being accurately $\frac{2}{2z.z+1} - \frac{5}{2z.z+1.z+2}$ which is of a due form. Whence (by Cor. 1, to Prop. 2, page 23) the sum will be $\frac{2}{2z} - \frac{5}{4z.z+1}$, or $\frac{4z-1}{4z.z+1}$. In which, if for z be wrote its first value, unity, we have $\frac{3}{4.2} = \frac{3}{8}$, the sum required.

Question 99, *by Mr. Thomas Leybourn.*

Required the sum of the series $\frac{1}{1.9.10} - \frac{1}{2.12.12} + \frac{1}{3.15.14} -$ &c. by Mr. Sterling's method.

Solution, *by Mr. Jonathan Mabbott.*

The terms of this series are assigned by $\overline{-1|}^{z-1}$ into $\frac{1}{z.\overline{3z+6}.\overline{2z+8}}$, the values of z being 1, 2, 3, &c. and $\overline{-1|}^{z-1}$ into $\frac{1}{z.\overline{3z+6}.\overline{2z+8}}$ converted into a series will be $\overline{-1|}^{z-1}$ into $\overline{\frac{1}{6z^3}-\frac{6}{6z^4}+\frac{28}{6z^5}-\frac{120}{6z^6}+\frac{496}{6z^7}}$ — &c. hence $A=o$, $B=1$, $C=-6$, $D=28$, $E=-120$, $F=496$, &c. then $a=o$, $b=1$, $c=-3$, $d=3$, e, f, g, &c. $=o$; consequently the series breaking off $\overline{-1|}^{z-1}$ into $\frac{1}{z.\overline{3z+6}.\overline{2z+8}}$ when reduced into a summable form, accurately becomes $\overline{-1|}^{z-1}$ $\overline{\frac{1}{6z.\overline{z+1}.\overline{z+2}}-\frac{3}{6z.\overline{z+1}.\overline{z+2}.\overline{z+3}}+\frac{3}{6z.\overline{z+1}.\overline{z+2}.\overline{z+3}.\overline{z+4}}}$, and comparing these terms, with those in the Theorem, in Prop. 3d. Sterling's Methodus Differentialis, page 30, we have $x=-1$, $n=-1$, $a=o$, $b=o$, $c=\frac{1}{6}$, $d=-\frac{3}{6}=-\frac{1}{2}$, $e=\frac{3}{6}=\frac{1}{2}$, then $A=o$, $B=o$, $C=\frac{1}{12}$, $D=-\frac{1}{8}$, E, F, &c. $=o$. Whence the sum will be $\overline{-1|}^{z-1}$ into $\overline{\frac{1}{12z.\overline{z+1}.\overline{z+2}}-\frac{1}{8z.\overline{z+1}.\overline{z+2}.\overline{z+3}}}$, which joined together becomes $\overline{-1|}^{z-1}$ into $\frac{2z+3}{24z.\overline{z+1}.\overline{z+2}.\overline{z+3}}$, in which, if the first value of z, (unity) be wrote, we have $\frac{5}{24.2.3.4}=\frac{5}{24^2}=\frac{5}{576}$, the sum required.

Question 100, *by Mr. Thomas Leybourn.*

Required the sum of the series $\frac{1}{1.6.10.21}-\frac{1}{2.8.12.24}+\frac{1}{3.10.14.27}-$ &c. by Mr. Sterling's method.

Solution, *by Mr. Jonathan Mabbott.*

The general term of this series is $\overline{-1|}^{z-1}$ into $\frac{1}{z.\overline{2z+4}.\overline{2z+8}.\overline{3z+18}}$ in which $z=1$, 2, 3, 4, &c. and $\overline{-1|}^{z-1}$ into $\frac{1}{z.\overline{2z+4}.\overline{2z+8}.\overline{3z+18}}$, reduced into a summable form is $\overline{-1|}^{z-1}$ into

$$\frac{1}{12z.z+1.z+2.z+3} - \frac{6}{12z.z+1.z+2.z+3.z+4} + \frac{15}{12z.z+1.z+2.z+3\ z+4.z+5}$$

$- \frac{15}{12z.z+1.z+2.z+3.z+4.z+5.z+6}$, which may be compared with the Theorem in Sterling's 3d Prop. page 30, thus $x = -1$, $n = -1$, $a = o$. $b = o$, $c = o$. $d = \frac{1}{12}$, $e = -\frac{6}{12} = -\frac{1}{2}$, $f = \frac{15}{12} = \frac{5}{4}$. $g = -\frac{15}{12} = -\frac{5}{4}$. Then A $= o$, B $= o$, C $= o$, D $= \frac{1}{24}$, E $= -\frac{1}{6}$. F $= \frac{5}{24}$, G, and all the rest $= o$; these values properly substituted in the Theorem, the sum will be $\overline{-1}|^{z-1}$ into

$$\frac{1}{24z.z+1.z+2.z+3} - \frac{1}{6z.z+1.z+2.z+3.z+4} + \frac{5}{24z\ z+1.z+2.z+3.z+4.z+5}$$

which joined together, becomes $\overline{-1}|^{z-1}$ into $\frac{z^2+5z+5}{24z.z+1.z+2.z+3.z+4\ z+5}$, in which, if for z its first value, unity, be wrote, we have $\frac{1+5+5}{24.2.3.4.5.6} = \frac{11}{17280}$, the sum required.

ARTICLE XIII.

A short Essay on Mechanics;

With practical Rules for finding the CENTER of GRAVITY of LINES, SUPERFICIES, and SOLIDS,

By Mr. JOHN BICKFORD, of Westminster.

AMONG all the branches of useful knowledge that of Mechanics seems to deserve our greatest attention. By the application of Mechanics to useful purposes, are derived the greatest part of the comforts of life :—What House can be built to shelter us from the inclemency of the weather, without the aid of Mechanics ?—What Mill to grind the corn that makes the bread we eat, without the application of mechanical powers ?—Or the very clothing we wear, is it not manufactured by machines of various constructions brought to their present perfection by the improvements the moderns have made in

Mechanics?—If we contemplate further on this part of natural knowledge we shall find, that the various operations of nature are carried on, by the Maker of the Universe, upon the most perfect mechanical principles: that the greatest exertions that we can make, and improvements accomplish, are only faint imitations of the mechanical laws of nature, which discoveries or improvements are most probably conveyed to us by the divinity at various periods of time, for the convenience and comfort of human life.

To instance some, amongst a great number of others, that have been made in the last centuries, may suffice. What amazing advantages are derived to the Merchant and to mankind in general, from the discovery of the Mariner's Compass! Great are the improvements which have been made in the properties of Air, since the discovery of the Air-pump. What great improvements in the properties of nature does the Electrifying Machine afford us? A great many more might have been mentioned equally important—but as this may be thought a digression from the subject—I shall return to that part of Mechanics which I had in view.

I am convinced that no part of Mechanics is more necessary to be understood by the young Artist than the knowledge of finding the center of Gravity of Lines, Superficies and Solids, particularly of the latter. In prosecuting this part, I intend giving the practical rules only, in as easy and concise a manner as possible, omitting the demonstrations; as a number of young Mechanics have an opportunity of pursuing the practical part only, who have not time to apply themselves to the theory. Those gentlemen who have time and inclination to demonstrate the propositions, will find much assistance from EMERSON'S or SIMPSON'S Fluxions; OZANAM'S Mathematics; DR. WALLIS'S Mechanics; PARKINSON'S Mechanics, &c.

REMARK.

A Mathematical Line or Superficies cannot be said to have thickness, and therefore void of gravity; but in practice they may be conceived to be extremely thin, and then we consider them as parts having gravity; which by the following Rules the center of gravity, of any simple or compound figure may be obtained.

DEFINITION.

The Center of Gravity of a Line Superficies or Solid is a certain point, upon which if the figure be suspended, all the parts will remain in equilibrium.

To find the Center of Gravity of Lines.

PROPOSITION I.

To find the Center of Gravity of a given Right-Line.

RULE. Divide the given right-line AB (Fig. 5) into two equal parts, the middle point C will be the center of gravity.

PROPOSITION II.

To find the Center of Gravity of two given Right-Lines.

Rule. Find the center of gravity F (Fig. 6) and E of each given line AB and CD by the last Prop. and draw the line FE, then AB + CD : FE :: DC : FG, then is G the center of gravity required*.

PROPOSITION III.

To find the Center of Gravity of three given Right-Lines.

Rule. Find the common center of gravity of two of the given lines, as AC (Fig. 7) and CD which will be at I, from I draw a line to G, the center of gravity of the line AB, then the line AB + AC + CD : GI :: AB : IH, then is H the center of gravity of the three given lines.

In like manner may the center of gravity of any number of right lines be found, by first finding the center of gravity of any two of them, and then connecting that center of gravity with the center of gravity of a third line, and then to a fourth line, &c.

PROPOSITION IV.

To find the Center of Gravity of the Periphery of a Square, Parallelogram, Rhombus, or Rhomboides.

Rule. Draw the two diagonals AB (Fig. 8 and 9) and CD, and the point of intersection G will be the center of gravity.

* The line FE is considered without Gravity, and the same is to be understood in the following propositions.

PROPOSITION V.

To find the Center of Gravity of the Periphery of a right-lined Triangle.

Rule. This may be done by Prop. 3, or more conveniently as follows:—Find the center of gravity of each side, by Prop. 1, then from these centers of gravity, draw the triangle EFD (Fig. 10) and bisect any two of the angles of this new triangle, and where the bisecting lines FH and EI cross each other, as at G, will be the center of gravity of the periphery of the given triangle.

PROPOSITION VI.

To find the Center of Gravity of the Periphery of any Quadrilateral Figure.

Rule. This may be done by the latter part of the Rule to Prop. 3. or by this method —Find the center of gravity of each of the four sides, as at F, G, H, E, (Fig. 11) by Prop. 1. from those parts draw the quadrilateral figure EFGHE, then bisect the angles A, B, C, D, by the lines AK, BN, CR, and DI, make HP = EI, HO = GR, FM = GN and FL = EK, then draw the lines LO and PM, where they intersect as at Q will be the center of gravity.

PROPOSITION VII,

To find the Center of Gravity of a given Arch of a Circle.

Rule. Draw the line CD (Fig. 12) to bisect the given arch, then as the arch ADB : its chord AB :: the radius CD : CE, so is E the center of gravity.

To find the Center of Gravity of a plain Superficies.

PROPOSITION I.

To find the Center of Gravity of a given Square, Parallelogram, Rhombus or Rhomboides.

RULE. Draw the diagonals as in Prop. 4, of their peripheries, and the point of intersection will be the center of gravity.

PROPOSITION II.

To find the Center of Gravity of a given Triangle.

Rule. Bisect any two of its sides as AB (Fig. 13) and BC in the points F and E, and draw lines from their opposite angles as AE and CF, the points of intersection G will be the center of gravity. Or it is two-thirds of either of these lines from the angular points that is CG is two-thirds of CF, or AG is two-thirds of AE.

PROPOSITION III.

To find the Center of Gravity of a given Trapezoid.

Rule.—Divide the given Trapezoid ABCD (Fig. 14.) into two triangles ADB and BCD and find the center of gravity of each by the last proposition, which will be at I and H, bisect the two parallel sides AB and DC in F and G and draw the line FG, then draw the line HI, and where this line cuts the line FG as at K is the center of gravity of the given trapezoid.

Or by this method, having divided the trapezoid into two triangles as before, and find the center of gravity of each, and also drawn the line HI joining these centers, then as the area of the trapezoid : the line HI : : the area of the triangle ABD : IK, which gives K the center of gravity as before.

PROPOSITION IV.

To find the Center of Gravity of a given Trapezium.

Rule.—First divide the given trapezium ABCD (Fig. 15.) into two triangles by drawing the diagonal AC then find the center of gravity of the triangle ABC by Prop. II. which is at E, also find the center of gravity of the triangle ADC by the same Prop. which is at F, and draw the line EF. Draw the diagonal BD, and in like manner find the center of gravity of the triangle DCB which is at H, and of the triangle DAB, which is at G, then draw the line GH, and where this line intersects the line EF as at I, is the center of gravity of the given trapezium.

The latter Rule of the last Prop. will hold equally true for this, for it will be as the area of the trapezium : the line GH : : the area of the triangle DAB : HI, which gives I the center of gravity as before.

PROPOSITION V.

To find the Center of Gravity of any given irregular Polygon.

Rule. Divide the given polygon ABCDEFGH, (Fig. 1, pl. 8) into triangles, and find the center of gravity of each by Prop. 2, which will be at I, K, L, M, N, O, and draw the line IK, then as the area of the triangle FED + the area of the triangle FDC : IK :: the area of the triangle FED : KP, so is P the common center of gravity of the two triangles FED and FDC. Draw the line PL, then as the area of the three triangles FEDCB : the line PL :: the two triangles FEDC : LQ, so is Q the center of gravity of the three triangles FEDCB, and thus you may proceed to find the center of gravity of the remaining triangles connected with the preceding, which will be at R, S and T, but T being the last will be the center of gravity of the whole figure.

PROPOSITION VI.

To find the Center of Gravity of a Semicircle.

Rule. Let ACBD (Fig. 2) be the given semi-circle, then as one fourth of the circumference of the circle AD : the radius CD :: two thirds of the said radius : CG, so is G the center of gravity of the semicircle.

PROPOSITION VII.

To find the Center of Gravity of a given Sector of a Circle.

Rule. Let ADBC (Fig. 3) be the given sector; first find the center of gravity of the arch ADB by Prop. 7, of the center of gravity of lines, which will be at F, then two-thirds of FC will give CG, so is G the center of gravity of the given sector.

PROPOSITION VIII.

To find the Center of Gravity of the Segment of a Circle.

Rule. Let ABD (Fig. 3) be the given segment, complete the sector ABCD and find its center of gravity G by the last proposition, and also the center of gravity E of the triangle ACD by Prop. 2, then as the area of the segment ABD : the triangle ACB :: EG the distance of the centers of gravity of the triangle and sector : GH so is H the center of gravity of the given segment.

PROPOSITION IX.

To find the Center of Gravity of a given Lune.

Rule. Let ACBDA (Fig. 4) be the given lune.—First find the center of gravity of the ſegment AFBD, by Prop. 8, which will be at E, then find the center of gravity of the other ſegment ACBF, by the ſame Prop. which will be at G, then as the area of the lune ACBD : the line EG : : the area of the ſegment ADBF : GH which gives the point H the center of gravity of the lune.

PROPOSITION X.

To find the Center of Gravity of a given Parabola.

Rule. Let ACB (Fig. 5) be the given parabola, then divide the abſciſſa DC into 5 equal parts, and ſet 2 of them from D to E, which gives the point E the center of gravity.

PROPOSITION XI.

To find the Center of Gravity of the Fruſtum of a Parabola.

Rule. To the greateſt double ordinate AB (Fig. 5) add twice the double ordinate in the middle between the given ordinates KL*, multiply the ſum by the ſquare of the height or abſciſſa HD, and divide the product by 6 times the area of the fruſtum, the quotient will be HI the diſtance of the center of gravity from the leſs end.

To find the Center of Gravity of Solids.

PROPOSITION I.

To find the Center of Gravity of a Cube, Parallelopipedon, Priſm, or Cylinder.

RULE. Divide the axis of theſe reſpective ſolids into 2 equal parts, and the middle point will be the center of gravity required.

* The middle diameter is found thus: To the ſquare of the leſs double ordinate FG, add the half difference of the ſquares of the given double ordinates FG and AB, the ſquare root of the ſum will be the double ordinate in the middle KL.

PROPOSITION II.

To find the Center of Gravity of a given Cone or Pyramid.

Rule. Let ABC (Fig. 6) be the given cone, CD the axis, then $\frac{1}{4}$ of CD being set from D to E will give the point E the Center of Gravity.

The very same Rule serves for the pyramid.

PROPOSITION III.

To find the Center of Gravity of a given Semisphere.

Rule. Let ABC (Fig. 7) be the given semisphere. Divide the radius of the sphere CD into 8 equal parts, set 5 of them from C to E, or 3 from D to E, which gives the point E the center of gravity.

The same Rule holds true for any segment of a sphere less than a semisphere, by dividing the height of the segment into 8 equal parts as before, and setting 5 from the top, or 3 from the base, will give the center of gravity of the segment.

But if the segment be greater than a semisphere, as HCI, (Fig. 7). Let the line FG be drawn at an equal distance from AB the diameter of the sphere to HI, then will the center of gravity of the zone HIGF be at the center of the sphere at D, and the center of gravity of the segment FCG will by the last Prop. be at L, then as the solidity of the segment HCI : the line DL : : the solidity of the segment FCG : HT, which gives T, the center of gravity of the given segment.

PROPOSITION IV.

To find the Center of Gravity of a given Sector of a Sphere.

Rule. Let ABCD (Fig. 8) be the sector of a sphere, the part ABCEA will be the segment of a sphere, and its center of gravity will be found by the last Prop. to be at F, the part ACD will be a cone, and its center of gravity, by Prop. 2, will be at G, then as the solidity of the whole sector ABCD : the distance of the centers of gravity FG : : the solidity of the cone : the line FH, and H is the center of gravity required.

PROPOSITION V.

To find the Center of Gravity of a Paraboloid.

Rule. Let ADB (Fig. 9) be the given Paraboloid. Divide the abciſſa DC into 3 equal parts, and ſet 2 of them from D to E, or 1 from C to E, which gives the point E the center of gravity of the paraboloid. The inſcribed plain triangle has the ſame center of gravity as the paraboloid.

PROPOSITION VI.

To find the Center of Gravity of the Fruſtum of a Paraboloid.

(3)

6.—*By Coſmopolite.*

If a point be taken any where within an equilateral triangle, and three ⊥'s. be let fall from that point on the three ſides; the ſum of theſe perpendiculars is equal to the ⊥ of the triangle. Required a new and conciſe geometrical demonſtration?

7.—*By Mr. John Griffith, of London.*

Let there be a right line, circle and parabola, all given in magnitude and poſition, and required to draw 2 right lines from the extremes of the lines, one to cut the circle and the other the parabola. So that joining the points in the curve of the circle and parabola, where the ſaid lines cut the curves the ſum of the three lines, ſo drawn ſhall be a minimum.

PROPOSITION I.

To find the Center of Gravity of two given Weights or Bodies, ſuſpended on a given Rod, ſuppoſed to be void of Weight.

RULE. As the ſum of the weights or bodies C (Fig. 10) and D: the length of the rod AB : : the weight D : AG, which gives G the center of gravity.

PROPOSITION II.

To find the Center of Gravity of a given Cone or Pyramid.

Rule. Let ABC (Fig. 6) be the given cone, CD the axis, then $\frac{1}{4}$ of CD being set from D to E will give the point E the Center of Gravity.

The very same Rule serves for the pyramid.

PROPOSITION III.

(4)

REMARKS TO CORRESPONDENTS.

MR. BICKFORD observes, that Rule to Prop. 3, page 136, is not general for finding the center of gravity of any segment of a sphere, but is true only in the semi-sphere, though asserted by Martin, in his Mathematical Institutions, Art. 1083; also by Ozanam in his Mathematics, and others.—The true Rule to find the center of gravity of the segment of a sphere is this:

From 8 times the radius of the sphere multiplied by the height of the segment, subtract 3 times the square of the height for a dividend: from 12 times the radius of the sphere, subtract 4 times the height of the segment for a divisor, the quotient arising will be the distance of the center of gravity from the vertex of the segment at C.

PROPOSITION IV.

To find the Center of Gravity of a given Sector of a Sphere.

Rule. Let ABCD (Fig. 8) be the sector of a sphere, the part ABCEA will be the segment of a sphere, and its center of gravity will be found by the last Prop. to be at F, the part ACD will be a cone, and its center of gravity, by Prop. 2, will be at G, then as the solidity of the whole sector ABCD : the distance of the centers of gravity FG : : the solidity of the cone : the line FH, and H is the center of gravity required.

PROPOSITION V.

To find the Center of Gravity of a Paraboloid.

Rule. Let ADB (Fig. 9) be the given Paraboloid. Divide the abciſſa DC into 3 equal parts, and ſet 2 of them from D to E, or 1 from C to E, which gives the point E the center of gravity of the paraboloid. The inſcribed plain triangle has the ſame center of gravity as the paraboloid.

PROPOSITION VI.

To find the Center of Gravity of the Fruſtum of a Paraboloid.

Rule. To the area of the greateſt end AB (Fig. 9) add twice the area of a ſection in the middle GF. Multiply the ſum by the ſquare of the height CH, and divide the product by ſix times the ſolidity of the fruſtum, the quotient will be HI, the diſtance of the center of gravity from the leſs end.

This Rule is general for the fruſtum of the hyperboloid; and the ſquare of the diameters above may be uſed inſtead of their areas.

The ſame Rule ſerves alſo for the fruſtum of a cone or pyramid, let the figure of their baſes be what they will.

To find the Center of Gravity of Bodies ſuſpended on Lines or Rods.

PROPOSITION I.

To find the Center of Gravity of two given Weights or Bodies, ſuſpended on a given Rod, ſuppoſed to be void of Weight.

RULE. As the ſum of the weights or bodies C (Fig. 10) and D : the length of the rod AB : : the weight D : AG, which gives G the center of gravity.

PROPOSITION II.

Given as in the last Prop. together with the Weight of the Rod on which the Weights are suspended.

Rule. Multiply the weight of the rod by half its length; to which add the product of the length of the rod multiplied by the less weight; divide this sum by the sum of the two weights, and the weight of the rod, and the quotient will be the distance of the center of gravity from the less weight.

PROPOSITION III.

To find the common Center of Gravity of a System of Bodies suspended on a given Rod, when the Weight of the Rod is considered.

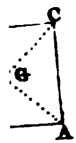
Rule. Multiply the weight of the rod (Fig. 11) by the distance of the center of gravity from the end A, (which if a prism will be in the middle of the length*) Multiply AB by the weight H, AC by the weight I, AD by the weight K, and AE by the weight L, &c. then add all those products together for a dividend. Add the weight of the rod to the sum of the weights suspended G, H, I, K, L, &c. for a divisor; the quotient arising will be the distance of the common center of gravity of all the weights and rod from the end at A.

PROPOSITION IV.

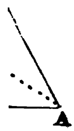
To find the Center of Gravity of a System of Bodies, when acted upon by their respective Attractions.

Rule. Let A, B, C, D, (Fig. 12) be the given bodies whose common center of gravity is required. Join the two centers of the bodies A and B with the line AB, and by Prop. 1st. aforegoing, find the center of gravity E, draw the line EC; then say—As the quantity of matter in the three bodies A, B, and C: the line EC :: the quantity of matter in A and B: the line CF, which gives the point F, the common center of gravity of the three bodies ABC. Draw the line FD, then as the quantity of matter in the four bodies A, B, C, D: the line FD :: the quantity of matter in the three bodies A, B, C; the line DG which gives the point G the center of gravity of the whole system; and so you may proceed, to find the common center of gravity, let the number of bodies be ever so many.

* If the rod or beam should be of any other form, its center of gravity may be found by the foregoing propositions of the center of gravity of solids.


C
G
A


A


A


A
B
P
D
C

ARTICLE XIV.

A Letter concerning the Influence of the Moon,

THE queen of night, with vast command,
Rules o'er the sea, and half the land;
And over moist and crazy brains,
In high spring tides at midnight reigns.

BUTLER.

Sir,

AS some observations of mine have appeared in No. 10, of the *Receptacle*, as an answer to a Query in No. 9, proposed by Mr. *Davis*, concerning the influence of the moon, which the most cursory reader must perceive, tend but little to elucidate that subject; yet, as it is worth considering more attentively, and requires a better explanation than is there given, I have, however incompetent to the task, thrown together the following supervenient observations. Whenever a person finds occasion to correct the opinion of another, he ought not to (except he wishes to gain the name of a cynic) use a single illiberal reflection: but, alas! it is too frequently the case in polemical writings for a multitude of these to be dispersed around; and then, as may naturally be expected, the opposite party retorts them with anatocism. For my part, I have too great a respect for Dr. *Sangrado*, and every other gentleman of genius, to entertain an idea of using illiberal treatment; my sole object is the pursuit of truth; for, as Dr. Jortin has elegantly observed, "These remarks are designed, slight and imperfect as they are, for the service of TRUTH, by one who would be glad to attend and grace her triumphs—*as her soldier*, if he have had the honor to serve successfully under her banner; or, *as her captive*, tied to her chariot wheels, if he have, though undesignedly, committed an offence against her."

We have the united and corroborating testimony of ages to evince, that lunatics are materially affected about the time of the new and full moons: perhaps, I may go too far, when I say that the first appearances of the catamenia, are influenced by a similar cause; under this head, much may be said both pro and con, and as it would be digressing from the immediate subject of this letter, I shall forbear enlarging upon it. I shall, most probably, be thought fanciful in a great degree, if I assert that a less quantity of liquor will render a person inebriated about the new and full moons, than would intoxicate the same person at any other period: but if facts may be depended upon, and "Facts are stubborn Things," this is undoubtedly the case.

The grand question is, how are these effects accounted for? Those who believe that "The pressure of the atmosphere will be lessened" at these periods, would do well to reflect upon the passage in *Bonnycastle's* Astronomy, which I have previously referred to: as I suppose very few of the readers of the *Delights* are without that elegant and useful introduction, I shall not copy the passage into this place. Besides, if the pressure of the atmosphere were lessened at these times, would it not be indicated by the barometer?—It is manifest that it would. The barometer is so intimately connected with changes in the atmosphere, that when its pressure is lessened, a depression of the mercury in the tube invariably happens: but from all the observations I have made, or heard of, I could never find that it was oftener depressed than elevated at these important changes of the moon. In some places within the Tropics, the mercury rises near half an inch in the day, and falls as much in the night, without any material variation at any period: but if the ærial tides lessened the pressure of the atmosphere, circumstances would be considerably different.

Others again, may suppose that lunatics are influenced by some particular motion in the air, at these periods: but such an hypothesis will be found to have but little weight, if what writers on fluids have mentioned be properly attended to. It has been observed in rivers, &c.* that the parts near the surface have been in a rapid motion, when at the same time, at the depth of four or five feet, the motion has been very gentle, and at the bottom there has been little or no motion. Hence may we not reasonably infer from analogy, that though the upper parts of the atmosphere are put into motion by the joint efficacy of the sun and moon, yet no such motions can be perceived in those parts near the earth's surface?

As none of these causes then have enough of reality to account for the effects produced, I shall here advance another—whether it be more consentaneous to reason, must be left for the impartial reader to determine. The incomparable Sir Isaac Newton, in the Principia, speaks of a *subtile spirit* or *etherial medium*, by the force and action of which, he imagines bodies attract each other, by its vibrations he supposes sensation is excited, and hence muscular motion is performed, &c. It is exceedingly probable that this elastic and electric spirit pervades the whole universe, and it would seem that it is the source of planetary attraction. If the existence of this subtile medium be allowed, surely it will not be difficult to account for the change in lunatics: for, if magnetic effluvia be known to pass with ease through gold, there can be but little room to doubt that this spirit will enter through the pores, &c. of a human body and find its way to the blood and fluids. And as this etherial medium will evidently have different force at different ages of the moon, it may very probably cause same peculiar intestine motion, by which the paraxysms

* Vide M. Marriot's Hydrostatics.

of the perſon (who we muſt allow, has the prediſpoſing cauſe of lunacy) may be occaſioned. Again, the vibrations of the medium, may be a mean of it's being conveyed through the capillaments of the nerves unto the brain, and will, on account of it's different power at the moon's changes, have ſuch an effect as may cauſe the Bacchanalian, who has *ſpun* out the evening, to *reel* home from his revel, at an earlier hour than he would otherwiſe have done.

With reſpect to the joint influences of the ſun and moon on the weather, I muſt confeſs I know not how to determine. As I have never read any Treatiſe deſcribing their influence on the human body, or on the weather, I am all the while groping in the dark: I am fearful if I proceed farther, I ſhall deviate from the right path.—With diffidence I advance it as my opinion, that if there be any change in the weather at the new and full moons, it muſt be principally occaſioned by the motion of the water in high tides promoting a greater quantity of vapour; but, it is certain the increaſe of evaporation will be very trifling; therefore this cauſe fails. When we conſider the heights at which clouds are uſually ſuſpended, it will not ſeem likely (remembering what has been before ſaid) that the ærial tides can affect them: With regard to my obſervations, I can ſafely ſay, that either in winter or ſummer, when I have looked for a change of weather ſoon after a new or full moon, I have been almoſt always diſappointed. But ſome advocates for the lunar influence have told me, that a change of the weather is almoſt ſure to occur within three days before or three days after a change or quarter of the moon: from one quarter to the next is not *eight* days, and theſe Gentlemen take *ſeven* into their limits of probability: thus is the lunar influence reduced to very preciſe and diſcriminating rules; I leave thoſe Gentlemen quietly to enjoy the honor due to ſuch ſagacity and penetration!

But I am afraid I have extended my remarks too far; I ſhall therefore draw to a concluſion after obſerving, that if any of your ingenious Correſpondents ſhould diſcover that I have advanced any thing falſely, I ſhall be exceedingly pleaſed to ſee a correction of my errors in a future Number: I have the cauſe of truth at heart, which I hope will ſuperſede the neceſſity of apologizing for ſending you this epiſtle.

I REMAIN,

SIR,

YOUR MOST OBLIGED SERVANT,

OLINTHUS GILBERT GREGORY.

ARTICLE XV.

Questions and Solutions.

Question 101, *by Cullen O'Conner.*

TRANSFORM an equation into another, the roots of which are the *m*th powers of the roots of the original equation.

Solution *by Mr. John Lowry, Excise-Officer, Solihull, near Birmingham.*

Rule. Substitute the *m*th root of some new letter, instead of the unknown quantity in the given equation, and it gives the equation required.

Example. Let $3y^3 + 6y^2 - 4y - 40 = 0$, and $m = 2$, then if for y we put $(z^{\frac{1}{m}})$ $z^{\frac{1}{2}}$ it becomes $(3z^{\frac{3}{m}} + 6z^{\frac{2}{m}} - 4z^{\frac{1}{m}} - 40 = 0.) = 3z^{\frac{3}{2}} + 6z - 4z^{\frac{1}{2}} - 40 = 0$, an equation whose roots are the square of the roots in the original equation.

Question 102, *by Mr. T. Leybourn:*

To find 3 biquadrate numbers, the sum of which shall be a square.

Solution, *by Mr. John Lowry.*

Assume $3x$, $4x$, and $1\frac{2}{5}x$ for the sides of the required biquadrates, then $(3x)^4 + (4x)^4 + (1\frac{2}{5}x)^4 = \left(\frac{481x^2}{25}\right)^2$ which is manifestly a square number, where x may be taken at pleasure. If $x = 1$, the numbers are, 81, 256, and $\frac{20736}{625}$. In the same manner may other answers be found adinfinitum.

Question 103, *by Mr. Thomas Leybourn.*

To find 3 numbers such, that if their sum be multiplied by the first, it shall be a triangular number, by the second a square, and by the third, a cube.

Solution, *by Mr. John Lowry.*

Assume $\frac{x}{2}$, $\frac{1}{2}$, and $\frac{x}{2} + \frac{1}{2}$ for the numbers, then their sum is $x + 1$, and $(x+1) \times \frac{x}{2} = \frac{x^2+x}{2}$ = a triangular number, $(x + 1) \times \frac{1}{2} = \frac{x+1}{2}$ = a square, and $\left(\frac{x+1}{2}\right)^2$ = a cube. Let $\frac{x+1}{2} = n^2$, then $x = 2n^2 - 1$, and $\left(\frac{x+1}{2}\right)^2$ becomes $= 2n^4$ which must be a cube number, or $2n \times n^3$ = a cube, or $2n$ = a cube $= a^3$, suppose then $n = \frac{a^3}{2}$ where a may be taken at pleasure, provided it be greater than 1. If $a = 2$, then $8\frac{1}{2}$, $\frac{1}{2}$, and 16 are the required numbers.

Question 104 *by Mr. James Stevenson.*

What proportion is there between the force of gravity at the surface of the earth and Georgium Sidus?

Solution *by Mr. J. H. Swale of Leeds.*

It is known from the observations of Dr. Hershal, that the nearest satelite of this planet performs a revolution round its primary in 8 days, 17 hours, and 1 minute (752460 seconds) of our time; at $16\frac{1}{2}$ semi-diameters distance therefrom ∴ putting the planet's semi-diameter $= a$; the satelites distance $= d$; time of its revolving $= n$; $3.1416 = m$; and $2x =$ the required measure of the force of gravity: we have $\sqrt{2ax} =$ the velocity per second in a circle at his surface; and $m\sqrt{\frac{2a}{x}} =$ time of revolution. Moreover from the universal laws of gravitation, $\sqrt{a^3} : \sqrt{d^3} :: m\sqrt{\frac{2a}{x}} : n$; then $n\sqrt{a^3} = m\sqrt{\frac{2ad^3}{x}}$: hence $x = \frac{2d^3m^2}{n^2a^2}$: ∴ the proportion as 6 is to 7; or as 1 to $1\frac{1}{6}$ extremely near, as required.

Question 105, *by Mr. John Griffith.*

Suppose the frustum of a cone, whose length and diameters are 30, 20 and 10 respectively to be divided into two equal parts by one cut, beginning in the middle. Required the direction of the saw, or the angle it must make with a line parallel to the base.

Solution *by Mr. John Ryley.*

Let ABCD (Fig. 2) be the given frustum of the cone, and ABV the cone compleated. By similar triangles VE is found $= 30$; ∴ by the question VO $= 45$, GH $= 15$. Now put ON $= x$ & GH $= d$; then by sim. triangles VO : GH :: VN : IL $= d + \frac{x}{3}$: But the content of the frustum GHCD is $=$ 1865.325 and half the content of the frustum ABCD is $=$ 2748.9 and the difference of these contents is the content of the ungula IHG $= 883.575 = a$. Hence Prob. 12, page 168, HUTTON's Mensuration we have

$$\frac{\overline{d+\frac{1}{3}x}\sqrt{\overline{d+\frac{1}{3}x}\times d-d^2}}{\frac{1}{3}x} \times .2618dx = a; \text{ or by reduction } \frac{\frac{(3d+x)^2}{9}\times\frac{3d+x}{3}}{}$$

$$\times d = \left(\frac{a}{.7854d}+a^2\right)^2; \text{ and from hence } x = \sqrt[3]{\frac{27}{a}\times\frac{a}{.7854d}+d^2}\Big|^2 - 3d =$$

9.5136, &c. $=$ HP; therefore IL $=$ 18.1712, IP $=$ 16.5856, and by Eu. 47. 1. IH $=$ 19.1204. Moreover by Trig. the sine of the angle LIH $=$ .4975625 $=$ the sine of $29^\circ\ 50\frac{1}{3}'$; which is the angle that the kerf of the saw must make with a line ‖ to the base of the frustum, in order to divide it into two equal parts.

Question 106, *by Mr. Stephen Hartly.*

A man was walking along the foot of an hill, and at the top of it he had the sight of a tower, bearing N. N. E. and by observation, its altitude was $18^\circ\ 20'$; travelling on E. S. E, for the space of 880 yards, he found it to bear N. W. by N. and its altitude only $12^\circ\ 40'$. How far was he from it when he first saw it? What was the height of it above the level, and the difference of height from each place of standing!

Solution *by Mr. John Ryley of Leeds.*

Let C (Fig. 9.) represent the hill, V the tower; C and D two places in the hill, which are upon an horizontal level with A and B; A being the place where he first observed the tower, and B the second; AE $=$ DC $=$ the difference of height between each place of observation. Now, because the bearing of the tower from A was N. N. E. and he afterwards travelled E. S. E. the angle BAC is a right angle, and as the bearing of the tower from B was N. W. by N. the angle ACB contains 5 points, or $56^\circ\ 15'$ and the angle ABC 3 points, or $33^\circ\ 45'$. Now as all the angles and side AB of the triangle ABC are given, we have per trig. AC $=$ 588 and CB $=$ 1058.36865 yards. Moreover in the triangle ACV, the angle VAC and side AC are given to find CV $=$ 194.842, hence AV $=$ 619.4412 $=$ the distance the man was from the tower when he first saw it. Now put BC $= a$, CV $= b$, tang. $\angle$ DBV $= t$, and CD $= x$. then BD $= \sqrt{a^2-x^2}$ and DV $= t\sqrt{a^2-x^2} = x+b$, and by involution $t^2a^2-t^2x^2 = b^2+2bx+x^2$; hence $x =$

$$\frac{\sqrt{t^2a^2+a^2-b^2}-b}{t^2+1} = 42.83 = \text{the difference of height between each place of standing.}$$

Question 107, *by Mr. Featherstonhaugh, Pupil to Mr. Howard.*

GODA is a given right line passing through the centres O, D of two circles GC, EB given in magnitude and position, it is required to draw a right line from one extreme A and cutting the circles in B and C severally, so that joining GC, GB the difference of the triangles GBA, GCB shall be given.

Solution *by the Proposer.*

Analysis. Suppose the thing done and AC (Fig. 3.) drawn as required, then since triangles whose heights are equal, are as their bases, consequently GBA : GCB :: BA : CB :: EA : GE by Apollonius) and EA—GE : EA :: GBA—GCB : GBA, therefore

$$GBA = \frac{EA \times \overline{GBA - GCB}}{EA - GE}$$ and consequently is known. Now, if upon GA a rectangle GAMI be constructed whose area is double the triangle GBA, the point of intersection of its opposite side IM with the circle EB will evidently determine the position of AC.

Question 108, *by Mr. W. Armstrong, Pupil to Mr. Howard.*

It is required to determine a point P, in the diameter AB, of a given semi-circle; so that if upon AP and PB, two semi-circles A*m*P P*n*B be described, the sum of their areas, may be to the space A*c*B*n* P*m*A, in any given ratio?

Solution *by the Proposer.*

Con. Let the given ratio be that of MN (Fig. 6.) to MR, produce MN to Q, so that NQ may be equal to MR; make BD ⊥ AB and take $AB\sqrt{\frac{1}{2}}$: BD :: MQ : MR draw DC ∥ AB meeting the circumference in C, demit CP perp AB & P is the point required.

Dem. Circles being to each other as the squares of their diameters $AB^2 - \overline{AP^2 + PB^2}$ = the space ACB*n*P*m* = (Bon. Geo. 11. 14 =) 2AP.PB = $2PC^2$ consequently $AB^2 : 2PC^2$ (:: $AB\sqrt{\frac{1}{2}}$: PC) :: MQ : MR and by division $AB^2 - 2PC^2$ (A*m*P + P*n*B) : $2PC^2$ :: MN (MQ—MR) : MR. Q. E. D.

Cor. Hence it appears that if upon the diameter of a semi-circle, two other semi-circles are described, and at their intersection a perp. be erected to meet the circumference, that a circle described thereon will be equal to the space included between their peripheries; for it is = 2 PC^2 = 2 semi-circles upon PC.

Question 109, *by Mr. W. Armstrong, Pupil to Mr. Howard.*

ACB is a given parabola, it is required to determine geometrically a point P in the base AB, so that drawing PQ ⊥ thereto, and meeting the curve in Q; AP + PQ may be a maximum.

Con. Draw the parameter LQ & demit the ⊥ QP & P will be the required point. (Fig. 10.)

Dem. Assume any other points KC and erect the ⊥s KD, GE, then if AR+KD be greater than AP+PQ, HD(KD—PQ) will be greater, then HQ (AP—AK but (Em. Con. B. iii. iv.) LQ : LH (LF+ID) :: HQ (FQ—ID : HD, but LQ is greater than LH ∴ HQ than HD : again if AG+GE be greater, then will NE (PG) be greater than NQ (PQ—GE.) but LQ : MN (LF+RE) :: NE (RE—LF : NQ & as MN is greater than LQ, NQ will be greater than NE. Q. E. D.

Question 110, *by Mr. John Lowry, Excise Officer, Solihull, near Birmingham.*

To determine a point in the curve of a given parabola, so that if a line be drawn from it perpendicular to the opposite side of the curve, the parabola may be divided in a given ratio by the said line?

Solution *by Mr. J. Hampshire, of London.*

Analysis. Let ABC (Fig. 13.) be given parabola, F the point from which the right line FD is to be drawn, dividing the parabola in the given ratio of $m^3 : n^3$ bisect FD in E, through which point draw the diameter EK meeting FG, an ordinate to the axis drawn from the point F in I, then area ABC : area FBD :: $CH^3 : FI^3$, (Cor. 2. 52. Sim. con. 5.) :: $m^3 + : n^3$ whence IF is given because CH is given, from D draw the tangent DO, which suppose ⊥ to FD meeting EK produced in O. Now because the parabola ABC is given, the parameter of its axis is given, which suppose P, then (Cor. 3, Hamilton's con. 3) $P \times EK = FI^2$, therefore EK is given, and by the property of tangents EK = KO therefore EO is given from D upon EO demit the ⊥ DR then the right-angled triangles DRE and FIE are sim. and equal, because DE=EF (by cons.) and the angle DER=FEI. Again the right-angled △s DOR and DRE being sim. we shall have $DR^2 = OR \times RE$ whence OR & RE=EI are given, because DR=IF & OE are given, and consequently DE=EF is given, again by another property of the parabola $DL \times P = FL \times LG$ whence GL is given, because DL=RI (by ‖ lines) =2EI=2ER by sim. △s and FL=2FI are given, hence this construction let $m^3 : n^3$ be given ratio of the area ADFC to the area DBF; upon CH take $Hv^3 : CH^3$ :: area DBF : area ABC :: $n^3 : m^3 + n^3$ find ek a third proportional to p the parameter of the axis BH and Hv which produce to o till $eo = 2ek$ which divide in r so that $er \times ro = Hv^2$, at the point r erect the ⊥ rd = Hv and join ed, do; make $2Hv \times 2vs = 2re \times p$ then on HC take $Hs = Hv + vs$ and at s the ⊥ sF meeting the curve in F, from which point draw the ordinate FG to the axis BH, upon FG take FL=2Hv and at L erect ⊥ LD cutting the parabola again in D, join DF and the thing is done.

Dem. Bisect DF in E, through which point draw the diameter EK meeting FG in I, from D let fall the ⊥ DR on EK and produce EK to O, so that EO may be equal to oe and join DO. Now by construction we have $Hv^3 : HC^3 :: n^3 : m^3 + n^3$ but $Hv = \frac{1}{2}$ FL (by const.) =DR=LI=IF by sim. △s because DE=EF therefore IF^3 (Hv^3) : HC^3 ::

$n^3 : m^3 + n^3$ :: area DBF : area ABC (Cor. 2, 52 Sim. con. 5) and by Div. and inversion $m^3 : n^3$ :: area ADFC : area DBF or the area ADFC : area DBF :: $m^3 : n^3$ in the given ratio by construction. Again (by cor. 3 Hamilton's con. 3) $p \times EK = FI^2 = Hv^2 = rd^2 = er \times ro = p \times ek$, whence EK $= ek = \frac{1}{2} eo = \frac{1}{2} Eo$, therefore EK $=$ KO $= ok$, and by the property of tangents OD is a tangent to the parabola, touching it in the point D; and by another property of the parabola $DL \times p = FL \times LG = 2Hv \times 2vs = 2re \times p$ (by Construction), ∴ DL $= 2re =$ RI by parallel lines $=$ 2RE $=$ 2EI by similar △s, because DE $=$ EF (by Construction), whence RE $=$ EI $= re$, and it has already been proved that DR $=$ LI $=$ IF $= rd$, whence the right-angled △s DRE EIF and *dre* are similar and equal, and consequently DR $=$ IF $= dr$, DE $=$ EF $= de$, and RE $=$ EI $= re$. Now because $eo =$ EO (by Construction), if from *eo* we take *er*, and from EO, ER we shall have $ro =$ RO, and RD has been proved equal to *rd*, therefore the right angled △s DRO and *dro*, are likewise similar and equal, and DO $= do$, whence the whole △ EDO is $=$ to the whole △ *edo*, but the △ *edo* is right-angled at *d*, (Eu. Cor. 8. 6.) whence the △ EDO must also be right-angled at D, therefore FD is ⊥ to the tangent DO, or to the curve in the point D.

Question 111, *by Mr. John Lowry.*

Given the sum of the perpendicular and arch bisecting the vertical angle of a spherical triangle; the sum of the difference of the sides, and difference of the segments of the base, and the difference of the angles at the vertex made by the perpendicular, to describe and determine the triangle?

Solution, *by Mr. J. Hampshire, of London.*

Construction. Upon the great circle *Aa* (Fig. 3) take *Aa* $=$ the sum of the ⊥ and arch bisecting the vertical angle, and thro' any point *b* taken therein, describe the lesser circle *be*, making the angle *Abc* $=$ to the given one; on *bc* take *bc* on the primitive $=$ to *ba*, and through *a* and *c* describe the great circle *ac* and through the point *c* the lesser circle *cq* ⊥ *cb* meeting *Aa* in *q*; ∥ to *qc* through A draw the great circle *Ap* cutting *acp* in *p*, through which point describe the great circle *p*B ∥ *bc* cutting *Aa* in B, upon *p*A take *pr* $=$ half the sum of the latter given differences, and at right-angles thereto, thro' *r* describe the great circle *ro* cutting *a*A in *o*, round which as a pole, with the distance *ro* describe the lesser circle *r*FD*o* to touch which through B, describe the two great circles BD and BF touching it in D and F and cutting A*r* in *m* and N, and MBN will be the triangle required.

Demonstration. Now it is evident that the ∠ AB*p* $=$ MB*p* $-$ *p*BN $= qbc$ by ∥ circles $=$ to the given one by Construction, because the ∠ MBA $=$ ABN, and because *bc* $=$ *ba* by Construction, the ∠ *bca* $=$ *bac* $=$ B*pa* by ∥ circles, therefore B*p* $=$ B*a*, and consequently AB $+$ B*p* $=$ AB $+$ B*a* $=$ A*a* $=$ to the first given sum by Construction. And again if from BD $=$ BF $=$ BM $+$ M*r* $=$ BN $+$ N*r* $=$ to half the perimeter of the triangle MBN we take BN $+$ N*p* we shall have *pr* $=$ to the sum of the given differences.

The same, *by the Proposer*.

Construction. By Question 621, *Gentleman's Diary*, describe the right-angled spherical triangle ICP (Fig. 8) having the sum of the Sides IC, CP = the given sum of the ⊥ and bisecting arch, and the included angle ICP = half the given difference of the angles at the vertex; on the arch PI continued, lay off PQ = half the given sum of the differences, erect the ⊥ QO, meeting CI produced in O; about O with distance OQ describe the small circle RQT, and from C draw the great circles CR, CT to touch it as at R and T, and intersect QP produced both ways in A and B: then is ACB the triangle required.

Demonstration. Bisect AB in D; then by the property of tangents CR = CT; RA = AQ, and TB = BQ; hence AC + AQ = CB + BQ, or AC − CB = BQ − AQ = 2 QD; also AP − BP = 2DP, consequently $\overline{AC - CB} + \overline{AP - BP} = 2 \cdot \overline{QD + PD} =$ 2QP = the given sum by Construction. Again since OR = OT, RC = TC and OC common, the angles RCO, TCO will be equal, and ∠ PCA − ∠ PCB = 2∠PIC = the given difference of the angles by Construction, and PC + CI = the given sum by Construction.

Question 112, *Mr. John Lowry*.

Given the sum of the base and perpendicular, the side of the inscribed square, and the rectangle under the sum of the sides, and radius of the inscribed circle, of a plane triangle to construct it?

Solution *by Mr. J. Hampshire*.

Suppose ABC (Fig. 8.) to be the triangle required, draw BI to bisect the vertical angle ABC; GI the radius of its inscribed circle, and ⊥ BD, and suppose FH the side of its inscribed square. Then by sim. triangles DB : AC :: BE : HF, or by composition DB + AC : AC :: BE + HF : = BD : HF but by a well known property of triangles AB + BC + AC : AC :: BD : GI therefore by equality of ratios AB + BC + AC : DB + AC :: HF : GI but $\overline{AB + BC + AC} \times GI = DB \times AC = \overline{DB + AC} \times HF$ but $\overline{DB + AC} \times HF$ is given, because DB + AC × HF are given, therefore DB × AC is given, but because the sum and rectangles of DB and AC are given, they are each of them given. Now if from $\overline{BC + CA + AB} \times GI$ we take $\overline{AB + BC} \times GI$ which is given, we shall have given AC × GI, but AC is given, therefore GI is given. Again, if from $\overline{AB + BC} \times GI - AC \times GI$ we shall have given 2GB × GI, because 2GB = AB + BC − AC, whence GB is given, and consequently the whole of the right-angled △ GBI is given; whence the angle GBI is given. Hence this

Construction. Let *pq* be = to the given sum of the base and perpendicular, *rs* the given side of the inscribed square, and the square of m = to the given rectangle of the sides and radius of the inscribed circle. Divide pq in a, so that $pa \times aq = pq \times rs$, then take gi a third proportional to pa, and the side of a square which shall be = $paq - m^2$; and $2gb$ a third proportional to gi and m; then with the given base gb and given ⊥ gi construct the right-angled Δ gbi, upon FH = rs describe the segment of a circle capable of containing an angle = $2gbi$, at H erect the ⊥ PH = $qa - rs$, thro' P draw PH ∥ FH cutting the circle in B, through which point and the points F and H draw the right lines BF and BH, from B demit the ⊥ BE, which continue BD = aq and through D draw DEA ∥ FEH meeting BF and BE produced in A and C, and ABC will be the required Δ. The Demonstration is evident from the Analysis.

The same, *by Mr. Lowry, the Proposer.*

Construction. Describe the given inscribed square BCQI (Fig. 5.) and on CB produced take BD = the difference between the given sum of the base and perpendicular, and side of the inscribed square, on CD describe a semi-circle DLC interfecting IQ produced in L; demit the perpendicular LA, and through Q draw AQH meeting IB produced in H: complete the rectangle BK, and make the rectangle RK = the given one, and the rectangle Bb = half the rectangle BK, join EF, and upon AB describe a segment of a circle ACB to contain twice the ∠ AEF, and touch or cut KH in C; join AC, CB, then is ACB the required triangle.

Demonstration. Demit the perpendiculars ON, SM, and draw the diagonal BQ; then by similar triangles AG : GQ :: QI (QG) : IH or AG.IH = $\overline{GQ}^2$. But by the property of the circle, AI^2. ($\overline{GQ}^2$) = AG × AD: therefore AG.IH = AC.AD, consequently AB + CP (BH) = BD + BI = the given sum of the base and perpendicular, by Construction. Again, since BQ bisects the angle ABH: BH or PC : BA :: HQ : AQ (Eu. 6. 3.) :: HL : IB or ON (Eu. 6. 2.) but CP : AB :: CO or HI : OS, therefore ON = OS = IB = the side of the given inscribed square by Construction.

Moreover, since ∠ AEF = half the vertical ∠ ACB by Construction, and BE.Eb = $\frac{AB.PC}{2}$; EB will be equal $\frac{AB + AC + CB}{2}$ by a well known property of plane triangles, and AF (Eb) = the radius of the inscribed circle; but (by Emerson's Geo. Cor. p. 30, b. 4.) $\overline{AB + AC + CB}$ × AF = AB × PC = rectangle BK; hence $\overline{AC + CB}$. AF = rectangle BK − rectangle BF (AB × AF) = rectangle RK = the given rectangle by Construction.

The same, *by Mr. John Fletcher.*

The rectangle of the given side of the inscribed square and given sum of base and perpendicular is (by Simp. G. 18 Edit.) = that of the base and perpendicular; the base and perpendicular then comes known (17, 5, 2d. Edit.) this rectangle is also known to be = the perimeter into radius of inscribed circle; if then from this the given rectangle of the sum of sides and radius of the inscribed circle be taken, there will remain that of the base and radius. Now as the triangle's area is to this last rectangle as the perpendicular to the radius, the radius comes known, and also the sum of the sides; hence the base, perpendicular, and sum of sides of the triangles being found, the problem will be elegantly constructed (by Prob. 77, p. 395, Simp. Algebra.)

Question 113, *by Mr. John Lowry.*

Given in a plane triangle, the base, one of the angles at the base, and the sum of the squares of the opposite side, and its alternate segment of the base made by the perpendicular to construct the triangle?

Solution, *by the Proposer.*

Construction. Let AB (Fig. 1.) be the given base, and ABC the given angle: make the rectangle AB × AF = the sum of the squares; from any point in BC as *a*, demit the perpendicular *ab* and divide AB in Q, so that AQ : QB :: $\overline{Bb}|^2 : \overline{Ba}|^2$. From Q to BC apply QC, a mean proportional between QB and QF, join AC, then ACB is the triangle required.

Demonstration. Demit the perpendicular CP, join CF and through the three points BFC describe a circle cutting AC produced in D, and join DB: Then since QC^2 = QB × QE (by Construction) QC touches the circle at C; hence (by Euc. 3, 32) ∠ QCB = ∠ CDB, and ∠ QCF = ∠ FBC; but the angle at Q is common to the △s QCF, QCB, therefore ∠ QFC = ∠ QCB. Draw AS parallel to QC, meeting BC produced in S; then by similar triangles, AC : CS :: BC : CD, or AC.CD = BC.CS, therefore AC.CD (BC × CS) : $\overline{SC}^2$ :: SC : BC :: AQ : QB :: $\overline{Bb}^2 : \overline{Ba}^2$ (by Const.) :: $\overline{BP}^2 : \overline{BC}^2$ and AC.CD = $\overline{BP}^2$, hence $\overline{AC}^2 + \overline{BP}^2 = \overline{AC}^2$ + AC.CD = AC.AD = AB.AF (Eu. 36. 3.) = the given sum of the squares by Construction.

Question 114, *by Mr. John Lowry.*

In a spherical triangle there is given the vertical angle, the perimeter, and the sum of the difference of the sides, and the difference of the segments of the base made by the arch bisecting the vertical angle, to describe and determine it?

Solution, *by the Proposer.*

Construction. Describe the great circles CR (Fig.) CT to include the given angle, and have CR, CT each = half the given perimeter; erect the perpendicular arches RO, TO meeting in C, join OC, and about O, with distance OR or OT describe the small circle RQT; lay off RS = half the given sum, make OI = OS, and through I draw AQIB to touch the small circle at Q, then is ACB the triangle that was to be described.

Demonstration. Since OI = OS, OQ = OR, and the angles at Q and R right angles IQ = RS: But by the Demonstration to question 1. AC ∓ CB = 2QD, and AI ∓ IB = 2ID; therefore $\overline{AC - CB} + \overline{AI - IB} = 2\overline{QD + ID} = 2IQ = 2RS =$ the given sum by construction; and ACB is the given vertical angle, and AC + CB + BA = CR + CT = the given perimeter by Construction.

Question 115, *by Mr. John Lowry.*

In a plane triangle there is given the difference between each side and its adjacent segment of the base made by the perpendicular, and the rectangle under the longest side, and its alternate segment of the base to construct it?

Solution, *by the Proposer.*

Construction. On the line AC (Fig.) lay off PI, PD = the given differences, and erect the ⊥ PBO, making IP.PC = the given rectangle; Divide IP in Q, so that 2IP.IQ = $\overline{IP^2 - PD^2}$, then by Simpson's Geo. 5. 18. make PC.QC = DP.PO, apply CB = CI, and join BD, then if AB be drawn making the ∠ ABD = ∠ ADB; ACB is the triangle required.

Demonstration. By reason of the equal angles AB = AD, and BC = CI by Const. ∴ AB − AP = PD, and BC − PC = PI = the given differences by Construction. Again $\overline{AB^2} - \overline{BC^2} = \overline{AP^2} - \overline{PC^2}$ (Simpson's Geo. 9, 2.) $= \overline{AB - PD}^2 - \overline{BC - PI}|^2$; hence $2PD.AB = 2IP.BC + \overline{IP^2} - \overline{PD^2} = 2IP.BC + 2IQ.IQ$; therefore PD : PI :: CI − IQ (QC) : AD :: PD.PO : PI.PO :: QC.PC : (AD.PC) AB.PC, but QC.PC = DP.PO by Construction; therefore AB.PC = IP.PO = the given rectangle by Construction,

Question 116, *by Mr. John Lowry.*

Given the sum of the squares of the base and perpendicular, the side of the inscribed square, and the rectangle contained under the segments of the greater side made at the point of contact of the inscribed square to construct the triangle?

Solution, *by Mr. John Fletcher.*

Construction. Produce BD (Fig.) one of the sides of the inscribed square ABCD to K, so that BD.DK is = the given rectangle of the segments of the longest side, on DK describe a semicircle, then (per Prob. 21, Simp. Geo.) apply the line IE passing through C, and meeting AB and BD produced in I and E such, that IE^2 will be = the given sum of squares of the perpendicular and base; from E draw EF perpendicular to DK to meet the periphery of the circle in T, from E through the angles of the square, draw TCH, EDG, and TGH is the required Δ constructed. For by the nature of the square and semicircle, the Δs GDR, KDT (conceiving TK joined) are similar, and GD-DE

= RD.DK; also because ED = EP : ER = PO :: CD : IB = HG, and $IE^2 = IB^2 + BE^2 = HG^2 + OE^2$. It is evident the same method of Construction will obtain, whether the rectangle of the segments of the longest or shortest side is given, or in cases wherein any other data is given, whereby the locus of the vertex may be described; also when the given square or given rhombus, and when the sum of the squares of base and perpendicular is a minimum, or the side of the square a maximum, the perpendicular and base are each to the side of the inscribed square as 2 to 1, in which case the Construction comes more simple.

Con. Perpendicular to the cord AB (Fig. 12) of the arc given, take BE = the side of the given square; then with center G, the middle of AB, and radius GF, describe an arc cutting a semicircle on AC in E, join CE and produce it to meet the arch AB produced in D, and the thing is done.

The same, *by the Proposer.*

Construction. Let BIDG (Fig. 14) be the given inscribed square, produce the diagonal BD, till $|KB|^2$ = the given sum of the squares of the base and perpendicular; on CB produced, let fall the perpendicular RS, and take LD such, that BL.DL be = RS^2, from L to BI produced, apply LQ = RS, and through GD draw QDA; also ‖ to AB draw CQM, making the rectangle QINM = the given rectangle of the segments, produce DI, CB to meet MV drawn ‖ QB in N and V, on GV describe a semicircle cutting BQ in O, to DI apply BF = BO, producing it to meet CQ in C; join CA, and ACB is the Δ required.

Demonstration. Continue IP to meet AC in H, and demit the ⊥s FE, CP and HP: then CB (BI) .BV = BO^2 (BF^2) by Construction; therefore BF^2 : IQ.QM :: BI : IQ :: BF : FC :: BF^2: BF.FC, hence BF.FC = IQ.QM = the given rectangle by Construction. Again, since BD bisects the angle ABQ.BQ (PC) : AB :: QD : DA (Eu. 6. 3) : : QI × : IB (FE), but CP : AB :: CX (QI) :: HF, therefore EF = FH = IB = the side of the given inscribed square.

Moreover since BL.LD = RS^2 = LQ^2 by Construction; therefore BL : LQ :: LQ : LD, and consequently the triangles LQB, LQD are similar, by Eu. 6. 6. therefore ∠ LQD = ∠ LBQ = ∠ RBS or ∠ BRS = ∠ LAQ, because a circle will pass through the points ABQL, and therefore the Δs ALQ, BSR are similar, but RS or BS = LQ or LA, consequently AQ = BR; hence $AB^2 + PC^2 = AB^2 + BQ^2 = AQ^2 = BR^2$ = the sum of the squares by Construction.

Question 117, *by Mr. John Lowry.*

Given the two sides of a spherical triangle to construct it, when the rectangles under the sines of the base and perpendicular is a maximum.

Solution, *by the Proposer.*

Construction. With the given sides as base and ⊥, construct a right-angled Δ, and it is that required.

For by Prop. 2, of my *Mathematical Lucubrations*; as radius : sine of half the vertical angle :: the rectangle contained under the sines of the sides : rectangle contained under the sines of the base and ⊥, but the first and third terms of this proportion are constant, therefore the fourth will be a maximum, when the second is so, *i. e.* when the vertical angle is 90°, its sine being then = radius, and the rectangle, under the sines of the sides equal to the rectangle under the sines of the base and ⊥.

If it had been for a given quantity instead of a maximum, the vertical angle is had from the above proportion, and it becomes the 4th case of spherical trigonometry.

Question 118, *by Mr. John Lowry.*

Given the vertical angle, and the sum of the base and perpendicular of a spherical Δ to determine it, when the rectangle under the sines of the two sides is a maximum.

Solution *by the Proposer.*

Lemma. If a given arch be divided into two parts, the rectangle contained under the sines of those parts will be a maximum, when they are equal. If they are not equal, let AP (Fig. 11) be their difference, join AB, PB, and from the center O, draw OE to bisect PB, OC to bisect AB, and demit the ⊥s BR, PI and AF; then it appears from P. 30, page 340, Simpson's Algebra, that RB × AF = AO ÷ 2 × RI, and by Trigonometry (AQ × BQ) = $\overline{AQ}^2$ = AO ÷ 2 × RA, but RA is greater than RI, ∴ AQ × BQ is greater than BE × AF, consequently I and P coincides with A and BC = CA.

Construction. With half the given sum as base, and the other half as perpendicular, and given vertical angle, construct a spherical Δ, by Question 680 *Gents. Diary*, and it will be the required one.

For since the vertical angle is constant, it is manifest from the proportion used in the preceding question, that the rectangle contained under the sines of the sides will be the greatest, when the rectangle contained under the sines of the base and perpendicular is greatest; which per Lemma, is when the parts are equal.

Question 119, *by Mr. John Lowry.*

To a given arch to add another arch, so that the rectangle under the sines of the whole and arch added, may be = to a given square.

Solution *by the Proposer.*

Construction. Let AC be the given arch to the radius CA; demit the perpendicular CI, and make the rectangle AC ÷ 2 × IP = the given square; erect the ⊥ PQ meeting the arch AC continued in Q, bisect QC in B, and BC is the part required.

Demonstration. Join QC,CB, and draw CS ∥ AO, and demit the perpendicular AE; then by similar △s QC = 2CL : CS = IP :: OA : AP, therefore CL × AP = IP × OA ÷ 2 = the given rectangle by Construction.

This problem is the same as having two given points in the circumference of a given circle, to draw a diameter of that circle, so that the rectangle under the perpendiculars falling thereon, from the given points may be = to a given square.

If the distance FL had been given, which is the same as Prob. 9, in Mr. Lawson's Dissertation on the Geometrical Analysis of the Ancients, then join CA, and upon it describe a semicircle, in which apply CG = the given line, through the center O draw OB ∥ CG, and it is the diameter required.

Question 120, *by Mr. John Lowry.*

Given the base, the vertical angle, and the difference of the sides of a spherical △ to determine it.

Solution, *by the Proposer.*

Suppose ABC the required △, (Fig. 3) then ED is = half the difference of the sides AC, CB, hence by Prop. 3, of *Mathematical Lucubrations,* tangent angle ACB ÷ 2 : 2R :: SAE × SBE : SAB × S. CP, where three terms being given, the fourth is also given, and consequently the perpendicular CP is given; hence there is given the base, vertical angle and ⊥ to construct the △, which is Question 680, *Gents. Diary.* Otherwise lay off CG = CB, then by the last Prop. of Dr. Simson's Euclid, 6th Edition, S^2. 2 angle $ACB : R^2 :: S. \frac{AB + GA}{2} \times S. \frac{AB - GA}{2} : S.AC \times S.BC$; consequently the rectangle of the sines of the sides is given; hence there is given the arch AG to add another arch CG, so that the rectangle under the sine of the whole arch AC, and parts added CG, or CB may be = a given square, which is the same as last Question.

Remark. I have a curious projection of this problem by me, which will be published at some future opportunity, with a great many curious properties of spherical triangles deduced therefrom.

Question 121, *by Mr. John Lowry.*

Given the line bisecting the base, the radius of the inscribed circle, and the difference between the sum of the sides and base of a plane triangle to construct it?

Solution, *by the Proposer.*

Construction. Let R (Fig. 7) = the given bisecting line, take AP = half the differences of the sum of the sides and base, and perpendicular thereto draw OP = the given radius, describe the circle IPR, and draw AP to touch it at I, and through O draw the indefinite line ACQ, make OF, FG, GI ⊥ to AO, AF, and AG respectively, and take the rectangle OK = R^2, producing KH to meet GI in I, lay off OS = OG, and OE = AS, and draw EL ∥ AO, then by P. 18, b. 5, Simp. Geo. find the point Q so that $\overline{2AS + OQ} \times OQ$ = the sum of the rectangles GH, EK, apply QB, QC each = QO, join BC, then is ABC the Δ required.

Demonstration. Draw QDT ⊥, and AT ∥ BC, and join AD, then since the ∠s ACQ ABQ = 180°, a circle will pass through the points BACQ and AQ^2 (QV.QT) : QT .2QD :: QV : 2QD, and by division, $AQ^2 : AQ^2 -$ QT.2QD :: QV : QV − 2QD, but OQ = QB by Construction; therefore AQ = AO + QB and $AQ^2 = \overline{AO + QB}^2$, hence $\overline{AO + QB}^2 : AQ^2 -$ QT.2QD :: QV : QV − 2QD (AG : AS) by similar Δs, but by Eu. 13, 2. $AD^2 = AQ^2 -$ QT.2QD + QD^2, hence $\overline{AO + QB}^2 : AD^2 - QD^2$ :: AG : AS; but QB^2 (QV.QD) : QD^2. :: QV : QD :: AG : OC; therefore $\overline{AO + QB}|^2$. AS = $\overline{AD^2 - QD^2}$. AG, and $QB^2 \times$ OG = QD^2. AG, or $AO^2 + \overline{2AS . QB}$. AS + QB . OG = AD^2.AC; hence $\overline{BQ + 2AS}$. BQ = $AD^2 \times$ AC ÷ AO − AO . AS.

Again by Construction, $\overline{QC + 2AS}$. CQ (= $\overline{BQ + 2AS}$. BQ) = rectangle GH + rectangle EK = rectangle GK − rectangle OL = (because AO : AG :: rectangle OK = R^2, rectangle GK = AC ÷ AO . R^2) = AC ÷ AO . R^2 − rectangle OL (= AO.AS) hence AG ÷ AO . AD^2 = AG ÷ AO . R^2, or AD = R = the given bisecting line by Construction, and BD = DC, and BC touches the circles at R, as is well-known, also the circle having the given radius is inscribed in the triangle ABC and BA + AC − BC = 2AP = the given difference by Construction. Q. E. D.

Question 122, *by Mr. John Lowry.*

Given in a spherical triangle, the difference of the angles at the vertex made by the ⊥, the difference between the difference of the sides, and difference of the segments of the base, and the radius of the inscribed circle to construct it.

Solution, *by the Proposer.*

Construction. Take EP = half the given difference (Fig. 3) and erect the perpendicular arches PC, Eb making Eb = the given radius of the inscribed circle, draw bG ⊥ to PC, and construct the right-angled spherical triangle bGC, having the angle bCG = half the given difference of the angles at the vertex, about b with distance bE describe the small circle ELF, and from C draw the great circles CLA, CFB to touch it as at L and F, and meet FP produced both ways in A and B, then is ACB the required triangle.

Demonstration. Bisect AB in D, then since bL = bF the ∠s at L and F are right angles, and bc common, the angles LCb, FCb will also be equal, hence angle ACP − angle BCP = 2∠bCG = the given difference of the angles, by Construction; and LFE is the given inscribed circle. Again since CL = CF, LA = AE; and FB = BE; AC − CB will be = AE − EB = 2DE, and AP − PB = 2DP, hence $\overline{AP - PB}$ − $\overline{AC - CB}$ = 2EP = the given difference by Construction. Q. E. D.

☞ *Fig.* 16, *Plate* 7, *belongs to the Solution of Question* 107.
Fig. 17, *belongs to the* 114*th.*
Fig. 18, *belongs to the* 116*th.*
Fig. 19, *belongs to the* 115*th.*
The above Figures were taken from Plate 9, *for want of room.*

ARTICLE XVI.

Queries and Solutions.

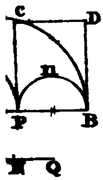
Query , *by the Rev. John Hurst, of London.*

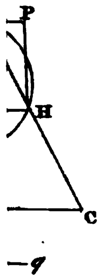
WHAT is the Cause of Earthquakes?

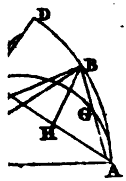
Solution *by the Proposer.*

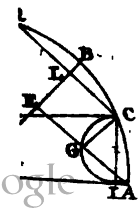
Many hypothesis have been formed by philosophers, to explain the causes of Earthquakes, but the generality, and with the greatest appearance of truth, agree, in deducing them principally from the violent force of the air contained in sulphureous bodies, which, together with that confined in the pores of the earth, being greatly compressed by the incumbent load, make a violent resistance. This is so far from implying any contradiction, that it is at once supported by reason, and confirmed by experience. But the apparent difficulty consists in explaining how the veins of the earth become again filled with air, after one concussion has happened; it being natural to think, that the quantity which struggled for vent, was thereby discharged; and that a long interval of time was


G
C
B


C
D
n
P
B
Q


P
H
C


D
B
G
E
A


L
B
L
E
C
G
A

neceſſary before another could be produced; and alſo why ſome countries are more ſubject to theſe terrible convulſions than others. Experience has abundantly proved, that ſome parts abound more with ſulphureous bodies than others, and extend themſelves in veins to a great diſtance from the baſes of the mountains, the natural depoſitories of theſe ſubſtances. And it is well known that the ſoil of Syria and Paleſtine is hollow and porous, and conſequently abounds with ſubteraneous waters, which being filterated through the pores of the mountains, are impregnated with univerſal particles, and thereby increaſe the quantity before lodged in the veins of the Earth, and during their paſſage through their ſubterraneous channels turn into a kind of paſte theſe ſulphureous and nitrous ſubſtances, which ferment to a certain degree, and take fire. They are not, indeed, here in ſufficient quantity to produce volcanoes; but ſufficient from their inflammatory quality to rarify the air contained in them, which eaſily incorporating itſelf with that confined in the innumerable pores, cavities, or veins of the Earth, compreſſes it by its greater expanſion, and at the ſame time rarifies by its heat; but the cavities being too narrow to admit its proper dilatation, it ſtruggles for a vent, and theſe efforts ſhake all the contiguous parts, till at laſt it forces itſelf a paſſage, where it finds the leaſt reſiſtance. This paſſage ſometimes cloſes as ſoon as the expanded air is eſcaped, and ſometimes continues open; as happened in an earthquake near Balbec, where it is ſaid there is now an opening in the earth, cauſed by this terrible convulſion, ſeveral fathoms wide, and twenty leagues long. When, on account of the reſiſtance being equal, it forces itſelf a paſſage in ſeveral parts, the chaſms are generally ſmaller, ſo that rarely veſtiges remain after the concuſſion. At other times, when the cavities are ſo large as to form ſpacious caverns, they not only rend the earth, and at every ſhock leave it full of chaſms; but alſo cauſe it to ſink into ſpacious hollows.

The loud ſubterranean noiſe preceding earthquakes, and which imitates thunder at a diſtance, both in its ſound and local motions, ſeem to correſpond with the above cauſe and formation of earthquakes, as it can only proceed from the rarefaction of the air, on the ignition of the exploſive ſubſtances being impetuouſly propagated through all the adjacent caverns of the earth; propelling, and, at the ſame time, dilating what is contained in them, till all the cavities being pervaded and no vent found, the efforts for a farther dilatation begin, and form the concuſſions with which it terminates.

The ſhocks are repeated at intervals of a few days, ſometimes of a few hours; proceeding from the matter being diſperſed in different places, and each in a different degree of aptitude for inflammation, one part ſucceſſively kindling after another, as each is more or leſs prepared. Hence alſo proceeds the different violences of the ſhocks, and the different intervals of time. For that quantity becomes firſt inflamed which has firſt contracted the ultimate aptitude, and its heat increaſes the diſpoſition of others, which were not ſo before; whence a part which would not have been ignited till after ſome days, becomes ſo, by means of this adventitious fire, in a few hours. Theſe ſecond are more violent, and cauſe a greater deſtruction than the firſt; for the fire of the portion of matter which is first inflamed, though in itself small, is sufficient to accelerate the fermentation of a much larger quantity, and consequently must be attended with more powerful effects. Thus have I endeavoured to explain the natural causes that produce those terrible events. But we should do well to remember, that the first and principal agent is omnipotence himself. It was he that originally formed the Earth, hollowed its subterraneous caverns, and placed its various strata; that fixed the laws of nature, and regulated them in such a manner, that they should produce their effects in the natural course of things exactly at such periods, which he foresaw would be most conducive to answer the great purposes of his moral providence and government.

ARTICLE XVII.

Questions and Solutions.

Question 123, by *Mr. Newton Bosworth.*

THE area of an ellipsis is 942.48, and its longest diameter 40; required the side of the inscribed square, the area of a circle inscribed in that square, and the side of an equilateral triangle inscribed in that circle?

Solution, *by the Proposer.*

The following Theorem will enable us to answer the first demand of the question, *viz.* Divide the rectangle of the squares of the two diameters of any ellipses by the sum of the said squares, and the quotient will be the area of the inscribed square; whose square root will consequently be the side thereof. Now $\frac{942.48}{.7854} \div 40 = 30 =$ the shortest diameter; then by the above Theorem, we have, $\sqrt{\frac{40^2 \times 30^2}{40^2 + 30^2}} = 24 =$ the side of the inscribed square. The diameter of a circle inscribed in any square, is evidently $=$ to the side of that square; therefore $24^2 \times .7854 = 452.3904 =$ the area of the inscribed circle. According to *Ward*'s Mathematician's Guide, Page 342, the side of an equilateral triangle is in proportion to the radius of its circumscribing circle, as 1 to .57735027; therefore as .57735027 : 1 :: 12 : 20.7846 = the side of the inscribed equilateral triangle.

Question 124, by *Mr. Newton Bosworth.*

The side of the greatest square that can be inscribed in the generating circle of a cycloid is 12; it is required to determine the content of the solid, formed by a revolution of the cycloid about its base?

Solution, by *Mr. James Gates, of Peterborough.*

The side of the inscribed square (as per question) is = 12. Then by *Eu.* 47. 1, the diameter of its circumscribing circle is = 16.9705 = the semi diameter of the cycloid, which multiplied by 3.1416 = 53.3145 the circumference of the circle, which is equal to the base of the cycloid generated by the circle. According to Dr. *Hutton's* Math. and Philo. Dict. page 355, the content of the solid formed by the rotation of the cycloid about its base, is in proportion to its circumscribing cylinder as 5 is to 8; therefore by a known rule the content of the cylinder = 48237.5616, $\frac{5}{8}$ of which is = 30148.476, the content of the solid required.

Question 125, by *Mr. John Lowry.*

To determine a point P, in a given parabola ABC, so that drawing PD parallel, and PQ perpendicular to the base AB; PD + PQ may be a maximum; also to determine the said point when the area of the part DPQ is a maximum; and lastly, supposing D the point P given, to draw two lines P*m*, P*n* including a given angle, so that the area of the space A*m*P*n* may be a maximum?

Solution, by *Mr. John Ryley, of Leeds.*

Let ABC (Fig. 1) be the given parabola: Draw DP parallel, and PQ perpendicular to AB; put CE $= a$; EB $= b$, and CF $= x$: then by the property of the parabola CE : EB :: CF : FP^2 $= \frac{b^2 x}{a}$. Therefore DP $= 2b\sqrt{\frac{x}{a}}$. But EF $=$ QP $= a - x$; consequently $a - x + 2b\sqrt{\frac{x}{a}}$ is to be a maximum by the question, whose fluxion is $\frac{b}{\sqrt{a}} . x^{\frac{1}{2}-1}\dot{x} - \dot{x} = o$; hence $ax = b^2$; therefore CF : EB :: EB : CE, and CP + PQ $= \frac{CE^2 + EB^2}{CE}$.

Secondly. In order to determine when the space ADPQ is a maximum, by retaining the symbols above specified, we shall have the area AEC $= \frac{2ba}{3}$, the area DFC equal $\frac{2bx}{3}\sqrt{\frac{x}{a}}$, and the area EFPQ $= \overline{a-x} \times b\sqrt{\frac{x}{a}}$. Consequently the ar-

ADPQ $= ab\sqrt{\frac{x}{a}} - bx\sqrt{\frac{x}{a}} + \frac{2ba}{3} - \frac{2bx}{3}\sqrt{\frac{x}{a}}$, which by the question is a maximum, therefore $3ax^{\frac{1}{2}} - 5x^{\frac{3}{2}}$ will be a maximum, whose fluxion is $\frac{3}{2}.ax^{-\frac{1}{2}}\dot{x} - \frac{15}{2}.x^{\frac{3}{2}-1}\dot{x} = o$, and by division $5x^{\frac{1}{2}} = \frac{a}{x^{\frac{1}{2}}}$; hence $x = \frac{a}{5}$; and the area ADPQ $= \overline{\frac{2}{3\sqrt{5}} + \frac{2}{3}} \times ab = \overline{\frac{1}{3\sqrt{5}} + \frac{1}{3}} \times$ AB.EC.

Thirdly. To determine the space AmPn, when a maximum, take CF $= r$, EF $=$ PQ $= v$, FP $=$ EQ $= c$, EB $= b$, the sine and tangent of the angle mPn $=$ S and T, tangent of the angle nPQ $= t$ and CN $= x$; then per Trig. nQ $= vt$; hence En $= c - vt$, and by similar triangles n Q : QP :: PF : FG $= c \div t$; also PQ : Qn :: PI : IV $= tx - tr$; therefore VN $= c + tr - tx$. But by the property of the parabola $\sqrt{\text{CF}} : \text{FP} :: \sqrt{\text{CN}} : \text{N}m = c\sqrt{x} \div \sqrt{r}$; hence mI $= c + c\sqrt{x} \div \sqrt{r}$; and by Eu 47. 1. $m\text{P} = \sqrt{\overline{c + c\sqrt{\frac{x}{r}}}\Big|^2 + \overline{x-r}\big|^2}$; and PV $= \overline{x-r} \times \sqrt{t^2+1}$. Therefore the area of the triangle mPV $= \frac{\text{S}}{2} \times \sqrt{\overline{c + c\sqrt{\frac{x}{r}}}\Big|^2 + \overline{x-r}\big|^2} \times \overline{x-r} \times \sqrt{t^2+1}$ the area of the trapezoid ENVn $= \overline{2c + tr - tx - tv} \times \frac{r+v-x}{2}$, and the area of the segment AmNE $= \overline{2b + \frac{2c^2x}{br + c\sqrt{rx}}} \times \frac{v+r-x}{3}$, and the sum of these areas is by the question a maximum.

Now the tangent of the sum of the angles mPV, VPI; or, the tangent of the angle mPI is $= \frac{1-\text{T}t}{\text{T}+t}$; hence $\frac{1-\text{T}t}{\text{T}+t} \times \overline{x-r} = c + c\sqrt{\frac{x}{r}}$, and by reduction $t =$

$$\frac{x - r - \text{T}c - \text{T}c\sqrt{\frac{x}{r}}}{c\sqrt{\frac{x}{r}} + c + tx - \text{T}r}$$, and $t^2 =$ .... [illegible]

$$\frac{x^2 - 2rx + r^2 - 2\text{T}cx + 2\text{T}cr - 2\text{T}cx\sqrt{\frac{x}{r}} + 2\text{T}cr\sqrt{\frac{x}{r}} + \text{T}^2c^2 + 2\text{T}^2c^2\sqrt{\frac{x}{r}} + \frac{\text{T}^2c^2x}{r} \ldots}{\frac{c^2x}{r} + 2c^2\sqrt{\frac{x}{r}} + c^2 + 2\text{T}cx\sqrt{\frac{x}{r}} + 2\text{T}cx - 2\text{T}cr\sqrt{\frac{x}{r}} - 2\text{T}cr + r^2x^2 - \ldots + \text{T}^2 \ldots}$$

therefore if these values of t, and t^2 be substituted in the above-found expressions for the maximum, it becomes $\frac{S}{2}\times\sqrt{\overline{c+c\sqrt{\frac{x}{r}}}\Big|^2+\overline{x-r}\Big|^2}\times\overline{x-r}\times$

$$\frac{\sqrt{\overline{x-r}\Big|^2+\overline{Tc+Tc\sqrt{\frac{x}{r}}}\Big|^2+\overline{c\sqrt{\frac{x}{r}}+c}\Big|^2+\overline{Tx-Tr}\Big|^2}}{c\sqrt{\frac{x}{r}}+c+Tx-Tr}+cr+cv-cx+$$

$$\frac{r^2-2rx+x^2-v^2}{2}\times\frac{x-r-Tc-Tc\sqrt{\frac{x}{r}}}{c\sqrt{\frac{x}{r}}+c+Tx-Tr}-\frac{2bx}{3}+\ldots\ldots\ldots\ldots$$

$\frac{2c^2vx+2c^2rx-2c^2x^2}{3br+3c\sqrt{rx}}$; or (putting $\frac{S}{2}\times\sqrt{\overline{T^2+1}}=m$) we have……………

$$\frac{m\times\overline{x-r}\Big|^2+c^2\times\overline{1+\sqrt{\frac{x}{r}}}\Big|^2\times\overline{x-r}}{c\sqrt{\frac{x}{r}}+c+Tx-Tr}-cx+\frac{r^2-2rx+x^2-v^2}{2}\times$$

$\frac{x-r\ Tc-Tc\sqrt{\frac{x}{r}}}{c\sqrt{\frac{x}{r}}+c+Tx-Tr}-\frac{2bx}{3}+\frac{2c^2vx+2c^2rx-2c^2x^2}{3br+3c\sqrt{rx}}$ a maximum; which put into fluxions becomes

$$\frac{3m\dot{x}\times\overline{x-r}\Big|^2+\frac{mc^2\dot{x}}{\sqrt{rx}}\times\overline{1+\sqrt{\frac{x}{r}}}\times\overline{x-r}+c^2\dot{x}\times\overline{1+\sqrt{\frac{x}{r}}}\Big|^2\times\overline{c\sqrt{\frac{x}{r}}+c+Tx-Tr}}{\overline{c\sqrt{\frac{x}{r}}+c+Tx-Tr}\Big|^2}-$$

$$\frac{\overline{\frac{c\dot{x}}{2\sqrt{rx}}+T\dot{x}}\times\overline{m.\overline{x-r}\Big|^3+c^2\times\overline{1+\sqrt{\frac{x}{r}}}\Big|^2\times\overline{x-r}}}{\overline{c\sqrt{\frac{x}{r}}+c+Tx-Tr}\Big|^2}-c\dot{x}+\overline{x\dot{x}-r\dot{x}}\times$$

$$\frac{x-r-Tc-Tc\sqrt{\frac{x}{r}}}{c\sqrt{\frac{x}{r}}+c+Tx-Tr}+\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\dot{x} - \frac{Tc\dot{x}}{2\sqrt{rx}} \times \overline{c\sqrt{\frac{x}{r}} + c + Tx - Tr} - \overline{\frac{c\dot{x}}{2\sqrt{rx}} + T\dot{x}} \times \overline{x - r - Tc - Tc\sqrt{\frac{x}{r}}}}{\overline{c\sqrt{\frac{x}{r}} + c + Tx - Tr}|^2}$$

$$\times \frac{\overline{r - x}|^2 - v^2}{2} - \frac{2b\dot{x}}{3} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\overline{2c^2v\dot{x} + 2c^2r\dot{x} - 4c^2x\dot{x}} \times \overline{3br + 3c\sqrt{rx}} - \tfrac{3}{2}c\dot{x}(\sqrt{r} \div \sqrt{x}) \times \overline{2c^2vx + 2c^2rx - 2c^2x^2}}{\overline{3br + 3c\sqrt{rx}}|^2}$$

$$= 0;\ \text{hence}\ 3m \times \overline{x-r}|^2 + \frac{mc^2}{\sqrt{rx}} \times \overline{1 + \sqrt{\frac{x}{r}}}\Big| \times \overline{x-r} + mc^2 \times \overline{1 + \sqrt{\frac{x}{r}}}\Big|^2 \times$$

$$\overline{c\sqrt{\frac{x}{r}} + c + Tx - Tr} - \overline{\frac{c}{2\sqrt{rx}} + T} \times m \times \overline{x-r}|^3 + c^2 \times \overline{1 + \sqrt{\frac{x}{r}}}\Big|^2 \times \overline{x-r}$$

$$- c + \frac{2b}{3} \times \overline{c\sqrt{\frac{x}{r}} + c + Tx - Tr}\Big|^2 + \overline{x-r} \times \overline{x - r - Tc - Tc\sqrt{\frac{x}{r}}} \times$$

$$\overline{c\sqrt{\frac{x}{r}} + c + Tx - Tr} + \overline{1 - \frac{Tc}{2\sqrt{rx}}} + c\sqrt{\frac{x}{r}} + c + Tx - Tr - \overline{\frac{c}{2\sqrt{rx}} + T}$$

$$\times \overline{x - r - Tc - Tc\sqrt{\frac{x}{r}}} \times \frac{\overline{x - r}|^2 - v^2}{2} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\overline{2c^2v + 2c^2r - 4c^2x} \times \overline{3br + 3c\sqrt{rx}} - \frac{2c}{3}\sqrt{\frac{r}{x}} \times \overline{2c^2vx + 2crx - 2c^2x^2} \times$$

$$\overline{3br + 3c\sqrt{rx}}|^{-2} \times \overline{c\sqrt{\frac{x}{r}} + c + Tx - Tr}\Big|^2 = 0.$$

From this general equation the value of x may be found in any circumstance; and in some cases, with no great degree of labour; for when the angle mPn is a right-angle, T will be infinite, and all the terms affected therewith will be so too; and consequently must be thrown out of the equation, when greater than a right-angle T will be negative, when $= 45°$, T $= 1$, &c. &c. &c.

Question 126, by *Mr. Joseph Dean, of London.*

Given the chord of an arch $= 10$, and versed sine $= 4$, to draw three equal tangents to it; one parallel to the chord, and the other two to meet the chord produced.

Solution, by *Mr. John Ryley.*

Algebraically. Let ABC (Fig. 2) be the given segment of a circle; IH, HE and EG three equal tangents; draw EF || DC, and put AB = 10 = $2a$; DC = EF = 4 = b, and GT = TE = EC, &c. = x; then GE = $2x$; and by *Eu.* 47. 1. FG = $\sqrt{4x^2 - b^2}$. Now because GT = TE, we have BG = $\sqrt{4x^2 - b^2} + x - a$, and AG = $\sqrt{4x^2 - b^2} + x + a$; but by the property of the circle, AG.GB = GT^2; hence by multiplication, &c. we have $4x^2 - b^2 + 2x\sqrt{4x^2 - b^2} = a^2$, and by due reduction we obtain $x = \dfrac{a^2 + b^2}{2\sqrt{2a^2 + b^2}}$; therefore EG = $\dfrac{a^2 + b^2}{\sqrt{2a^2 + b^2}} = \dfrac{41}{\sqrt{66}} =$ 5.04675, &c. = the length of each tangent: hence FG = $\dfrac{a^2}{\sqrt{2a^2 + b^2}} = \dfrac{25}{\sqrt{66}} =$ 3.077287, &c. Consequently as the length of each tangent and the point G are determined, the tangents may be readily drawn.

Geometrically. Bisect the versed sine DC in the point O, and parallel to AB draw NT to cut the arc in the points N, T; so shall the points N, C, T, be those to which the tangents must be drawn. For by a known property of the circle, CE = ET, and by similar triangles EL : LF :: ET to TG; but the antecedents EL, LF are equal, by Construction; consequently the consequents ET, TC must be so too.

Calculation. Let AB and CD be denoted as above, and by the property of the circle DM = $\dfrac{a^2}{b}$; therefore CS = ST = $\dfrac{a^2 + b^2}{2b^2}$, and DS = PQ = $\dfrac{a^2 - b^2}{2b}$; also OS = TQ = $\dfrac{a^2}{2b}$, and by similar triangles, TQ : QS :: TP : PV = $\dfrac{b^2}{2a^2} \times \sqrt{2a^2 + b^2}$; also VP : PT :: PT : PG = $\dfrac{a^2}{2\sqrt{2a^2 + b^2}}$. Therefore FG = $\dfrac{a^2}{\sqrt{2a^2 + b^2}}$, and EG = $\dfrac{a^2 + b^2}{\sqrt{2a^2 + b^2}}$, the same as before.

Question 127, by *Mr. James Stevenson.*

Given $x^{\frac{m}{n}} - y^{\frac{m}{n}} = a$, and $\overline{xy}|^{\frac{m}{n}} = b$, to find x and y; and to give an example when $a = 640$, $b = 27200$, $m = 16$, and $= 11$?

Solution, by *Mr. John Ryley, of Leeds.*

If the square of the first equation be taken from the second, we shall have $2x^{\frac{m}{n}} y^{\frac{m}{n}} = b - a^2$. Now if this equation be added to the second, we get $x^{\frac{2m}{n}} + 2x^{\frac{m}{n}} y^{\frac{m}{n}} + y^{\frac{2m}{n}} = 2b - a^2$; and by extracting the root of both sides of this equation, we have $x^{\frac{m}{n}} + y^{\frac{m}{n}} = \sqrt{2b - a^2}$; which being added to the first of the original equations, the result is $2x^{\frac{m}{n}} = \sqrt{2b - a^2} + a$, hence $x = \left(\frac{\sqrt{2b - a^2} + a}{2}\right)^{\frac{n}{m}} =$ (in the present case), 8.725506, and $y = \left(\frac{\sqrt{2b - a^2} - a}{2}\right)^{\frac{n}{m}} = 3.60281$.

Question 128, *by Mr. O. Gregory.*

On the 8th of April, 1795, in a certain North latitude, under the Meridian of Greenwich, the true altitude of the sun's center, at six o'clock, P. M. solar time, was 5° 51′ 29″: when it was six o'clock P. M. solar time, on the same day, at another place, on the same parallel of latitude, having 90° of West longitude—What was the sun's altitude—and what was his Meridian altitude at each place?

Solution, by *the Proposer.*

The sun's declination for the first given time under the Meridian of Greenwich, is 7° 23′½ North; therefore, by spherics, as sine of 7° 23′½ : radius :: sine of 5° 51′ 29″ alt. : sine of 52° 30′ lat. But in 90° of West longitude, the time is 6 hours behind that at Greenwich; therefore when it is six o'clock P. M. at the former place, it will be midnight at the latter; and then will the declination be 7° 29′ North. Consequently as radius : sine of 7° 29′ :: sine of 52° 30° : sine of 5° 55′ 50″ suns altitude at six o'clock, longitude 90° West. Again, 37° 30′ + 7° 23′½ = 44° 53′½, Meridian alt.—lat. 52°½, long. 0°, and 37° 30′ + 7° 29′ = 44° 59′, Meridian alt.—lat. 52°½, long. 90° West, April 8th, 1795.

Remark. If the longitude had been more, the difference of the altitudes would have been greater; and when the longitude is large, and astronomical operations are required to be accurate, this difference will always deserve notice, except it be near the time of the solstices.

Question 129, by *Mr. O. Gregory.*

The distance on the abscis of a parabola from the focus to the vertex, is three inches. Now, if tangents to the curve be drawn from the end of the right Parameter, until they meet each other, it is required to find the content of the solid generated between these tangents and the curve, whilst a revolution is performing round the parameter as an axis?

Solution, by the *Proposer*.

Dr. *Hutton*, and other writers on Conics demonstrate, that the right parameter CD (Fig. 3) of a parabola, is equal to 4 times the focal distance, BF, that is CD $= 4 \times 3 = 12$; also, that the distance from the focus to any point in the curve, is equal to the distance from the focus to the place where a tangent from that point meets the abscis produced, that is FD $=$ FA $= 6$.

Now, the tangents CA and DA in revolving round CD, will describe a solid in form of two equal cones whose bases join, in which AB the diameter of their bases will be 12, and each of their altitudes CF and FD will be 6. Therefore $12^2 \times 6 \times 2 \times \frac{1}{3}$ of $.785398 = 452.389248$ content of the cones described by the tangents.

The parabola CBD whilst performing a revolution round the parameter, will describe a parabolic spindle whose greatest diameter 2FB will be 6, and its length CD $= 12$;— consequently $6^2 \times 12 \times \frac{8}{15} \times .785398 = 180.955699$, its content. Hence we shall have $452.389248 - 180.955699 = 271.433549$ for the content of the solid generated between the tangents and the curve. W. W. K.

Question 130, by *Mr. O. Gregory*.

There is an ellipse, the transverse and conjugate of which are 12 and $4\sqrt{5}$ inches.— If tangents to the curve be drawn from the ends of the principal Latera-recta, until they meet each other, and form a parallelogram: It is required to find the difference of the contents of the solids generated by these tangents, when the revolution is performed round the longer and shorter diagonals of the parallelograms respectively?

Solution, by the *Proposer*.

Writers on Conics have shewn, that tangents drawn from the ends of the Latera-recta of an ellipse, will meet in a point Q (Fig. 4) where CQ $=$ CA, the semi-transverse $= 6$. Also, that PCF : CA :: CA : CP, P being the point where the tangent meets the transverse produced; but CF $= \sqrt{AC^2 - CD^2} = \sqrt{36-20} = 4$; hence as $4 : 6 :: 6 : 9 =$ CP $=$ CR; and consequently the diagonals PR and QS, are 18 and 12 respectively. When the revolution is performed round the longer diagonal, the generated solid will be in form of two cones, the diameters of whose bases are each $=$ QS, and the altitude of both together is PR; therefore $12^2 \times \frac{18}{3} \times .785398 = 678.583872$. When the revolution is performed round the shorter diagonal, the diameter of the cone's bases, which constitute the generated solid will be PR, and the altitude of both together is QS; consequently, $18^2 \times \frac{12}{3} \times .785398 = 1017.875808$. Whence it appears that the contents of the solids are as 2 to 3, and their difference is one half of the less, or one third of the greater, that is, 339.291936 is the required difference.

Remark. The contents of the solids generated, will be universally in the reciprocal ratio of the diagonals of the parallelogram. For if D and d be the fixed diagonals, the contents will be as $Dd^2 : D^2d$, that is as $d : D$.

Question 131, by *Mr. O. G. Gregory.*

The breadth of a piece of timber is 12 inches, and the depth is such, that the weight it will bear when its narrower face is parallel to the horizon, is in proportion to the weight it will bear when its broader face is parallel thereto, as 3 to 2. If the transverse and conjugate axes of a prolate spheroid be equal to the breadth and depth of this piece of timber—What will be the contents of the greatest square prism and square pyramid that can be inscribed therein; and what is the difference between the contents of the greatest cylinder that can be inscribed in the pyramid, and the greatest cone that can be inscribed in the prism?

Solution, by *Mr. John Ryley, of Leeds.*

Put the breadth of the piece of timber = 12 inches = b and x = the depth; then; because the strength of a beam is as the breadth multiplied into the square of the depth, we have per question $b^2x : bx^2 :: 3 : 2$; hence $3bx^2 = 2b^2x$, and $x = \frac{2b}{3} = 8$ inches, the conjugate axe. (Fig. 5)

Now let MNEF be a section of the prism, and put AB $= a$, CD $= c$, and BK $= x$, then AK $= a - x$, and KH = the length of the prism $= a - 2x$; also, by the property of the figure, $a^2 : c^2 :: x \times \overline{a - x} : \frac{c^2}{a^2} \times \overline{ax - x^2} = FK^2$; therefore by the question $\frac{c^2}{a^2} \times \overline{ax - x^2} \times \overline{a - 2x}$ is to be a maximum, which being put into fluxions and reduced, we have $x = \frac{a}{2} - \frac{a}{2\sqrt{3}}$, hence $KF^2 = \frac{c^2}{6}$; but the area of a square inscribed in the circle EF is $= 2KF^2 = \frac{c^2}{3}$: Moreover HK $= \frac{a}{\sqrt{3}}$; therefore, the content of the prism is $= \frac{ac^2}{3\sqrt{3}} = \frac{256}{3}\sqrt{3} = 147.798$.

But this is not the greatest prism that can be inscribed in the spheroid; for let a and b remain as before, and put LD $= x$; then LC $= c - x$, and LI $= 2x$. But by the property of the ellipse, $c^2 : a^2 :: x \times \overline{c - x} : \frac{a^2}{c^2} \times \overline{cx - x^2} = FL^2$, and $\overline{cx - x^2}$ ×

$\overline{c - 2x}$ by the question is a maximum, which being fluxed, &c. we shall find $x = \frac{c}{2} - \frac{c}{2\sqrt{3}}$; and $LF^2 = \frac{d^2}{b}$, and the area of a square in the circle $FN = \frac{a^2}{3}$; also, $LI = \frac{c}{\sqrt{3}}$, and the content of the prism $= \frac{a^2c}{3\sqrt{3}} = 128\sqrt{3} = 221.7024$.

Again, let MBN, NCF be sections of pyramids inscribed in the spheroid, and take a and c, the same as before, and $BH = x$; then $a^2 : c^2 :: x \times \overline{a - x} : \frac{c^2}{a^2}\ \overline{cx - x^2} = HN^2$, and $ax^2 - x^3 =$ a maximum; which being put into fluxions and reduced, we get $x = \frac{2a}{3}$; $NH^2 = \frac{2c^2}{9}$, the area of the pyramid's base $= \frac{4c^2}{9}$, and its content $= \frac{8ac^2}{81} = 75\frac{23}{27}$.

But this is not the greatest pyramid that can be inscribed in the spheroid. For if AB $= a$, CD $= c$, and CL $= x$; then $c^2 : a^2 :: cx - x^2 : \frac{a^2}{c^2} \times \overline{cx - x^2} = LF^2$, and $cx^2 - x^3$ a maximum; which being fluxed, &c. $x = \frac{2c}{3}$ the area of the pyramid's base $= \frac{4a^2}{9}$, and its content $= \frac{8a^2c}{81} = 113\frac{7}{9}$; which exceeds the first content by $37\frac{25}{27}$ cubic inches.

Now, in order to determine the greatest cylinder that can be inscribed in the pyramid NCF, put $.7854 = n$; then as the area of the pyramid's base is $= \frac{4a^2}{9}$; the area of a circle inscribed in the said base, is $= \frac{4na^2}{9}$; and it is also well known, that when the inscribed cylinder is a maximum, its altitude will be $\frac{1}{3}$ of the altitude of the pyramid; wherefore, by similarity of figures, $\frac{4c^2}{9} : \frac{4na^2}{9} :: \frac{16c^2}{81} : \frac{16na^2}{81} =$ the area of the cylinder's base, which being multiplied by $\frac{2c}{9}$, the product is $\frac{32na^2c}{729} =$ the content of the cylinder.

Again, from what has been done above, we have the area of the cone's base inscribed in the prism $= \frac{na^2}{3}$, and its content $= \frac{na^2c}{9\sqrt{3}}$. Consequently, $\frac{na^2c}{9\sqrt{3}} - \frac{32na^2c}{729} =$

$\frac{81na^2c - 32\sqrt{3}\,nac^2}{729\sqrt{3}} = 18.325$, the required difference between the cone and cylinder.

Question 132, by *Mr. W. Armstrong.*

ACB is a given parabola, it is required to determine, geometrically, a point P in the base AB, so that drawing PQ ⊥ thereto, and meeting the curve in Q; AP + PQ may be a maximum?

Solution, by *Mr. John Ryley, of Leeds.*

Construction. From one end Q (Fig. 6) of the parameter DQ, draw PQ ⊥ to AB, and P will be the point required.

Demonstration. Let WZ be the directrix, WXI the square, and IGF the string by which the parabola is generated. Now it is evident, that when AP + PQ is a maximum, EP + PQ or FQ + QP will be so too; because AE is constant; also IG + GF = PQ + QF; but IG + GH = EI + IG is evidently less than IG + GF; and if HG was drawn on the opposite side of DQ, the same consequence would follow; therefore AP + PQ is the required maximum.

Question 133, by *Mr. James Stevenson, of Heath.*

Given the vertical angle and the sum of the including sides, to determine the triangle, when the square of the perpendicular is = to the area.

Solution, by *Mr. John Ryley, of Leeds.*

Construction. On any line RS (Fig. 7) describe the segment of a circle to contain the given vertical angle; apply PT ⊥ to, and = $\frac{1}{2}$ RS: join R,T, and S,T; in RT produced, take TV = TS, and join V,S; in VR (produced if necessary) take VA = the given sum of the sides; draw AB ∥ to RS meeting VS produced in B, and ABC will be the triangle which was to be determined.

Demonstration. Demit the perpendicular CD. Now, as TV is = to, and CB parallel to TS; therefore BC = CV, (*Eu.* 4. 6) consequently AC + BC = AV, and the angle ABC = ∠ RTS; wherefore the triangles ABC, RTS are similar in all respects.— Moreover, the area of the triangle RTS is = $\frac{1}{2}$RS × TP; but $\frac{1}{2}$RS = PT, by construction; therefore $\frac{1}{2}$RS × PT = PT^2. But it has been proved that the triangle ABC is similar to RTS; consequently $\frac{1}{2}$ AB × CD = CD^2 = the area of the triangle ABC. Q. E. D.

Question 134, by *Mr. Richard Judson, of Beverley.*

I say, the altitude of the conical part of the up-heaped Winchester bushel, is = to the depth of the half-bushel; and that of the half-bushel to the depth of the peck; and so of the rest to the pint. Required a demonstration?

Solution, by the late *Mr. Lamb, of Sproatley.*

Put x = the height of the cone on the pint, a = the area of the base; hence $16a$ = the area of the cone's base on the bushel; then $\frac{ax}{3}$ = the content of the cone on the pint; $\frac{2ax}{3}$ = the content on the quart; $\frac{4ax}{3}$ = the content on the $\frac{1}{4}$ peck—proceeding thus, I find $\frac{64ax}{3}$ = the content of the cone on the bushel, and $\frac{64ax}{3} \times \frac{3}{16a}$ $= 4x =$ VP (Fig. 8) the height of the cone on the bushel, ABCD. Put now a = AP = 9.25 the semi-diameter of the bushel ABCD; b = PI = 8 the depth; c = WC = EI = 11.654 the semi-diameter of the measure EFGH, containing two bushels; d = 10,079385 the depth, IL. Then by similar triangles, as VP : AP :: AC : WC, *viz.* $4x : a :: b : c$ ∴ $4cx = ab$, and $4x = ab \div c = 6.3496$ inches the height of the cone on the bushel, which is exactly the depth of the half bushel. Q. E. D.

That WC = EI the semi-diameter of HA, or measure containing two bushels, may be thus proved. Put x = WC, then as AP : AC :: WC : WΓ, *viz.* As $a : b :: x : d$; ∴ $bx = ad$, $x = ad \div b = 11.654$ = WC = EI; hence WE = AP.

Question 135, by *Cosmopolite.*

If a point be taken any where within an equilateral triangle, and three ⊥s let fall from that point on the three sides, the sum of these ⊥s is = to the ⊥ of the triangle.—— Required a new and concise geometrical demonstration.

Solution, by *the Proposer.*

Let ABC (Fig. 11) be the equilateral triangle, O the point, FO, DO, and EO the perpendiculars. Draw the dotted lines as per figure; then, BC × FO = twice the area of the triangle BOC; AC × DO = the double area of AOC; and AB × EO = twice the area of BOA, the sum of these is = double the area of the whole triangle:— hence BC × FO + AC × DO + AB × EO = BC × AP; and by dividing by BC = AB = AC, we get FO + DO + EO = AP. Q. E. D.

Question 136, by *Mr. James Wolfenden, Hollinwood, near Manchester.*

Given the base and vertical angle to determine the triangle, when the difference of the perpendicular and line bisecting the vertical angle is the greatest possible.

Solution, by *the Proposer.*

Upon the given base AB (Fig. 9) let a segment of a circle be described to contain the given ∠; then its evident that the vertex of the required Δ will be somewhere in the periphery thereof, which suppose at C; and having completed the circle, draw the diameter EF ⊥ AB meeting it in H; join FC cutting AB in *y*, and let also the chord F*a* be supposed drawn indefinitely near FC, cutting AB in *e*; on F*ea* let fall the ⊥s *yr*, C*x*, on EF the ⊥ CP, and on AB the ⊥s CD and *as*, the last of which cuts PC in *n*, and join CO (O being the center). Now C*y* — CD being a maximum per question; *ae* — *as* will be ultimately = to it, that is, *er* + *ax* = *an*. But the Δs C*ax*, *yre* are similar, ∴ *ax* : *re* :: C*x* : *yr* :: CF : *y*F :: FP : FH, ∴ *ax* : *ax* + *er* = *an* :: FP : FP + FH; also per similar Δs C*an*, CPO and C*ax*, CPF, we have C*a* : *na* :: CO : PC, and C*a* : *xa* :: CF : CP ∴ *ax* : *an* :: CO = FO : CF; consequently FP : FP + FH :: FO : CF, and by division, FP : FH :: FO : CF — FO, or FH × FO = PF × $\overline{CF - FO}$: hence this

Construction. Having described the circle, &c. as above, through the end of the diameter F, draw GS ⊥ thereto, take FG = FO, draw GR ∥ FE, then with axis GR, vertex G, and parameter = FE, describe a parabola G*b*, intersecting the hyperbola *wb*, described with the asymptotes FE and FS; and having a power = FH × FO, in *b*; through *b* ∥ AB draw *b*C, intersecting the circle in C, and join AC, BC, and ACB will be the triangle required.

For produce *cb* to meet FE and GR in P and R; then by the property of the parabola, and construction $\overline{bR}|^2$ = RG × EF = PF × EF = $\overline{FC}|^2$ by the property of the circle, hence R*b* = FC and P*b* = FC — OF, RP being by construction, = FO, ∴ FP × $\overline{FC - FO}$ = *b*P × PF = power of the hyperbola = FO × FH per construction. Q.E.D.

Question 137, by *Mr. James Wolfenden.*

Given the diameter AF, and the perpendicular ordinate PE of a semi-circle AEF; to draw geometrically, the chord AC, cutting PE in B, so that if ED be perpendicular to PE, the area of the right-angled triangle BCD may be a maximum.

Solution, by *the Proposer.*

Analysis. Let AEF be the given semi-circle, AF the diameter, and PE the ⊥ ordinate; and conceive ABC to be the required chord, and A*ra* another, drawn indefinitely near it, meeting PE in *r*; on PE let fall ⊥s CD, *am*, and CH ⊥ AF, cutting *am* in *n* and A*a* in *x*; draw, also, CF, and the radius CO of the semi-circle.

Now BD × DC being a maximum, per question, $rm \times ma$ will, in its ultimate state, be = to it, and consequently BD : DC or HC : HA :: $Dm - rB : an$, or as the increment of the base to the ⊥. But it is easy to prove that the △s OFC, Cax are similar; as likewise the △s CHO, Can; whence $Ca = Cx$ and $Cx : Br$ :: CA : BA :: AH : AP ∴ $Cx : Br$ :: AH : AP; and since $Cn : Ca = Cx$ :: HO : OC, we therefore have $Cn : Br$ :: HO × AH : CO × AP, and by division, $Dm = Cn : Dm - Br$ :: HO × HA : HO × HA − CO × AP.

Moreover, per similar △s, $Cn = mD : na$ :: HO : HC :: HO × HA : HC × HA, wherefore AH × HO − AO × AP : HC × HA :: $Dm - Br : na$ :: (by what has been proved above) HC : HA :: HC^2 : HA × HC, ∴ AH × HO − AO × AP = HC^2 = AH × HF, hence AO × AP = AH × $\overline{HO - HF}$ = $\overline{AO + HO}$ × $\overline{2HO - AO}$ = AO × HO − AO^2 + $2HO^2$, ∴ AO × $\overline{AO + AP}$ = $\overline{AO + 2HO}$ × HO; or (making $Ay = yO$) HO × $\overline{HO + yO}$ = yO × $\overline{AO + AP}$. Hence this

Construction. To yO let HO be added, so that the rectangle under the whole and part added may be = yO × $\overline{AO + AP}$; draw the semi-chord HC, and join AC, which will evidently be the chord required.

PRIZE—Question 138, by *Mr. James Wolfenden.*

Required the sum of the infinite series $\frac{x}{2.3} + \frac{3^2.x^5}{2.3.4.5\,6.7} + \frac{3^2.5^2.7^2.x^9}{2.3\ldots\ldots.11} + \frac{3^2\,5^2.7^2 9^2.11^2 x^{13}}{2.3\ldots\ldots\ldots\ldots.15}$, &c. by means of circular arcs and logarithms.

Solution, by *the Proposer.*

It is well known that the infinite series $x + \frac{x^3}{2.3} + \frac{3^2.x^5}{2.3.4.5} + \frac{3^2.5^2.x^7}{2.3.4.5\,6.7}$ + &c. is = circular arc, sine x and radius 1; and also the series $x - \frac{x^3}{2.3} + \frac{3^2x^5}{2.3.4.5} - \frac{3^2.5^2\,x^7}{2\,3\,4.5.67} + \frac{3^2.5^2.7^2.x^9}{2.3\ldots\ldots\ldots9}$ − &c. = hyperbolical logarithm of $x + \sqrt{1 + x^2}$: call the said arc and logarithm a and m respectively: then if we take half the sum of these series, we have $x + \frac{3^2.x^5}{2.3.4.5} + \frac{2^2.5^2.7^2.x^9}{2.3.4\ldots\ldots9}$ &c. $= \frac{a}{2} + \frac{m}{2}$. Let now both sides of this equation be multiplied by $\dot{x}$, and the fluents give $\frac{x^2}{2} + \frac{3^2x^6}{2.3\,4\,5.6} +$

$\frac{3^2.5^2.7^2.x^{10}}{2.3.4\ldots\ldots10}$ + &c. $= \frac{ax}{2} + \frac{1}{2}\sqrt{1-x^2} + \frac{mx}{2} - \frac{1}{2}\sqrt{1+x^2}$. Again, multiply both ſides of this laſt equation by $\dot{x}$, and take the fluents, ſo ſhall $x^2 \times$: $\frac{x}{2.3} + \frac{3^2.x^5}{2.3.4.5.6.7} + \frac{3^2.5^2.7.^2x^9}{2.3\ldots\ldots11}$ + &c. $= \frac{2x^2+1}{8}a + \frac{mx^2}{4} + \frac{3}{8}x\sqrt{1-x^2} - \frac{3}{8}x\sqrt{1+x^2} - \frac{m}{8}$: which expreſſion divided by x^2, will evidently be the ſum of $\frac{x}{2.3} + \frac{3^2x^5}{2.3..7} + \frac{3^2.5.^27^2.x^9}{2.3\ldots\ldots11} + \frac{3^2.5^2.7^2.9^2.11^2.x^{13}}{2.3\ldots\ldots\ldots\ldots15}$ + &c. as required.

Corollary. If we take half the difference of the original ſeries above, we ſhall have $\frac{x^3}{2.3} + \frac{3^2.5^2.x^7}{2.3\ldots7} + \frac{3^2.5^2.7^2.9^2.x^{11}}{2.3\ldots\ldots\ldots11}$ + &c. $= \frac{a}{2} - \frac{m}{2}$. And by multiplying both ſides of the equation by $\dot{x}$, and taking the correct fluents, we get $\frac{x^4}{2.3.4} + \frac{3^2.5^2.x^8}{2.3\ldots.8} + \frac{3^2.5^2.7^2.9^2x^{12}}{2.3\ldots\ldots\ldots12}$ + &c. $= \frac{ax}{2} + \frac{1}{2}\sqrt{1-x^2} + \frac{1}{2}\sqrt{1+x^2} - \frac{mx}{2} - 1$. Let this laſt equation be alſo multiplied by $\dot{x}$, the fluents taken, and x tranſpoſed, and we obtain the infinite ſeries $x + \frac{x^5}{2.3.4.5} + \frac{3^2.5^2x^9}{2.3\ldots9} + \frac{3^2.5^2.7^2.9^2.x^{13}}{2.3\ldots\ldots\ldots13}$, &c. $= \frac{2x^2+1}{8}a + \frac{3}{8}\sqrt{1-x^2} + \frac{3}{8}\sqrt{1+x^2} + \frac{m}{8} - \frac{mx^2}{4}$.

ARTICLE XVIII.

An Essay on the Golden Number, Cycle of the Sun, Roman Indiction, the Julian & Dionysian Periods, &c.

By the late Mr. W. BROWN, of CLEOBURY.

Communicated by Mr. O. G. Gregory, of Yaxley.

OF THE GOLDEN NUMBER.

THE Golden Number, Prime, lunar Circle, or Enneadecaterides, is a revolution of 19 years; in which time the new moons, &c. return, (within 1½ hour nearly), to the same day they were on before; and indeed *Meton*, an old *Athenian*, the inventor of this number, who lived long before our Saviour's nativity, and the fathers of the primitive church after it, by reason of their unskilfulness in calculations of this sort, thought that the moon's change and full, fell precisely again on the same day of the month that they did 19 years before; hence, at the time of the *Nicene* Council, when the way of settling the time for observing the feast of *Easter* was established, the numbers of this cycle were inserted in the first column of the calendar, in golden letters, from whence came the names of Prime, Golden Number; but, as has been observed, the lunations in every 304 years, anticipating one whole day, by reason of the 1½ hour's return in 19 years, occasions their happening almost 5 days sooner now than at the council of *Nice*, which was in the year of our Lord, 325.

OF THE CYCLE OF THE SUN.

The cycle of the sun is a revolution of 28 years, in which time the same dominical letter comes about again, in the same order, and leap years expire: this cycle serves to find the dominical letter for any year.

OF THE ROMAN INDICTION.

The Cycle of Indiction is a revolution of three lustrums, or 15 Years, after which, those who used it began it again; this is more ancient than the two preceding cycles, and hath nothing to do with the heavenly motions, being established by *Constantine*, A. D. 312, who substituted them in the room of the *Olympiads*. They were so called, because they denoted the year that tribute was to be paid to the republic.

Having just defined the three foregoing Cycles, it may be proper to shew how to find either of them for any Year of *Christ*. And here it is to be observed that the Year before the Christian Æra the golden Number was 1, the Cycle of the Sun 9, and the Roman indiction 3; therefore as these three Cycles consist, respectively, of 19, 28, and 15, Years, we have this easy Rule: Add 1 to the Year, and divide by 19, for the golden Number; add 9 to the Year, and divide by 28, for the Cycle of the Sun, and add 3 to the Year, and divide by 15, for the Indiction: the Quotient will shew the Number of Cycles of each since Christ, and the Remainder' after each Division, the required respective Cycle.

OF THE DYONYSIAN PERIOD.

From the Cycles of the Sun and Moon (before explained) multiplied into one another, arises another Period of 532 Years (for $28 \times 19 = 532$) called the *Dionysian* Period; after the Completion of which, not only the new and full Moons return to the same Days of the Month, but the Days of the Month return also to the same Days of the Week; and therefore the dominical Letters, and moveable Feasts return again in the same Order, whence this is called the great paschal Cycle.

To find the Year of the Dionysian Period for any Year of Christ.—To the current Year, add the Number 457 (the Year of that Cycle when the Christian Æra commenced) and divide the Sum by 532, the Remainder after the Division is the Year of the Period required.

OF THE JULIAN PERIOD.

From the Multiplication of the lunar Cycle of 19 Years, the solar of 28 Years, and the Indiction of 15 Years, arises the great *Julian* Period of 7980 Years (for $19 \times 28 \times 15 = 7980$) which is supposed to have it's Beginning 764 Years before the Creation; it comprehends all other Periods, Cycles and Epochas; for there is but one Year in this whole Period, which has the same Numbers for the three Cycles of which it is made up: the Year before Christ happened in the 4713th Year of this Period; therefore to find the Year of the Julian Period for any Year of Christ, to the current Year, add 4713, the Sum will be the Year of the Julian Period.

Here let the foregoing Rules be illustrated by finding what is the golden Number, Cycle of the Sun, Roman Indiction, Dionysian and Julian Periods for the Year 1747.

To 1747
Add 1
19)1748(92
171
38
38
0 Since the Remainder is 0, the golden Number is the same with the Divisor, viz. 19.

To 1747
Add 9
28)1756(62
168
76
56
20 the Cycle of the Sun.

To 1747
Add 3
15)1750(116
15
25
15
100
90
10 the Roman Indiction.

To 1747
Add 457
532)2204(4
2128
76 the Dionysian Period.

To 1747
Add 4713
6460 the Julian Period.

Now, if the Cycles of the Moon and Sun, are given, to find the Year of *Christ*; or if the Cycles of the Moon, Sun, and Indiction are given, to find the year of *Christ*; they become problems of a different and more complex nature, but may be solved as follows.

Problem I.

Required the year when the Prime is 5, and the cycle of the sun 6?

Solution.

Put x for the required year of Christ; then (by the nature of the question) we have $\frac{x+1-5}{19} = b$, and $\frac{x+9-6}{28} = a$; which will be whole numbers; but $x = 28a - 3$, and x also $= 19b + 4$, consequently $19b + 4 = 28a - 3$, and $b = \frac{28a-7}{19} = a + \frac{9a}{19} - \frac{7}{19}$, $\therefore \frac{9a-7}{19} = m$, some integer, again $a = \frac{19m+7}{9} = 2m + \frac{m}{9} + \frac{7}{9}$ &c. $\therefore \frac{m+7}{9} = k$, an integer, from this substituted for m in the equation $a = \frac{19m+7}{9}$, gives $a = 19\,k - 14$; and this put for a in $x = 28\,a - 3$, gives $x = 532\,k - 395$, the required year.

Here suppose k = any integer, as 1, 2, 3, or 4, &c. then will $x = \overline{532 \times 1} - 395 = 137$, the year: or $x = \overline{532 \times 2} - 395 = 137 + 532 = 669$, the year; or $x = \overline{532 \times 3} - 395 = 669 + 532 = 1201$, the year; or lastly, $x = \overline{532 \times 4} - 395 = 1201 + 532 = 1733$, the required year.

Or, Universally,

Put x for the year of our Lord; m = the Golden number, and s = the cycle of the sun; then we have $\frac{x+1-m}{19} = b$; and $\frac{x+9-s}{28} = a$; and by a process similar to the above, we shall get $x = \overline{532\,p + 57s} - \overline{56m + 457}$ the required year; where $p = 1, 2, 3$, &c. Hence this general

Canon.

To once, twice, thrice, or four times, 532, add 57 times the cycle of the sun; from that sum subtract the sum of 457, + 56 times the Golden number; and the remainder will be the year of Christ.

Example.

If the Cycle of the Sun be 20, that of the Moon 19—Query, the year of our Lord? Here, by the general Canon we have (making $p = 4$) $\overline{532 \times 4} + \overline{57 \times 20} - \overline{56 \times 19 + 457} = 1747$ the year required.

N. B. If 1, 2, or 3, be put for p the general equation will give 151, 683, and 1215, respectively, for the years.

Corollory.

From the universal equation, $x = 532\,p + 57s - 56m - 457$, we have by transposition $x + 457$, $= 532p + 57s - 56m$, but $x + 457$ = the Dionysian Period: hence, we obtain the following general rule for finding the Dionysian Period, from the circles of the sun and moon given.

Rule.

Multiply the Cycle of the Sun into 57, and the Cycle of the Moon into 56; divide the difference of their products by 532; the remainder after the quotient, will give the year of the Dionysian Period.

☞ When the latter product exceeds the former, then subtract the remainder after the quotient, from 532, for the Dionysian Period.

Example I.

What is the Dionysian Period, when the Cycle of the Sun is 20, and the Cycle of the Moon 19? Here $\overline{57 \times 20} - \overline{56 \times 19} = 76$, (which cannot be divided by 532) the Dionysian Period.

Example II.

What is the Dionysian Period, when the Cycle of the Sun is 1, and the Cycle of the Moon 19? In this case $\overline{57 \times 1} - \overline{56 \times 19} = -1007$; also $\frac{1007}{532}$, quotes 1, and leaves 475, which subtract from 532, leaves 57 for the Dionysian Period.

Problem II.

When the Prime is 8, the Solar Cycle 9, and the Roman Indiction 14,—What is the Year of our Lord?

Solution.

Putting x (as before) we have, by the nature of the question, $\frac{x+1-8}{19} = b$; $\frac{x+9-9}{28} = a$; and $\frac{x+3-14}{15} = c$; to find x? Here (1) $x = 19b + 7$, (2) $x = 28a$, and (3) $x = 15c + 11$. Hence $19b + 7 = 28a$, and $b = \frac{28a-7}{19} = a + \frac{9a}{19} - \frac{7}{19}$ $\therefore \frac{9a-7}{19} = m$ some integer, and by proceeding as in Problem I. we get $a = 19k - 14$ (where k, an integer). Therefore (by 2 Equa.) $x = 532k - 392$, but (by 3. Equa.) $x = 15c + 11$, $\therefore 15c + 11 = 532k - 392$, and $c = \frac{432k - 403}{15} = 35k + \frac{7k}{15} - 26 - \frac{13}{15}$; consequently, $\frac{7k-13}{15}$ must be an integer: and of course, k must be some integer, that multiplied into 7, and 13 taken from the product, may be divisible by 15. In this case, k is found to be 4 (for $\frac{7 \times 4 - 13}{15} = 1$) which substituted for it in $x = 532k - 392$, will give $x = \overline{532 \times 4} - 392 = 1736$, the year, and so for any other.

Or, Universally.

Put x = year of Christ; m = lunar Cycle; s = solar Cycle, and r = roman Indiction; then by the nature of the question $\frac{x+1-m}{19} = b$; $\frac{x+9-s}{28} = a$; & $\frac{x+3-r}{15} = c$, will be whole numbers; and by pursuing the above method, we shall get $x = 4845s - 3780m - 1064r - 4713 - 7980k$ = the required year: Whence this general

Canon.

From 4845 times the Cycle of the Sun; take the sum of 4713, added to 3780 times the Cycle of the Moon and 1064 times the roman Indiction: this remainder divided by 7980 will quote the value of k, and the remainder, after the division will be the required year.

☞ When the sum of the negative products exceed the positive one, divide the difference by 7980, and take the remainder, (after division) from 7980 for the year.

Example I.

If the lunar Cycle be 19, the solar Cycle, and the roman Indiction 10; Query, the Year of our Lord? Here, by the general Canon we have $\overline{4845 \times 20} - \overline{3780 \times 19} - \overline{1064 \times 10} - 4713 = 9727 \therefore \frac{9727}{7980}$ quotes 1, for k; and the remainder is 1747 the year required.

Example II.

If $m = 19$; $s = 1$; $r = 6$; Query x? In this case $\overline{4845 \times 1} - \overline{3780 \times 19} - \overline{1064 \times 6} - 4713 = -78072 \therefore \frac{78072}{7980}$ quotes 9, and leaves 6252, which taken from 7980, there remains 1728 for the year required.

Corollary.

From the universal equation $x = 4845s - 3780m - 1064r - 4713 - 7980k$, we have by transposition $x + 4713 = 4845s - 3780m - 1064r - 7980k$; but $x+4713$ the Julian Period: hence we obtain the following general Rule for finding the Julian Period, from the Cycles of the Sun, Moon, and Indiction given.

Rule.

Multiply the solar Cycle into 4845, the lunar Cycle into 3780, and the roman Indiction into 1064; divide the difference betwixt the former, and the sum of the two latter products, by 7980, and the remainder, after division, will be the year of the Julian Period.

☞ When the sum of the two latter products, exceed the former; then subtract the remainder, after division, from 7980, for the Julian Period.

Example I.

What is the Julian Period, when the solar Cycle is 20, lunar Cycle 19, and the roman Indiction 10? Here $\overline{4845\times20}-\overline{3780\times19}-\overline{1064\times10}=14440 \therefore \frac{14440}{7980}$ quotes 1, and leaves 6460 for the Julian Period.

Example II.

What is the Julian Period when the solar Cycle is 1, lunar Cycle 19, and roman Indiction 6? In this case, $\overline{4845\times1}-\overline{3780\times19}-\overline{1064\times6}=-73359 \therefore \frac{73359}{7980}$ quotes 9, and leaves 1539, which taken from 7980, gives a remainder of 6441 for the Julian Period.

ARTICLE XIX.

Queries with their Solutions.

Query 1, by *Mr. Newton Bosworth*, of Peterborough.

IT has been proved by repeated experiments, that neither spirits of wine, or any other inflammable liquor, can be set on fire by any burning-glass yet made use of: how is this curious phænomenon to be accounted for?

Solution, by *Mr. O. G. Gregory*, of Yaxley.

So long as it was the generally received opinion that the Sun was a body of fire, and that the solar rays were hot, it must appear a very singular phænomenon that spirits of wine could not be set on fire by a burning-glass, when the same means would fuse iron and gold, calcine fossils, and vitrify tiles and pumice-stones. I know not what methods have been hitherto made use of to account for this; nor can I readily conceive how it can be done satisfactorily, without acceding to the opinions lately proposed concerning the nature of the Sun.

Several weighty arguments have been offered by Dr. Herschel and others, to shew that in all probability, the Sun is but little hotter than the earth we inhabit, and that of course, the rays which are emitted from him (or more probably, from his atmosphere) are not hot. They produce heat, only when acting upon some peculiar kinds of matter, as the collision of flint and steel produce fire, though neither of them be hot

Bodies are more heated from the action of the Sun's rays, in proportion as they are more dense; or as they are more rough: they are also more heated in proportion as their colour deviates from white, and the more opaque bodies are, the more they are affected by this method of producing heat.

Perfectly white and perfectly transparent substances, have very little, if any, heat produced in them by the action of the Sun's rays. These observations are the result of repeated experiments: but to account for the different dispositions of various substances (as deduced from these experiments) is a task that cannot very readily be performed, without a further acquaintance with the arcana of nature than has yet fallen to the lot of short-sighted mortals.

The Great Architect has, we may be assured, for wise purposes, given to different substances, different qualities and powers, and with this we perhaps ought to rest satisfied until the time arrive when His *people* shall, separated from this veil of mortality, view Him and the *grand plan* of all his works, as designed from eternity, with enlarged faculties, rapture undescribable, and gratitude unbounded!

Query 2, by *Mr. Goodwin*, Postgrove, near Philadelphia.

What is the proper criterion by which we may distinguish animals from vegetables?

Solution, by *Mr. O. G. Gregory*, of Yaxley.

To every unprejudiced, reflecting mind, I think, the exuberant goodness of the Supreme Being must be manifest, not only in the multitude, but the diversity of living creatures. The whole chasm in nature from the lifeless clod to man, is so well managed and husbanded, that a contemplation on the regular gradations that may be traced out in the varieties of vegetables and animals, raises the most sublime ideas of the transcendent wisdom of the adorable Creator.

With what a gradual process the animal creation sinks down from the creature complete in all its senses, to these which are scarcely a step above inanimate matter! Some animals are without the sense of smelling; others, of seeing; others, of hearing; others, have no senses but those of feeling and tasting: some living creatures, as polype, can scarcely be distinguished from vegetables; others, as some kinds of shell-fish, can hardly be discovered to have life, such are those which are fixed to the surface of rocks, and never move after being severed from the place to which they grow. To distinguish between animals which in some respects appear nearly similar to vegetables, and vegetables which one would almost imagine are possessed of animal sensation, is a speculation which has long engaged the attention of philosophers. Dr. Sibly, in his "Key to "Physic," has handled this subject more at large, than any other author that I have met with, and as his reasoning is ingenious, I shall extract a passage from it, for the perusal of the readers of your publication.

"The power of vegetation" says the Doctor, "is as perfect in an onion or a leek, "as in a dog, an elephant, or a man; and yet, though you threaten a leek or an "onion ever so much it pays no regard to your words, as a dog would do; nor, "though you wound it, does it avoid a second stroke. It is this principle of *self-"preservation* in animals, which, being the most powerful one in nature, is generally "taken, and with very good reason, as the true characteristic of animal life. This "principle is undoubtedly a consequence of sensation; and, as it is never observed "to take place in vegetables, we have a right to say that the foundation of it, namely, "sensation, belongs not to them. There is no animal, which makes any motion in "consequence of external impulse where danger is threatened, but what puts itself in a "posture of defence; but no vegetable whatever does so. A muscle, when it is touched, "immediately shuts its shell; and, as this action puts it in a state of defence, we conclude "that it proceeded from the principle of self-preservation. When the sensitive plant con-"tracts from a touch, it is no more in a state of defence than before; for whatever would "have destroyed it in its expanded state will also do it in its contracted state. The motion "of the sensitive plant, proceeds only from a certain property called irritability; and "which, though our bodies possess it in an eminent degree, is a characteristic neither of "animal or vegetable life, but belongs to us in common with brute matter. It is certain "that an electrified silk thread shews a much greater variety of motions than any sensitive "plant. If a bit of silk thread is dropt on an electrified metal plate, it immediately "erects itself; spreads out the fibres like arms; and, if not detained, will fly off. If "a finger is brought near it, the thread seems greedily to catch at it. If a candle "approaches, it clasps close to the plate as if afraid of it. Why do we not conclude "in this case that the thread is really afraid of the candle? For this plain reason, that "its seeming flight is not to get away from the candle, but to get towards the elec-"trified metal, and, if allowed to remain there, will suffer itself to be burnt without "offering to stir. The sensitive plant, in like manner, after it has contracted, will "suffer itself to be cut in pieces, without making the least effort to escape. The case "is not so with the meanest animal. An hedge-hog, when alarmed, draws its body "together, and expands its prickles, thereby putting itself in a posture of defence. "Throw it into water, and the same principle of self-preservation prompts it to expand "its body and swim. A snail, when touched, withdraws itself into its shell; but if "a little quick-lime is sprinkled upon it, so that its shell is no longer a place of safety, "it is thrown into agonies, and endeavours to avail itself of its locomative power in "order to escape the danger. In muscles and oysters, indeed, we cannot observe this "principle of self-preservation so strongly, as nature has deprived them of the power "of progressive motion: but, as we observe them constantly to use the means which "nature has given them for preservation, we can have no reason to think that they "are destitute of that principle upon which it is founded."

If I were to extract any more of Dr. Sibly's arguments, I should render this answer too diffusive to be inserted in your entertaining Miscellany: I must therefore refer the curious reader who wishes for more on the subject, to the book from whence the above extract is taken. See "Key to Physic: from page 49 to 56."

Query 31 by *Mr. O. G. Gregory*, of Yaxley.

Thunder storms it is now generally conjectured are electric phænomena; agreeable to this hypothesis it is required to advance a satisfactory reason why they should occur more frequently at any place in summer than in winter?

Solution, by the *Proposer*.

Though philosophers have anxiously laboured for a considerable time to obtain a thorough acquaintance with the nature of the electric matter, the subject is yet in such a state as to leave plenty of room for conjecture. Some persons suppose that the electric fluid is the same with the ether of Newton; others have recourse to the element of fire; and others suppose it is a fluid *sui generis*; whilst others conclude that it is either phlogiston, or intimately connected with it. But the opinion which will help most satisfactorily to explain the phænomena spoken of in the query, is, that the electric fluid is a peculiar modification of the sun's rays, or, that the solar rays when absorbed by the earth, become united with some substance, which union produces the fluid: I suppose though, that it may also be produced by other means; but this is the most general.

Hence then, a particular part of the earth and adjacent air receiving more of the sun's rays when it is summer at that place, than when it is winter; becomes much more frequently saturated with the electric matter in the former season than in the latter. Being then electrified positively, the redundant fluid will at length rush towards a part of the earth or air which is in a negative state of electricity, and thus cause the thunder-storm by endeavouring to restore the equilibrium. And, as any particular part of the earth will, according to this hypothesis, by means of the copious receipt of the solar rays therein, be frequently overcharged with the fluid in hot seasons: we may reasonably expect the more frequent recurrence of these phænomena in summer than in winter.

ARTICLE XX.

Mathematical Questions and Solutions.

PRIZE—Question 139, by *Mr. Robert Phillips.*

If the equation of a curve be $x = \sqrt{b^2 + y^2} + b \times$ h. l. of $\overline{y + \sqrt{b^2 + y^2}}$, it is required to find its area; together with the content of the solid generated by the rotation of the said curve round its axis: supposing that when $x = o$, $y = o$.

Solution, by *Mr. R. Taylor.*

By taking the fluxion of both sides of the given equation and multiplying the same by y, we get $y\dot{x} = \frac{y^2\dot{y}}{\sqrt{b^2 + y^2}} + \frac{by\dot{y}}{\sqrt{b^2 + y^2}}$; the fluent of which, (when corrected) is $=$ $\frac{1}{2} \times y \times \sqrt{b^2 + y^2} - \frac{1}{2} \times b^2 \times$ h. l. of $\overline{y + \sqrt{b^2 + y^2}} + b\sqrt{b^2 + y^2} - b^2 + \frac{1}{2}b^2 \times$ h. l. b. the area required. For the content of the solid we have $py^2\dot{x} = \frac{py^3\dot{y}}{\sqrt{b^2 + y^2}} + \frac{bpy^2\dot{y}}{\sqrt{b^2 + y^2}}$, and the fluent (when corrected according to the nature of the question) is $=$ $\frac{1}{5} \times p \times \overline{b^2 + y^2}|^{\frac{3}{2}} \times \frac{3y^2 - 2b^2}{3} + \frac{1}{2} bpy \sqrt{b^2 + y^2} - \frac{1}{2}b^3 p \times$ h. l. of $\overline{y + \sqrt{b^2 + y^2}} + \frac{2}{15} b^5 + \frac{1}{2} b^3 p \times$ h. l. of b, the content of the solid required; where $p = 3.14159 +$.

Corollary. The subtangent of the curve is equal to $\frac{y^2 + by}{\sqrt{b^2 + y^2}}$.

Question 140, by *Mr. Thomas Leybourn.*

What number is that which being any how divided, the square of one part when added to the other shall always be a square number.

Solution, by *Mr. John Ryley, of Leeds.*

Let x and y be the parts into which the number is to be divided; then by the question $x^2 + y$ and $y^2 + x$ are to be squares. Assume $x^2 + y = (m - x)^2 = m^2 - 2mx + x^2$, and $y^2 + x = (n - y)^2 = n^2 - 2ny + y^2$; hence $x = n^2 - 2ny$, and $y = m^2 - 2mx$. But by substitution $y = m^2 - 2mn^2 + 4mny$, and $x = n^2 - 2nm^2 + 4mnx$: Hence $x = \frac{n^2 - 2m^2n}{1 - 4mn}$, and $y = \frac{m^2 - 2mn^2}{1 - 4mn}$; therefore the required number, or $x + y$ is $= \frac{m^2 - 2mn^2 - 2nm^2 + n^2}{1 - 4mn}$, where m and n may be taken at pleasure; hence $x^2 + y = \frac{n^4 + 4n^3m^2 + 4n^2m^4 + m^2 - 2mn^2 - 4nm^3}{1 - 8mn + 16m^2n^2}$, and $y^2 + x = \ldots\ldots \frac{m^4 + 4m^3n^2 + 4m^2n^4 + n^2 - 2m^2n - 4mn^3}{1 - 8mn + 16m^2n^2}$ which are both squares.

If $m = n = 1$, $x = y = \frac{1}{3}$, $x + y = \frac{2}{3}$, $x^2 + y = y^2 + x = \frac{4}{9}$.

If $m = 1$ and $n = 2$, $x = 0$, $y = 1$, $x + y = 1$, $x^2 + y = y^2 + x = 1$.

If $m = n = 2$, $x = y = \frac{4}{5}$, $x + y = \frac{8}{5}$, $x^2 + y = y^2 + x = \frac{36}{25}$.

If $m = \frac{1}{2}$, and $n = -\frac{1}{3}$, $x = \frac{1}{6}$, $y = \frac{5}{12}$, $x + y = \frac{3}{12}$, $x^2 + y = \frac{16}{36}$, and $y^2 + x = \frac{7}{144}$.

If $m = 0$, and $n = 1$, $x + y = 1$, $x^2 + y = y^2 + x = 1$.

If $m = 0$, and $n = 2$, $x + y = 4$, $x^2 + y = 16$, and $y^2 + x = 4$.

If $m = 0$, and $n = 3$, then $x^2 + y = 81$, and $y^2 + x = 9$, &c. &c. &c.

Question 141, by *Mr. James Stevenson.*

Shew how to find the square roots of 875 and 87.5 at one operation on a sliding rule, without having recourse to the line D.

Solution, by *Mr. J. H. Swale, of Leeds.*

Invert the slider B. then to the given number on A set unity on B: observing what number on A coincides with the same on B; and it will be the root required: in the present case set unity on B to 875 on A, then we find 29.58 on B against the same on A, the root required; the slider remaining as above, and calling the number against unity 87.5 instead of 875; the root found as before is 9.354 as required.

Question 142, by *Mr. James Gates, of Peterborough.*

A candle 20 inches high was placed, perpendicularly, upon an elliptical table, whose transverse diameter is = 26 inches, conjugate diameter = 18 inches, and height = 36 inches, it is required to find the area on the floor, darkened by the shadow of the table.

Solution, by *Mr. J. H. Swale.*

The plane of the table being parallel to the horizon, and the candle ⊥ thereto; the shadow will evidently be projected into an ellipse similar to the table: hence we shall have 20 : 56 :: 13 : $36\frac{2}{5}$ :: 9 : $25\frac{1}{5}$. Hence the transverse of the shadow = $72\frac{4}{5}$; conjugate = $50\frac{2}{5}$; and area = $72\frac{4}{5} \times 50\frac{2}{5} \times .7854 = 2881.726848$ square inches as required.

Question 143, by *Mr. Thomas Barlow.*

To divide geometrically a given right line into two parts, such, that their rectangle may be to the sum of their squares in a given ratio.

Solution, by *Mr. John Fletcher.*

Let AB (Fig. 1) be the given line, and the given ratio that of m to n: make BF ⊥ AB; and equal to the side of a square expressing the magnitude of a fourth proportional to $n + 2m$, m, and AB^2: draw FP ∥ AB meeting a semi-circle described on AB in P, then PD drawn ∥ to BF will divide AB in D as required. For by construction $n+2m : m :: AB^2 : BF^2 (= DP^2 = AD \times DB)$ whence $n + 2m : 2m :: AB^2 (= AD^2 + DB^2 + 2AD \times DB) : 2AD \times DB$, and by division $n : 2m :: AD^2 + DB^2 : 2AD \times DB$, ∴ $AD \times DB : AD^2 + DB^2 :: m : n$.

Question 144, by *Mr. Wolfenden.*

Given the base and vertical angle, to determine the triangle, when the sum of the squares of the ⊥ and difference of the sides is the least possible.

Solution, by *the Proposer.*

Construction. Upon the given base AB (Fig. 2) let a segment of a circle ACB be described to contain the given angle, and draw the diameter Pw cutting AB at right angles in y; then make Dy = 2Py, and draw DC ∥ AB cutting the periphery in C, join AC BC; so shall ABC be the triangle required.

Demonstration. Draw Cw and CP meeting AB in e, and the ⊥ Cr on AB; and let any other triangles AdB, AxB be formed in the circle, the ⊥s, of which, or their equals ym, yn, are the one greater and the other less than Cr the ⊥ of ABC. First (by Simpson's Geo. IV. 18.) AC : BC :: Ae : Be, and AC + BC : AC − BC :: AB : 2ye; (by comp. and div.) also AB : AC + BC :: AC − BC : 2DC; hence we have $AB \times \overline{AC+BC} : AB \times \overline{AC+BC} :: \overline{AC-BC}^2 : 4DC \times ye = 4Py \times Dw$ (because $Dw \times Py = DC \times ye$ from the similar triangles Pey, DCw) $= Dw \times 2Dy$ (per constr.) Whence it follows that $\overline{AC-BC}|^2 + Dy^2$ is $= Dw \times 2Dy + Dy^2$, $\overline{Ad-Bd}|^2 + my^2$

$= mw \times 2Dy + ym^2 = \overline{Dw - Dm} \times 2Dy + my^2 = Dw \times 2Dy + Dy^2 + Dm^2$, and $\overline{Ax - Bx}|^2 + yn^2 = wn \times 2Dy + yn^2 = \overline{wD + Dn} \times 2Dy + yn^2 = wD \times 2Dy + Dy^2 + Dn^2$; but the two last of these quantities evidently exceed the first by the squares of Dm and Dn, consequently $\overline{AC - BC}|^2 + Dy^2$ is the least possible. Q. E, D.

Corollary. Hence when the sum of the squares is to be of a given magnitude P, if we make $Dy = 2Py$, and from the triangle ACB as before, and take Dm^2 or $Dn^2 = P - \overline{AC - BC}|^2 - Dy^2$, we shall have ym, yn the perpendiculars of the triangles AdB, AxB, both of which will manifestly answer the required conditions. It also appears that in any plane triangle ACB (things remaining as in the above construction) $\overline{AC - BC}|^2 = 4DC \times ye = 4Dw \times Py$.

The same, by *Mr. R. Taylor.*

Construction. On AB (Fig. 3) the given base, describe an arch of a circle ACB to contain the given ∠; ⊥ to AB draw the diameter DG, intersecting AB in E; then in ED take $EP = 2EG$, and draw PC ∥ AB; join AC and BC, and ABC will be the △ required.

Demonstration. Draw DC, AG, BG, and CG cutting AB in F; and on DG and AC demit the ⊥s CP, GZ: then will AZ = half the difference of AC and BC. Now as the ∠ CFB (EFG) is = to FAC + FCA = FAC + FAG = ZAG, the right-angled triangles AZG, GEF, and GPC, as well as the triangles GBF and GBC are sim. and PC : AZ :: GC : AG (BG) :: BG (AG) : GF :: AZ : EF, ∴ $AZ^2 = PC \times EF = GE \times PD$ (because of the similar △s GEF and PCD) and $\overline{AC - BC}|^2 = 4GE \times PD$: but when $4GE \times PD + PE^2$, or its equal $4GE \times DE - 4GE \times PE + PE^2$, is a minimum, then will $PE^2 - 4GE \times PE + 4GE^2$, and consequently $PE - 2GE$ be a minimum also; in which case PE must be = 2GE, as per construction.

Corollary. When the sum of the squares is to be of a given magnitude M, take VN = 4EG, and divide it in Q, so that VQ × QN may be $= AB^2 - M$; then will either of the parts VQ, QN be the perpendicular of a △ answering the question.

A fluxional Solution, by *Mr. Barlow.*

This Gentleman after having proved the equality of GE × PD and AZ^2, proceeds as follows. Put $ED = a$, $GE = b$, and $EP = x$; then will $PD = a - x$, and $GE \times PD = AZ^2 = ab - bx$, consequently $4ab - 4bx + x^2$ is to be a minimum, (per question) in fluxions $2x\dot{x} - 4b\dot{x} = 0$, ∴ $x = 2b$.

Question 145, by *Mr. John Knowles, of Liverpool.*

In a given circle, the diameter of which is 20 inches, it is required to inscribe, by elementory principles, a trapezoid, when the sum of the squares of the parallel sides is to the difference of their squares as 7 to 5, and the least part of the diameter included to the sum of the parts excluded as 5 to 4.

Solution, by *Mr. Richard Nicholson, Teacher of the Mathematics in the Rev. C. Vincent's Academy, Leeds.*

Suppose the thing done, ABCDE (Fig. 4) the given circle, ABCD the required trapezoid, whose parallel ends AB and CD cut the diameter EF in G and H; now $AB^2 + DC^2 : AB^2 - DC^2 :: 7 : 5$ per question; $2AD^2 (= 8AG^2) : 2DC^2 (= 8DH^2) :: AG^2 : DH^2 :: 12 : 2 :: 6 : 1$, by compounding and dividing; again, by dividing and taking $EI = FG$; $AG^2 - DH^2$ $(= \overline{GH + HE}.\overline{HI - HE} - \overline{EF - HE}.HE = GH.HI + HE.HI - GH.HE - HE^2 - EF.EH + HE^2) : EF.EH - HE^2 :: 5 : 1$, by multiplying means and extremes; $GH.HI + HE.HI - GH.HE - EF.EH = 5EF.EH - 5HE^2$, which by subtracting $\overline{HE.HI - GH.HE - EF.EH}$ becomes $GH.HI = \overline{6EF.EH - 5HE^2 - HE.HI + GH.HE} = \overline{6EF - 5HE + GH - HI}.HE$, or $\frac{1}{5}GH.HI = \frac{6}{5}EF - HE + \frac{GH - HI}{5}.HE = \overline{\frac{6EF + GH - IH}{5} - HE}.HE$; but $HG : GF + EH (= HI) :: 5 : 4$ per question, therefore $HG + IH (= EF) : 9 :: GH : 5$; but EF is given by the question, consequently GH and $HG + IH$ are given; hence this

Construction. Upon $GI = 20$ describe the semi-circle ILG, take $GH = 11\frac{1}{9}$, and erect the ⊥ HL; then from the point of intersection of which, and the semi-circle draw LN parallel to EF cutting the semi-circle, described upon $EK = \frac{6EF + GH - IH}{5} = 122\frac{2}{9}$ in L, and the ⊥ LH being demitted will determine H. For $GH.HI = HL^2 = \overline{EK - EH}.EH$, the rest is evident from the Analysis.

Question 146, by *Mr. James Stevenson, of Heath.*

It is required to determine two lines in a given ratio, so that if a given line be added to each, the sum of the squares of the compound lines shall be of a given magnitude.

Solution, by *Mr. John Ryley.*

Perpendicular to the indefinite right line AB (Fig. 5) take BC = AB = the given line; join AC, which produce indefinitely, and parallel thereto draw the line BF; from any point in which draw GH ∥ to BC, cutting AD in I: take GI to IK in the given ratio, and LK = AB; through B and K draw BM, and ∥ thereto draw LN; then from A to LN apply AP = the side of a square, which is equal to the given sum of the squares; from P, and ∥ to BC, draw PS, cutting BM, BD, BF and AE in the points M, D, F, S; and MD, DF are the required lines.

Demonstration. By construction, and the nature of parallel lines, AB = BC = KL = PM = ES; and by similar triangles GI : IK :: FD : DM the given ratio; and by Eu. 47. 1. $PA^2 = PD^2 + DA^2 =$ (by construction) $PD^2 + DS^2 =$ the given sum of the squares.

Question 147, by *Mr. James Stevenson.*

It is required to determine the greatest ellipses that can be inscribed between the peripheries of two similar and concentric ellipses, whose principal diameters are given.

Solution, by *Mr. John Ryley.*

Let ADBC, (Fig. 6) EHFG be the given concentric ellipses; GPI the greatest semi-ellipse that can be inscribed between the peripheries, LT a tangent and IT an ordinate drawn to the point of contact of the ellipses and tangent. Now in such a case as this, it has been proved by writers on fluxions, that GK = KL; therefore, put OB = r, OC = c, OG = m and OK = x; then by the property of the ellipse $OC^2 : OB^2 ::$ $DK \times CK : KT^2 = \frac{a^2}{c^2} \times \overline{c^2 - x^2}$; also OK : OC :: OC : OL $= \frac{c^2}{x}$; therefore OL $-$ OK $=$ LK $=$ GK $= \frac{c^2}{x} - x = x - m$, hence $x = \frac{m + \text{or} - \sqrt{8c^2 + m^2}}{4}$ consequently KT will become known, and the point to which the tangent must be drawn to touch both curves determined.

Question 148, by *Mr. O. G. Gregory, of Yaxley, Author of "Lessons Astronomical and Philosophical, for the Amusement and Instruction of British Youth," just published*.*

A Surveyor is desired to set out a piece of land in form of a right-angled triangle, its content to be 9 A. 2R. 14P. and to contrive it in such a manner that a road straight across from the right angle to the middle of the hypothenuse shall be just 10 chains long. Pray young gentlemen help him out, for the wiseacre is sadly puzzled!

Solution, by *Mr. James Gates, of Peterborough.*

Let ABC (Fig. 7) represent the triangle, EB = 10, as per question: complete the parallelogram ABCD, and draw the diagonals AC, BD. Now it is evident that 2BE = AC, therefore we have given the hypothenuse and the content to find the perpendicular FB = 9.5875. Also by 47. 1. Euc. the segments EF, FC = 2 8425 and 7 1575 respectively, and $AF^2 + BF^2 = AB^2$, likewise $AC^2 - AB^2 = BC^2$. Hence AC = 20, AB = 16 0265, and B = 11.9641 the sides of the Δ required.

* The Editor has read this Work and very much approves of it; he has introduced it into his own school, and recommends it to school-masters as a very proper book to be put into the hands of youth.

Question 149, by *Mr. James Gates, of Peterboro'.*

Suppose a straight rod 13 inches long was placed perpendicularly before a candle 18 inches high, and 10 inches distant from the bottom of the candle. It is required to find the length of the shadow of the rod; its position when the shadow is a minimum, with the length of the shadow and position of the rod when a maximum?

Solution, by *Mr. John Dawes, of Birmingham.*

Upon EB (Fig. 8) erect the $\perp$s BC and *da* (making B*d* = 10) which make = 18 and 13. Now it is evident that the shadow is a maximum when the ray becomes a tangent to the circle drawn with the radius *da*, and a minimum when in the position *dg*, therefore say by similar Δs, as C*b* : *ba* :: CB : BA. with C*d* and *df* find *d*C*f* + *d*CB = BCE = *fd*E, with which and *df* find *d*E, then is *d*A = 26 inches, B*d*C = 60° 56′ 40″, *d*E = 35.014 inches, and *fd*E = 68° 12′ 20″.

Question 150, by *Mr. Barrow.*

Suppose a globe whose diameter is 9 inches, be placed on an upright pin $\frac{3}{4}$ feet high in 40° of north latitude, when the Sun enters cancer: What will be the area of the shadow of the globe on the plane of the horizon in the space of three hours time?

Solution, by *Mr. John Dawes.*

Lemmata. Let the Azimuths of the sun be calculated for (at least) every half hour within the limited time. Let the altitudes of the sun be calculated when upon those azimuths.

Construction. Let BP (Fig. 9) represent the pin, *p*P the semi-diameter of the globe; BAP = altitude of the sun, BA = co-tangent of it, making B*p* + P*p* radius *aa* = co-secant of it making *ae* radius, which is = the diameter of the globe. Let (Fig. 10, answering to Fig. 9) BA, BB, BC, &c. be the co-tangent of the altitudes as before, and the intermediate angles the azimuths: then in the calculation there is enough given to find AE, which being multiplied by *xy*, and half each external ellipses being added will give the true answer.

Demonstration. The shadows of globes upon the plane of the horizon are perfect ellipses, unless the sun is in the Zenith, then they are circles; hence confect: *mn*, *mn*, *mn* the conjugate diameters are all equal to each other, and to the diameter of the globe, and the right lines bisecting the ellipses *xy*, *xy*, *xy*, are equal to each other.

Remark. There is wanted in the data, the time of the day; or at least the year of our Lord to determine the area required.

Question 151, by *Mr. John Griffith.*

IT happen'd so upon a day,
Two boys inclin'd to have some play,
Procur'd a pole just twelve feet long,
A tapering one both straight and strong;
And in the middle made a dent,
Which they did fix upon a point:
The lighter boy, he did intend
To ride one foot from the small end;
His weight exactly was five stone,
The other pounds just eighty-one,
And he must ride just two feet on;
Which being done, the case was plain,
An equilibrium did remain;
The pole was oak, both dry and sound,
Whose small end measur'd one foot round;
The greater end, pray find out at your leisure—
Perhaps you'll be required the same to measure.

Solution, by *Mr. J. H. Swale, of Leeds.*

Let ADBC (Fig. 11) represent the pole moving on the point P, or suspended freely from the point F: O the center of gravity of the pole; N the common center of gravity of the boys and pole; which must evidently be the case when an equilibrium obtains: put IK $= a$; AD $= 2d$; IA $= m$; 5 stone $= 920$ oz. $= w$; 81lb. $= 1296 = W$; .5318 oz. $= s$; .7854 $= n$; and greater diameter $= 2x$: then IN $= 2m$; and per mensuration, $\overline{x^2 + dx + d^2} \times \frac{4}{3}an =$ solidity of the pole; and $\overline{x^2 + dx + d^2} \times \frac{4}{3}ans$ $=$ weight. Again, per mechanics, IO $= \frac{3x^2 + 2dx + d^2}{x^2 + dx + d^2} \times \frac{1}{4}a$: hence, from a known theorem in mechanics, on the common center of gravity; we have......................

$$\frac{\overline{3x^2 + 2dx + d^2} \times \frac{1}{3}a^2 ns + \overline{2W + w} \times m}{\overline{x^2 + dx + d^2} \times \frac{4}{3}ans + W + w} = \frac{1}{2}a$$

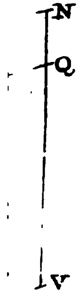
from which equation we get $x =$

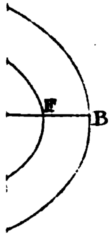
$$\sqrt{\left(\frac{\overline{W + w} \times \overline{a - 2m} \times \overline{2W + w}}{a^2 ns} \times \frac{3}{2} + d^2\right)} = 6.6568$$

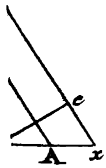
inches; then the required diameter $= 13.3136$ inches, as required.

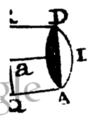
Question 152, by *Mr. James Stevenson.*

Given the solidity of a right cylindroid $= 60000$ inches, whose ends are equal and similar ellipses, the transverse and conjugate of which are in the ratio of 9 to 7; to determine the diameters, circumference, and length, when the whole surface is a minimum.


N
Q
V


F
B


e
A
x


D
I
a
A

Solution, by *Mr. J. H. Swale.*

Put 60000 = a; .7854 = m; 8+$\sqrt{65}$ = n; transverse axe = $9x$; and the length = y; then, per mensuration, $63mx^2$ = area of either base; $2mnx$ = circumference; $2mnxy$ = the convex surface; and $126mx^2 + 2mnxy$ = the whole external superficies: Again, $63mx^2y = a$; then $y = \frac{a}{63mx^2}$: which value substituted above in the expression for the minimum, and put into fluxions is $252mx\dot{x} - \frac{2an\dot{x}}{63x^2} = o$: from which x equal $\sqrt[3]{\frac{2an}{15876m}}$ = 5.4 nearly; therefore, transverse = 48.6; conjugate = 37.8; circumference = 136.141236; and the length = 43.34 inches, as required.

Question 153, by *Mr. William Robinson, of London.*

In a quadrangular field, the bearings of the opposite angles from each end of the south side is N. W. by N. and E. N. E. also the bearing and length of the same side is W. by N. 1045 links, and the distance of the N. E. corner from the intersection of the diagonals is 845, and of the N. W. corner from the said intersection 690—Query, the area of the field?

Solution, by *Mr. J. H. Swale.*

Let ACGB, (Fig. 12) represent the rectangular field; AB the given side; then the area of any plane triangle being equal to the sine of any ∠ multiplied by ½ the rectangle of the including sides, we shall have the area of the trapezium ACGB = AO × OB + OG × OB + OC × OG + OC × AO × ½ sine ∠ AOB = BO + OC × AO + OG × ½ sine ∠ AOB = BC × AG × ½ sine ∠ AOB = 1004625.574939 = 10A. 0R. 7.40092P. as required.

ARTICLE XXI.

Queries with their Solutions.

Query 1, by *Dr. Sangrado.*

WHAT is the reason, that in the yellow jaundice the patient has often a pain in the left shoulder?

Solution, by *the Proposer.*

The nerve that goes to the liver from the brain turns again upwards towards the left shoulder; and as all our sensations are carried on by means of the nerves, so all that affects the liver, affects the shoulder also.

Query 2, by *Dr. Sangrado.*

Why do the joints of wearied and aged people crack when they move?

Solution, by *the Proposer.*

Between the joints there is a liquor called the synovia, which serves, like the oiling a wheel or an hinge, to keep the joints supple and easy. Now, as hinges and wheels crack and move heavily when destitute of the proper lubricating fluid, so will the joints of the weary and the aged, this liquor in them being exhausted by frequent use, or strong exercise.

Query 3, by *Dr. Sangrado.*

Why are people taller in the morning than at night; which is a known fact?

Solution, by *the Proposer.*

This proceeds from somewhat a similar cause with the foregoing. The trunk of the body rests upon a number of small bones, which go to compose the spine, or back-bone. Between each of these bones there is a joint, and also a liquor. In the morning this liquor is in large quantities, and consequently the bones are kept at greater intervals from each other, by which means the trunk of the body becomes taller; but as the liquor wastes towards night, these bones lie closer upon each other; and therefore the spine thus contracting, the whole trunk of the body contracts, and the person becomes shorter. This difference between morning and evening stature is very sensible; in some men it is an inch or more. It was not, however, remarked among the faculty till some years ago, when by accident it was discovered by a recruiting serjeant, who had enlisted a number of men, whom he supposed equal to the royal standard; but his captain, upon examination, found them under-size. This created no small embarrasment in the serjeant, but willing to be as certain as possible, he measured his men the morning following, and they proved to be of exact size. By this accident it was discovered that all men are taller in the morning than at night.

Query 4, by *Mr. J. J. Thompson, of Brighton.*

Whether a smoke-jack will go best with 4, 6, or 8 wings: also, why it will go better with a clear fire, than where there is a quantity of smoke?

Solution, by *Mr. William Marrat, Writing-Master, at the Grammar-School, Lincoln.*

A smoke-jack will go better with 8 wings than with 4 or 6; because the current of air setting up the chimney will not lose its influence on one wing, before it begins to act upon another; as may be exemplified by a great or small number of teeth in a wheel, in regard to friction. It will also go the quickest when there is the least smoke; for the hotter and clearer the fire is, the less smoke there is, but the degree of heat is greater; and consequently the current of air occasioned thereby will be greater also; for the smoke is no agent in driving the jack; but is itself driven by the current of air that sets from the contiguous room.

The same, *answered by the Proposer.*

Whether a smoke-jack will go best with 4, 6, or 8 wings, I believe no general rule can be given. Conjectures may be advanced both *pro* and *con*. First, in regard to 4 wings, it has its advantages and disadvantages. In a large chimney, where there is generally a greater passage for the air to ascend than in the more modern-built ones, I suppose 4 flies to be best, because there are only 4 passages for the air to pass through; consequently the pressure must be greatest on the wings, and so force them round the faster; but in a close built chimney, where there is only just room for the flies to turn round, I should think that 6, 8, or more were requisite—the narrower the chimney, the greater the number of wings; or else the chimney will smoke on account of there not being a sufficient number of passages for the smoke to ascend, and very probably that is the reason most of the London smoke jacks are made with 6 flys, this being a medium between the two. The reason it will go best with a clear fire is, the clearer the fire, the greater the heat, whereby the air is rarified, and its mobility augmented; so that it must ascend the chimney much faster than if it be less heated and clogged with smoke; consequently it must force the wings round faster.

Query 5, by *The Rev. J. Shakleton.*

We find that a *tufa*, or *golden ball*, is now an usual sign of majesty, and has been so for a long time past. Pray how did it take its rise, at what place, and in what king's or emperor's days?

Solution, by *the Proposer.*

The British soldiers in Roman pay, saluted their countryman *Constantine*, Emperor at York, and presented him with a *tufa*, or *golden ball*, as a symbol of his sovereignty over the Island of Britain. He was much taken with the emblem, and upon his conversion to christianity, placed a cross upon it, and had it carried before him in all processions whatsoever. After which time it became the usual sign of majesty, and was used by all other christian princes. *See Churchill's Div. Britan.*

Query 6, by *Mr. James Osmond.*

What is beauty—and what is taste?

Solution, by *the Proposer.*

Beauty in the most general sense, is the harmony of things, and their ends: physical beauty, hence is the exact correspondence of all the various parts of an individual, with the end of his existance, as a body: moral beauty, the same correspondence of all his powers, considered as a moral being. Hence it is clear, that every thing existing is susceptible either of moral or physical beauty; from the meanest animal to man, and from man to God. Taste seems to be the faculty of feeling harmony in things and actions, though the word itself implies only the signification of the faculty of distinguishing one thing from another. Whether this faculty be simple feelings, or comparing judgment, or a composition of both, is another question.

Query 7, by *Dr. Sangrado.*

How do we prove that there is a circulation of blood carried on through the bones?

Solution, by *the Proposer.*

From an obvious experiment of the madder root, which has been found to turn the bones of those animals that feed upon it, red. It has been observed, that there are veins and arteries through the bones, as through all other parts of the body, but which, in some measure, escaped the eye from their exceeding minuteness. The tincture of madder, however, enters these, and dyes them of a red colour; so that a pig fed three days upon madder, and then interchangeably three days upon its usual food, will have its bones, when sawed transversely, take the appearance of a tree sawed in the same manner, a ring of red, and then a ring of white, and so on in proportion to the number of times it was thus kept upon the madder root.

Query 8, by *Mr. Archer.*

What is the reason that when two viols are tuned in unison, one of them being touched the other will answer, though at a distance?

Solution, by *the Proposer.*

Because air is the proper vehicle and conveyance of sounds; and accordingly, as the air is driven with greater or less violence, it affects all objects that it meets with. A cittern perhaps is by its make as much accommodated for the reception of sounds, as any other instrument, therefore we'll consider the effects of a repercussive air upon that: an ordinary noise will beat the air every way, and that which meets with these strings, will move them all into a distinguishable audibility, proportionable to the shrillness or smallness of the voice; this is universally granted by such as have made the experiment.— Now, since this voice, in what key or note soever it is delivered, does effect the sounding of so many notes at once, and that these notes are proportionable to a greater or less agitation of the air; then the efficient cause is the motion of the air, and not the unison, as is generally thought: though we cannot deny the sympathy to be more effective than in different keys; as is evident by a piece of paper which will violently tremble upon a string that is an unison, when it lies almost still upon the other keys, and all by the same agitation of air.

Query 9, by *Mr. Durand.*

In Antichell Grey's debates in the House of Commons, mention is made of a motion made by Mr. Mallet for bringing in a bill to repeal the act of King James I. entitled, "Felony to marry a second husband or wife, the former being living." On which motion Mr. Waller said, "Let the gentleman that would bring in this bill tell us, whether his dove-house is not better stored when one cock has but one hen, than his yard, where one cock hath many hens." Now, the member proposing the bill, pretended its design was for peopling the nation, and preventing the promiscuous use of women; and it is certainly owing to our ineffectual provisions respecting matrimony, that polygamy, tho' prohibited by law, is so much practised in fact. *Query*—Whether polygamy tends to depopulation or not? And whether its being permitted under any restrictions whatever, would in fact tend to prevent the promiscuous use of women; which is undoubtedly detrimental to population?

Solution, by *the Proposer.*

Almost all the arguments that have been advanced in favor of polygamy, a practice renounced by the general consent of the christian world, have been founded on a misapplied and misconstrued maxim, "That an increase of population is advantageous to a state." There is doubt to be made of the truth of the maxim itself; but the reason why populousness is an advantage to the state, is the very reason why it becomes populous: an increase of people proceeding neither simply from monogamy or polygamy, but from an increase of employment, by which the numerous poor may reap a comfortable subsistence. For, in a country where such employment increases, the people will of course increase, even though polygamy were ever so strictly forbidden; while, on the other hand, where the means of subsistence decrease, the people will of course decrease too, even though polygamy, and what is still more, the adoption of foreigners, were permitted in their utmost latitude. Those who cannot find the means of subsistence, it is plain, must either starve or emigrate, or rob, and be hanged or transported. Admitting therefore, that a man may beget more children by having four or five wives, than by having only one; the question immediately occurs, can he maintain and provide for them?—This will depend upon his circumstances; and must limit the practice of polygamy (as it really is in those countries where it is established) to the opulent only. In most countries of Europe, if a sober industrious couple marry young, they will probably have as many children as they can possibly maintain by their labour, even where there is plenty of provisions and employment; and more would only be a burthen to them and to the community. Again, the increase of the children of the rich and powerful might be so great as to become a nuisance to society, as they are generally brought up in idleness and luxury; but were it otherwise, the wealth of the parents divided among such a numerous offspring, would in a generation or two reduce them all to poverty, so at once put an end to the practice contended for. In a word, a full and final answer may be given to this question, by taking a comparative view of the different states where monogamy and polygamy are established.

The same, *answered by Mr. Henry Johnson.*

It is remarkable that two great men, though of very different principles, should agree in opinion about polygamy, that it was favored, as well as permitted by the law of nature. I mean the late Lord Bolinbroke and Bishop Berkeley: not that their opinion, though one and the same, was deduced from the same principles: what Lord Bolinbroke's were, I shall not pretend to enquire; but certainly the Bishop of Cloyne's opinion was founded on his great and sincere regard for christianity; having adopted it to shew that the christian law was more perfect than the law of nature. But, with due deference to the character and memory of so great a man, the law of nature is undoubtedly the law of God: and the bishop certainly would have admitted that the christian law is the law of God. A contradiction will hence arise, that shews the bishop had a very vague idea of the law of nature, or at least thought it a vague and indeterminate thing. Numerous, however, are the arguments that have been brought, and founded in what is called the law of nature, to prove both the moral and political propriety of monogamy. Mr. Hume conceives the first principle of it to lie in the natural desire of exclusive property in a desirable object. Others have deduced it from the old principle of doing as one would be done by; appealing to experience, that men do not like any participation with them in female enjoyment. These reasoners indeed admit that custom may have a wonderful effect in destroying this principle; and therefore it may not be unjustly objected to as a law of nature.

Some casuists deduce an argument against polygamy from the equality, which is said naturally to subsist in every country, between the number of males and females brought into the world: so that hence, if some men have more than one wife, others must necessarily have none at all. This would be something of an argument, if celibacy among the males were not so notorious: but till there be a law, obliging men to marry some one woman, this can be no argument against those who do marry having more than one.

One of the best arguments deduced from the law of nature, and which affords therefore a more pertinent answer to the question; (which if I understand it right is merely political) is this, It notoriously requires a long time after a child is born before it can provide for, or take care of itself; so that there is evidently a necessity for the industry of a father, and the care of a mother to bring up their young. Now, that the preservation of our young is as much a law of nature as the propagation of them, I conceive nobody will dispute. It is indeed, a general law among all those animals whose young cannot provide for themselves as soon as brought forth. Nay, it is certain, that in breeding time, most animals, whose nature is not vitiated by their becoming domestic, enter into a kind of *vinculum matrimoni*. Instead of living together promiscuously, as they do at other times of the year, they separate into pairs, and each makes choice of its mate; the contract thus made between them being kept more inviolable, I believe among the brutes than among the human species, till their young are capable of providing for themselves, and need no longer their parental assistance. Supposing mankind therefore on a footing with the brutes in this respect; when is the matrimonial contract between a man and woman to be dissolved? Before the first child is capable of providing for itself the woman brings forth another; and if she be commonly fruitful, as most women would be were it not for political inconveniencies, before their last child is grown up, both father and mother are grown so old as to be hardly able to provide any longer for themselves. Hence it appears, that not only the matrimonial union into pairs, but the continuance of that union is dictated by the law of nature.

Now, I presume that if monogamy be deducible from an express law of nature, it must of course be the best and surest method to population; as the very end of our creation appears to be agreeable to the first command said to be given to our species, *Increase and multiply*.

Query 10, by *Mr. C. Hastings*.

Is the passion of jealousy to be imputed most to *physical* or *moral* causes? That is, to the effects of constitution and climate, or those of education and opinion?

Solution, by *the Proposer*.

It is generally supposed that jealousy prevails most in warm climates, where the inhabitants are said to be of a more amorous constitution than those of colder countries. A slight retrospect, however, of the history of the several nations of the world, will show the fallacy of this supposition. The cold and barren climates have produced people strongly addicted to amorous pleasures; nay the religion of some hath been founded solely on the gratification of this sensual propensity. On the contrary, there hath been found among the inhabitants of some southern and warm climates, a coldness and abstinence, that appears astonishing to the people of the north. Should it be allowed indeed that a hot climate inflames this passion; yet the same warmth causes a waste of animal spirits, that must equally operate againſt the deſire of gratification, and render the climate a matter of indifference.

The paſſion of jealouſy is ſtill leſs affected by climate than that of love. The ſpace of a few leagues on the continent of Africa, is known to ſeparate a people ſubject to all the extravagance and fury of this paſſion from another, who glory in proſtituting their wives both to friends and ſtrangers. Again, cuſtoms the moſt oppoſite to jealouſy, are eſtabliſhed all over India. The women there are perfectly at liberty, and boaſt of their amours without giving the leaſt umbrage. Nay huſbands themſelves ſeek out gallants for their wives; and the girls of greateſt merit are ſuch as prove the earlieſt ripe for the embraces of men. The Guebers and Armenians, though they reſide among people that are extravagantly jealous, are not ſo themſelves. The inhabitants of Cachemis bring their wives voluntarily to their princes, in order to boaſt of illuſtrious blood in their families. The modern Italians are jealous; yet their anceſtors were not ſo. In fine, the caprices of this paſſion are carried ſuch lengths, that, in a nation the moſt jealous in the world, a man who would think himſelf the moſt highly diſhonored by the illegal intimacy of his wife or daughter with his equals, ſhall abandon both, without remorſe, to the incontinence of the prieſts.

The motive to jealouſy appears to be this. There is a natural inſtinct in man to his own preſervation, which ſtrongly attaches him to every thing he thinks a property. If therefore, an amiable woman appears to him neceſſary to his happineſs, he will doubtleſs deſire the excluſive poſſeſſion of her. In the wild and early ſtate of things, when men laboured under the inconveniences of a precarious ſubſiſtence, there were many things, which are now become property, which were held in common. The paſſion of love bore the form only of a phyſical neceſſity, and was ſatisfied in as groſs a manner as hunger and thirſt. In proportion as ſocieties were better eſtabliſhed, manners grew mild, and a ſpirit of property introduced itſelf. It then became as natural to deſire the ſole property in an agreeable woman, as in a convenient houſe, or a fertile plantation. The deſire of ſatisfying mere phyſical wants, no longer engroſſed all the faculties of the mind; but men becoming ſenſible of the comforts of ſociety, began to cheriſh the ſocial virtues; and the animal inſtinct, uniting with the ſentiments of friendſhip, made the paſſion of love between the ſexes put on a more decent appearance. To this decency, was of courſe annexed a ſpirit of jealouſy, which in that caſe was natural.

When cuſtom alſo, made the conqueſt of engaging the affections of a fine woman honorable, to ſuch as have merit to effect it, our natural deſire to be preferred to others, naturally induced us to ſeek ſo flattering a diſtinction; and we become jealous through vanity. But, as the abuſe of theſe two ſpecies of inſtinct, regarding property and preference, generates avarice and ambition, ſo it occaſions different degrees of jealouſy in different minds. Nay there are ſome people of ſo miſerable a diſpoſition, that, without either right or pretenſion, they are jealous of all mankind, through envy at ſeeing any creature happy. Thus, ſelf-love, vanity, and envy, form the compound of jealouſy; a paſſion to which all mankind are more or leſs ſubject.

ARTICLE XXII.

Mathematical Questions and Solutions.

Question 154, by *Mr. Hewitt, Spitalfields.*

SUPPOSE the national debt 200 millions sterling; what annuity will be sufficient to discharge the same in 20 years, at 5*l.* per cent. per annum, compound interest?

Solution, by *Mr. John Ryley.*

Put $a = 200000000$, $r = 1.05$ the amount of 1*l.* for a year; $t = 20$ years, and $x =$ the required annuity; then from the nature of compound interest, we have $x = \frac{ar - a}{r^t - 1}$, or in numbers $\frac{1.05 \times 200000000 - 200000000}{\overline{1.05}|^{20} - 1} = \frac{10000000}{1.6532977051444201339 45 \&c.}$ $= 6048517l.\ 8s.\ 9d.$ the annuity required.

Question 155, by *Mr. George Dixon.*

A meadow in form of an oval does lie,
Just ninety-nine acres, which nine men did buy;
The numbers* in th' margin the ratio express
The greater diameter bears to the less;
The acres to which the first person lays claim,
I shall not, ye artists, now unto you name:
The second man's acres, by two do exceed
The number of acres the first hath indeed;
The third has two acres just more than the second,
And two acres more for the fourth must be reckon'd;
So their shares in progression arithmetic run,
Until the elliptical meadow is done.

* 7 : 5

The shares of the buyers, by parallel lines,
To the lesser diameter must be assign'd;
At the transverse's end the first acres commence,
Then the second man's acres, and so they go thence:
The parallel distances now I would know,
How far first and last are from each end also:
Now, gents, by arithmetic let it be done,
And I'll treat when we meet at the sign of the sun.

Solution, by *Mr. John Ryley.*

First—For each man's share. There is given the common difference = 2, the number of terms = 9, and the sum of the series = 99 of an arithmetical progression, to find the first term, which by the known rule for that purpose is $= \frac{2 \times 99 + 2 \times 9 - 2 \times 9^2}{2 \times 9} = 3$. Therefore 3, 5, 7, 9, 11, 13, 15, 17, and 19, are each man's acres respectively. *Secondly*—For the axes. If we suppose the dimensions to be taken (Fig. 1) in chains, $\frac{990}{.7854} = 1260.5$ = the number of circular inches in the whole field, and $\sqrt{5 \times 7} : 7 :: \sqrt{1260.5} : 42$ = the transverse axe $:: 5 : \sqrt{1260.5} : 30$ = the conjugate axe, nearly.—*Thirdly*—For the abscisses.

	Tabular seg. area.	Corresponding versed sine.	
$\frac{30}{42 \times 30} = \frac{1}{42}$	= .02380,	.0692 × 42 =	2.9106 = Aq; 39.0894 = Bq†
$\frac{80}{42 \times 30} = \frac{4}{63}$	= .06349,	.1352 × 42 =	5.6784 = Ar; 36.3216 = Br
$\frac{150}{42 \times 30} = \frac{5}{42}$	= .11904,	.2089 × 42 =	8.7738 = As; 33.2262 = Bs
$\frac{240}{42 \times 30} = \frac{4}{21}$	= .19047,	.2916 × 42 =	12.2472 = At; 29.7528 = Bt
$\frac{350}{42 \times 30} = \frac{5}{18}$	= .27777,	.3841 × 42 =	16.1322 = Au; 25.8678 = Bu
$\frac{480}{42 \times 30} = \frac{8}{21}$	= .38095,	.4883 × 42 =	20.5086 = Av; 21.4914 = Bv
$\frac{360}{42 \times 30} = \frac{2}{7}$	= .28571,	.3922 × 42 =	25 5276 = Aw; 16.4724 = Bw
$\frac{190}{42 \times 30} = \frac{19}{126}$	= .15080,	.2468 × 42 =	31.6344 = Ax; 10.3656 = Bx

Fourthly—For the parallel distances*.

Aq	=	2.9106
qr	=	2.7678
rs	=	3.0954
st	=	3.4734
tu	=	3.8850
uv	=	4.3764
vw	=	5.0190
wx	=	6.1068
Bx	=	10.3656
Sum		42.0000 whole axe

(Aq to Bx: Chains)

† Remark, one half of the abscisses are found by subtracting that first found from the axe.

* The parallel distances are found by subtracting the the first abscissa from the second, the second from the third, &c.

Fifthly—For the ordinates.

$42 : 30 :: 2\sqrt{2.9106 \times 39.0894} : 15.2377 = ab$
$42 : 30 :: 2\sqrt{5.6784 \times 36.3216} : 20.5162 = cd$
$42 : 30 :: 2\sqrt{8.7738 \times 33.2262} : 24.3914 = ef$
$42 : 30 :: 2\sqrt{12.2472 \times 29.7528} : 27.1873 = gh$
$42 : 30 :: 2\sqrt{16.1322 \times 25.8678} : 29.1829 = ij$
$42 : 30 :: 2\sqrt{20.5086 \times 21.4914} : 29.9918 = kl$
$42 : 30 :: 2\sqrt{25.5276 \times 16.4724} : 29.2944 = mn$
$42 : 30 :: 2\sqrt{31.6344 \times 10.3656} : 25.8689 = op.$

Question 156, by *Mr. Porter, Suffolk-Street, Birmingham.*

Given the diameter of a spinning wheel 68 inches, diameter of the spoal 12 inches, and the distance of their centers 75 inches; to determine the length of the wheel-band?

Solution, by *Mr. John Ryley.*

First, (Fig. 2) CD — cd = 34 — 6 = 28 = CE. And by Euc. 47. 1. $\sqrt{\overline{Cc}|^2 - CE^2}$ = Ec = Dd = $\sqrt{75^2 - 28^2} = 69.5773$, this doubled gives D$d$ + Ff = 139·1546. Now, Cc : (rad.) 1 :: CE : s. ∠ CcE = BCD = bcd = 21° 55′, and the double of this added to the greater semi-circle, and subtracted from the less gives $223^\circ\frac{5}{6}$ and $136\frac{1}{6}$ the number of degrees in each circle occupied by the wheel-band. Then, as 360° : 75 × 3.1416 :: $223^\circ\frac{5}{6}$: 146.4989 inches = FHD; also, as 360° : 12 × 3.1416 :: $136^\circ\frac{1}{6}$: 14.2593 inches = arc fd. Now, if the sum of these be collected, the result is 299.9128; or 300 inches nearly, the length of the wheel-band.

Question 157, by *Mr. T. Keith, of London.*

At a village in Yorkshire, on the brink of the Swale,
Where cowslips and violets their sweet odours exhale;
Dwells a maid of real beauty, and with wisdom refin'd—
She's the pride of her sex, and the hope of my mind.
From th' equations* below, learned Gents pray unfold
The name of this fair one, whom I prize above gold.

$$
\left.\begin{aligned}
{*}\ \frac{w+x}{2} + yz &= 79\\
\frac{x+y}{3} + zw &= 90\\
\frac{w+z}{5} + xy &= 22\\
\frac{y+z}{7} + wx &= 24
\end{aligned}\right\}
$$

When w, x, y, and z shew the letters in the alphabet composing her name.

Solution, by *Mr. John Ryley.*

By putting a, b, c, and d, for the respective numbers in the given equations, and multiplying we get $w+x+2yz=2a$, $x+y+3zw=3b$, $w+z+5xy=5c$, and $y+z+7wx=7d$. Hence $w=2a-x-2yz=5c-z-5xy=\frac{3b-x-y}{3z}=\frac{7a-y-z}{7x}$; and by expunging y, we have $y=\frac{2a-5c-x+z}{2z-5x}=\frac{15cz-3z^2-3b+x}{15xy-1}$ and $y=\frac{6az-3xz-3b+x}{6z^2-1}=\frac{21dz-21bx-3z^2+7x^2}{3z-7x}$. Now, by expanding the first of these equations we get $30cz^2+6z^3-6bz+2xz-75cxz+15z^2x+15bx-5x^2=30axz-75cxz-15x^2z+15xz^2-2a+5c+x-z$; or $\overline{15z-5}\times x^2+\overline{2z+15b-30az-1}\times x=5c-2a-z+6z^3-30cz^2+6bz$; hence $x^2+\frac{2z+15b-30az-1}{15z-5}x=\frac{5c-2a-z+6z^3-30cz^2+6bz}{15z-5}$, and by compleating the square, &c. $x=\frac{30az-2z-15b+1}{30z-10}-$ .... $\sqrt{\frac{5c-2a-z+6z^3-30cz^2+6bz}{15z-5}+\frac{4z^2+60bz+225b^2-120az^2-900abz-4z-30b+900a^2z^2+60az+1}{30z-10}}$;

or, in numbers $x=\frac{2368z-1349}{30z-10}-\frac{\sqrt{360z^4-39720z^3+5652964z^2-6402524z+1820761}}{30z-10}=\frac{2368z-1394-\sqrt{x}}{30z-10}$ by substitution. Again, if the second of the above equations be expanded, we shall have $126dz^3-126bxz^2-18z^4+42z^2x^2-21dz+21bx+3z^2-7x^2=18az^2-9xz^2-9bz+3xz-42axz+21zx^2+21bx-7x^2$; or, by division, $\overline{14z-7}\times x^2+\overline{3z-42bz+14a-1}\times x=6az+6z^3-3b-42dz^2+7d-z$, and from hence $x^2+\frac{3z-42bz+14a-1}{14z-7}x=\frac{6az+6z^3-3b-42dz^2+7d-z}{14z-7}$, and by compleating the square, &c. we get $x=\frac{42bz-3z-14a+1}{28z-14}-$ .... $\sqrt{\frac{6az-3b-42dz^2+6z^3-7d-z}{14z-7}+\frac{9z^2-252bz^2-1764b^2z^2+84az-6z-1176abz-196a^2+84bz-28a+1}{28z-14}}$;

or in num^s $x = \frac{3777z - 1105}{28z - 14} - \frac{\sqrt{336z^4 - 56616z^3 + 14320441z^2 - 8366126z + 1223881}}{28z - 14}$ $= \frac{3777z - 1105 - \sqrt{m}}{28z - 14} = \frac{2368z - 1349 - \sqrt{n}}{30z - 10}$; which by multiplication becomes $47006z^2 + 4z - 7836 = \overline{30z - 10} \times \sqrt{m} - \overline{28z - 14} \times \sqrt{n}$; or squared $2209564036z^4 + 376048z^3 - 736678016z^2 - 62688z + 61402896 = \overline{900z^2 - 600z + 100} \times m + \overline{784z^2 - 784z + 196} \times n - \overline{1680z^2 - 1400z + 280} \times \sqrt{m} \times \sqrt{n}$. Now by actually multiplying the values of m and n into their respective co-efficients, and subtracting the first side of the equation from the sum of their products, dividing the difference by $1680z^2 - 1400z + 280$, we shall obtain $348z^4 - 48864z^3 + 8992538z^2 - 7728493z + 1492337 = \sqrt{m} \times \sqrt{n}$, which equation being squared, and the values of m and n actually multiplied together, &c. we get $z^8 - 1956z^7 - 4562814\frac{1}{8}z^6 + 68176354z^5 - 185480011z^4 - 78500223\frac{1}{3}z^3 + 86251408z^2 + 1118959 2z = 9201888$, which being resolved we get $z = 4$; also from the above $w = 21$, $x = 1$, and $y = 17$; therefore WARD is the name of this beautiful and wise lady.

Question 158, by *Mr. Thomas Whiting.*

A person fired a cannon from the top of a tower 100 feet high, with the initial velocity of 120 feet in a second, and the ball was observed to fall at the bottom of a conical building at the same instant the sound reached the top. Required the height of the building, and elevation of the piece, the horizontal range and solidity of the building being the greatest possible.

Solution, by *Mr. John Ryley.*

Let AC (Fig. 3) represent the height of the tower, CIB the path of the projectile, and BFG the conical building; then, because the initial velocity, or the number of feet uniformly described in a second, with the velocity of projection, is given, the space CS, described from rest to acquire this velocity, is known, being $= \frac{\overline{120}|^2}{4 \cdot 16\frac{1}{12}} = 223\frac{5}{6}$: Now by Prob. III, Simpson's Select Exercises, if CV be the direction of projection, B the point where the ball impinges on the horizon, and BV drawn perpendicular thereto; CB joined, and VE perpendicular to CB; then CE = VD = twice the impetus $447\frac{2}{3}$; ∴ VB is given $= 547\frac{2}{3}$, which when the range is a maximum, is = CB. Hence, by Eu. 47. 1. CD=AB is found = 538.46, and per Trig. the angle BCD is found = 10° 32′; consequently as the angle SCB is bisected by the direction of projection CV, we get the ∠ of elevation DCV = 39° 44′. Now the time of flight is equal to the time of descent through VB $= \sqrt{\frac{547\frac{2}{3}}{16\frac{1}{12}}} = 5.83$ seconds; therefore CG $= 1142 \times 5.83 = 6658$ feet = the distance that sound would fly during the time of the ball's flight.

Now, put CG $= 6658 = a$; CD $=$ AB $= 538.46 = b$; HR $=$ BD $= 100 = c$; and CR $= x$; then by Eu. 47. 1. GR $= \sqrt{a^2 - x^2}$, HC $= \sqrt{a^2 - x^2} + c$, and DR $=$ BH $= x - b$; therefore $\overline{x-b}|^2 \times \sqrt{a^2 - x^2} + c \times \overline{x-b}|^2$ by the question is a maximum, and its fluxion $= 2\dot{x} \times \overline{x-b} \times \sqrt{a^2 - x^2} + \frac{-x\dot{x}}{\sqrt{a^2 - x^2}} \times \overline{x-b}|^2 + 2c\dot{x} \times \overline{x-b} = 0$, hence by reduction, $3x^2 - 2a^2 - bx = 2c\sqrt{a^2 - x^2}$, and by involution and transposition,

$$\left.\begin{array}{l} 9x^4 - 6bx^3 - 12a^2x^2 + 4a^2ba + a^4 \\ \qquad\quad + b^2x^2 \qquad\qquad - 4c^2a^2 \\ \qquad\quad + 4c^2x^2 \end{array}\right\} = 0;$$

which being put into numbers and resolved, we find $x = 5549$ very nearly; hence DR $=$ BH $= 5010.54$; BF the diameter of the cone's base $= 10021.08$; GH its perpendicular altitude $= 3778.8$, and its content $= 993605549577$ feet.

Question 159, by *Ferdinando.*

From one of the angular points C, of a given parallelogram ABCD, to draw geometrically, a right line CEF, cutting DA in E, and BA produced in F; so that the sum or difference of the triangles CDE, EAF may be of a given magnitude.

Solution, by *Mr. John Ryley.*

Construction. Make (Fig. 4) the parallelogram; GCDH $=$ twice the given sum of the triangles; draw BI so, that the angle IBC may be half a right angle, from the point of intersection K of IB, and a semi-circle described upon CG, let fall the perpendicular KL, draw LE parallel to AB, and E is the point through which CF must be drawn.

Demonstration. Since the parallelogram CGHD is $=$ twice the triangle CDE (LEDC) $+$ twice the triangle AEF by construction, it follows that the parallelogram GLEH is $=$ to twice the triangle AEF, and consequently AF $\times$ AE $=$ GL $\times$ GH (AB); or AF : AB :: GL : BL :: (per similar triangles) BL : LC; therefore GL $\times$ LC $=$ BL^2 $=$ (by the circle) $=$ LK^2; hence BL $=$ LK and the angle IBC $= \frac{1}{2}$ a right one, as per construction. The construction will not be materially different when the difference of the triangles is given, for in that case BN : BC :: BL : LC, hence the point E will be determined.

Question 160, (PRIZE) by *Cassia Broomwoot.*

In the midst of a gentleman's garden that's square†,
Is a circular fountain* of water that's clear;
The gardener has orders a shrubbery to make.
From the pond to two walks‡, and the area to take ¶

† Side 200 yards.
* Diam. 81 yards.
‡ The walks are pa-

But it puzzles him quite—for in curvature space
Not being acquainted—so he begs for a place
In your fam'd *Delights*—and whoever unites
This knot, with your leave, will be sure of the prize.

rallel to the garden-walls & distant therefrom 47.24771 yds.

¶ If right lines be drawn through A, the extremity of the diameter of the pond that is parallel to the walks, and from the points M, M, &c. where these lines cut the circum. tangents, be drawn to cut the diameter produced in T, T, &c. then if from T, ⊥s be demitted upon the first-mentioned lines cutting them at right-angles in Q, Q, &c. these points Q, Q, &c. shall be in the fence of the shrubbery. He desires your ingenious correspondents to describe the fence, and give the area.

Solution, by *Mr. John Ryly.*

Let BCDF (Fig. 5) be the garden, GH, IK the walks, AMB the circular fountain, and LBN the fence of the shrubbery. Put AB $= a$, BC $= x$, and CQ $= y$; then by Euc. 47. 1. AQ $= \sqrt{\overline{a+x}|^2 + y^2}$, and by similar triangles AC : CQ :: CQ : CT $= \frac{y^2}{a+x}$; therefore AT $= \frac{\overline{a+x}|^2 + y^2}{a+x}$, and BT $= \frac{\overline{a+x}.x + y^2}{a+x}$. Again, by the property of the circle TA.TB $=$ TM$^2 = \frac{y^2 + \overline{a+x}|^2}{a+x} \times \frac{y^2 + \overline{a+x}.x}{a+x}$; also, AQ : AC :: AB : AM $= \frac{\overline{a+x}.a}{\sqrt{\overline{a+x}|^2 + y^2}}$; hence AQ $-$ AM $=$ MQ $= \frac{ax + x^2 + y^2}{\sqrt{\overline{a+x}|^2 + y^2}}$. Moreover, by Euc. 47. 1. $\frac{y^2 + \overline{a+x}|^2}{a+x} \times \frac{y^2 + \overline{a+x}.x}{a+x} = \frac{\overline{\overline{a+x}.x + y^2}|^2}{\overline{a+x}|^2 + y^2} + \frac{y^4}{\overline{a+x}|^2} + y^2$; which equation being properly reduced, we obtain $y^4 = \overline{x+a}|^3.x$, the equation of the curve, which shows it to be a line of the fourth order. Now as the curvelinear fence LN is intercepted by the walks GH, IK, it is evident that the greatest value of y will be when $y =$ CL $= 52.75229$; in which case we have $x = 10.2067$.

Quadrature. From the equation of the curve we have $\dot{x}y = \overline{a+x}|^{\frac{3}{4}}.x^{\frac{1}{4}}\dot{x}$; but it does not appear that this fluxion belongs to any of those forms which admit of a perfect fluent; therefore, we must have recourse to infinite series, consequently $\dot{x}y = a^{\frac{3}{4}}x^{\frac{1}{4}}\dot{x} + \frac{3.x^{\frac{5}{4}}\dot{x}}{4.a^{\frac{1}{4}}} - \frac{3.x^{\frac{9}{4}}\dot{x}}{4.8.a^{\frac{5}{4}}} + \frac{3.5.x^{\frac{13}{4}}\dot{x}}{4.8.12.a^{\frac{9}{4}}} - \frac{3.5.9.x^{\frac{14}{4}}\dot{x}}{4.8.12.16.a^{\frac{13}{4}}} + \frac{3.5.9.13.x^{\frac{21}{4}}\dot{x}}{4.8.12.16.20.a^{\frac{17}{4}}}$, &c. and by taking the fluent of each term, &c. we get $xy = \frac{x^{\frac{1}{2}}}{a^{\frac{1}{2}}} \times : \frac{4ax}{5} + \frac{x^2}{3} - \frac{3x^3}{104a} + \frac{5x^4}{544a^2} - \frac{15x^5}{3584a^3} + \frac{117x^6}{51200a^4}$, &c. Now as this series converges very swiftly, the area may be readily approximated to any assigned degree of exactness; the two first terms only give the area within less than $\frac{3}{10}$ths of an unit, and the sum of the first six terms is 414.531, &c. the double of which is 829.062 = the area LBNL. But it does not appear by the question, whether the shrubbery is to occupy the space LBNL only, or that, together with the parallelogram LHKN; if this last-mentioned place is to be included in the shrubbery, the area of the whole space LBNKH is = 6029.7309 square yards; and if a similar shrubbery be made on the opposite side of the fountain, the contents of both will be 12059.4618 square yards. But if the space LRAMBNSA be the shrubbery, we shall have the area GHKI — the sum of the areas of the fountain, and twice the space LBNKH = 3888.4448 for the area required.

Question 161, by *Mr. Richard Elliot.*

Suppose the length, breadth, and depth of a cistern to be 16 (x), 12 (y), and 20 (z), feet respectively, and that there are 2 circular holes (each 1 inch in diameter) one placed in the bottom, the other in the side close to the bottom. Now, if it was filled with water, and both holes open, in what time will the whole be exhausted, supposing the velocity equal that generated by gravity through the whole height above the apertures?

Solution, by *Mr. John Ryley.*

The required time of exhausting the cistern may be found nearly, by supposing the area of the aperture in the bottom of the cistern, to be equal to both; as that in the side is placed close to the bottom, it is evident that the time of exhaustion will be nearly the same as if both holes had been at the bottom. These things being premised, put $m = 16\frac{1}{12}$, the force of gravity, $n = 1.5708$ the area of both apertures; v any indefinite altitude of the surface of the fluid above the bottom of the vessel, and $a = 27648$ inches the area of the descending surface. Then by the laws of descending bodies, and the nature of the question $\sqrt{m} : \sqrt{v} :: 2m : 2\sqrt{mv} =$ the velocity of the issuing fluid; but the velocity at the aperture, and that of the descending surface, will be inversely as their areas; therefore $a : n :: 2\sqrt{mv} : \frac{2n}{a}\sqrt{mv} =$ the velocity per second of the descending surface. But in descending the space $\dot{v}$, the velocity may be considered as uniform; consequently $\frac{2n}{a}\sqrt{mv} : \dot{v} :: 1'' : \frac{a\dot{v}}{2n\sqrt{mv}} =$ the fluxion of the time, in which the cistern will be exhausted; when $v = z$, becomes $\frac{a}{n}\sqrt{\frac{z}{m}} = 19628$ seconds $=$ 5 hours, 27 minutes, and 8 seconds; the required time nearly.

But in order to determine the *exact* time in which the cistern will be exhausted; the time of emptying the cistern till the water is only one inch deep, by the orifice in the bottom, will be expressed by $\frac{a}{n} \cdot \frac{\sqrt{z} - \sqrt{1}}{\sqrt{m}}$; and the time of emptying the cistern to the aforesaid depth by the aperture in the side by $\frac{a\sqrt{z-1}}{n\sqrt{m}}$. Therefore the time of emptying the cistern until the water is only 1 inch deep by both apertures; (taking $n =$ .7854) will be $= \frac{a \cdot \overline{\sqrt{z} - \sqrt{1}} \times \sqrt{z-1}}{n\sqrt{m} \cdot \overline{\sqrt{z} - \sqrt{1} + \sqrt{z-1}}} = 18954$.

Now, to find the time in which the cistern will be exhausted, when the water is only 1 inch deep; it is proved by writers on hydrostatics, that only $\frac{2}{3}$ds of the quantity of water, will run out at the aperture in the side, that runs out at an equal orifice in the base; therefore the time of exhaustion in this case will be expressed by $\frac{3a}{5n\sqrt{m}} = 1520$; which being added to that above-found, the sum is $20474'' = 5$ hours. $41'$ $14''$, the whole time of exhaustion; which exceeds that first found, by $14'$ $6''$ only.

Question 162, by *Mr. John Brookes.*

Given the difference of the sides, the difference of the segments of the base made by the perpendicular, and the radius of the inscribed circle to construct the triangle.

Solution, by *Mr. John Ryley.*

Construction. Take DN (Fig. 6) = the given difference of the segments of the base; bisect DN in E, and take EF so, that EF × ED = D^2 (D being half the difference of the sides) at D erect an indefinite perpendicular, in which take DH = the given radius of the inscribed circle, and HC so, that HC × EF = D × HD; join CF, and draw HO parallel to DN, about O as a center and radius = DH, describe a circle, and draw CA, CB to touch it, and meet DN produced in A and B, and ABC will be the triangle required.

Demonstration. It is well known that ED : D :: D : EF; also that $\overline{AB + AC + BC} \times HD = AB \times \overline{DH + HC}$; hence $\overline{AC + BC} \times HD = AB \times HC$; therefore, AC + BC : AB :: HC : HD. Likewise by a known property of triangles, AC + CB : AB :: AC — CB (2D) : 2EF; therefore AC + BC : AB :: 2D : 2EF :: HC : HD :: D : EF; the rest is evident from the Construction.

The same, by *Mr. John Dawes, Surgeon.*

Upon any indefinite right line, as AB (Fig. 7) lay off ED = half the difference of the segments of the base, $Eg = \frac{1}{2}$ the difference of the sides, & $Eb = \frac{\overline{Eg}|^2}{ED}$: erect the given perpendicular *fg*; draw *bf* until it meets the perpendicular DC in C, from which, the circle *nbg* being described, draw the tangents CB and CA, so is ABC the Δ required.

It has been proved in this work, that C*i* (ED) : A*k* (*eg* = *fm*) :: A*k* : E*b*, therefore $Eb = \frac{\overline{Eg}|^2}{ED}$. *Q. E. D.*

Question 163, by *Mr. R. Elliott, of Liverpool.*

The fluxion of the tangent of 60° is = to twice the fluxion of the tangent of 45°.— Required the investigation by a general theorem, that will exhibit the ratio of the fluxion of any tangent to that of its corresponding arc.

Solution, by *Mr. John Ryley.*

Let ABC (Fig. 8) be a quadrant of a circle, BT a tangent at B; join AT, and draw A*t* infinitely near AT, and T*m* perpendicular thereto. Put AB = *r*, BT = *t*; T*t* = $\dot{t}$ BR = *x*, and R*n* = $\dot{x}$; then by Euc. 47. 1. AT = $\sqrt{r^2+t^2}$. Now, as the triangles ABT, T*mt*; AR*n*, AT*m* are similar, AT : AB :: T*t* : *m*T = $\frac{r\dot{t}}{\sqrt{r^2+t^2}}$; also, AT : T*m* :: AR : R*n* = $\dot{x} = \frac{r^2\dot{t}}{r^2+t^2}$; or, $\dot{t} = \frac{r^2+t^2}{r^2}.\dot{x}$, which is a general expression for the fluxion of any tangent and its corresponding area; but if *r* = 1, when the arc contains 45°, then *t* = 1; and when 60°, $t = \sqrt{3}$; therefore in the first case $\dot{t} = \frac{1^2+1^2}{1^4}.\dot{x} = 2\dot{x}$ and in the latter $\dot{t} = \frac{1^2+3}{1^2}.\dot{x} = 4\dot{x}$. *Q. E. D.*

Question 164, by *Mr. John Knowles.*

Given the height of the eye, its distance from the picture, and the position of an original point, to find its perspective representation geometrically, without introducing the point of sight, or station point.

Solution, by *Mr. John Ryley.*

Let P (Fig. 9) be the original objective point; ABCD the picture, and E the eye. Draw PF, PG in any manner at pleasure, to meet the bottom of the picture if F and G;

also from the eye at E draw EH parallel to FP, and from the same point E, draw EK parallel to PG. Lastly, FH and GK to intersect each other at P, and P will be the perspective representation of the original point P, as was required.

Question 165, by *Mr. John Brookes, of Leeds.*

Let BZ be an indefinite perpendicular to a given line AB, to which from A, draw any line APC, and take the point P such, that AC multiplied by PC may be $=$ to AB^2.—Required the properties of the curve, which is the locus of P.

☞ *This question has been proposed before, but not publicly answered, that I know of.*

Solution, by *Mr. John Ryley.*

Put (Fig. 10) $AB = a$, $AD = x$, and $DP = y$; then by Euc. 47. 1. $AD = \sqrt{x^2+y^2}$, and by similar triangles, $AD : AP :: DB : PC = \frac{a-x}{x}\sqrt{x^2+y^2}$; also, $AD : DP :: AB : BC = \frac{ay}{x}$; therefore, $AC = \frac{a}{x}\sqrt{x^2+y^2}$, and by the question $\frac{a-x}{a}\sqrt{x^2+y^2} \times \frac{a}{x}\sqrt{x^2+y^2} = a^2$, which by reduction becomes $x^3 = \sqrt{a-x} \cdot y^2$, the equation of the curve. Hence it appears that the curve is the Cissoid of Diocles, and BR its asymptote, whose properties are well known.

Question 166, by *Mr. James Ashton, of Harrington, near Liverpool.*

Given the ratio of the base to one of the sides of an isosceles $\triangle$, as 1 to r, and the area of its greatest inscribed ellipsis $= a$. It is required to find the dimensions of both, and give a demonstration of the process.

Solution, by *Mr. John Ryley.*

In order to *demonstrate the process*, and make the solution to this question as intelligible as possible, it seems necessary to shew under what circumstances an ellipse inscribed in such a triangle as is described in the Question will be a maximum. Now, in order to shew when the ellipse is a maximum, Put GD (Fig. 11) $= x$; then as $AB : AC :: 1 : r$; CD will be as $\sqrt{r - \frac{1}{4}}$, and $CG = \sqrt{r^2 - \frac{1}{4}} - x$, and by similar triangles $CD : DB :: CG : GE = \frac{\sqrt{r^2-\frac{1}{4}}-x}{2\sqrt{r^2-\frac{1}{4}}}$; but by the property of the ellipse $GE \times DB =$ the square of the semi-conjugate $= \frac{\sqrt{r^2-\frac{1}{4}}-x}{4\sqrt{r^2-\frac{1}{4}}}$; therefore $2\sqrt{\frac{\sqrt{r^2-\frac{1}{4}}-x}{4\sqrt{r^2-\frac{1}{4}}}} \times x$

is a maximum; which being put into fluxions and reduced gives $x = \frac{2}{3}\sqrt{r^2 - \frac{1}{4}}$; hence it appears that the transverse axe of the ellipse is $\frac{2}{3}$ of the perpendicular of the triangle.

Now put $c = .7854$, and BD $= x$, then BC $= 2rx$, CD $= \sqrt{4r^2x^2 - x^2} = mx$, by substitution; therefore CG $= \frac{1}{3}mx$, GD $= \frac{2}{3}mx$, GE $= \frac{x}{3}$, and the conjugate axe $= \frac{2x}{\sqrt{3}}$, all which is evident from what has been done above.

Now by the question, $\frac{2cx}{\sqrt{3}} \times \frac{2}{3}mx = a$; hence $x = \sqrt{\frac{3a\sqrt{3}}{4cm}} = \sqrt{\frac{3a\sqrt{3}}{c\sqrt{4r^2-1}}}$

AB $= \sqrt{\frac{3a\sqrt{3}}{c\sqrt{4r^2-1}}}$; CD $= \sqrt{\frac{3a\sqrt{3} \times \sqrt{4r^2-1}}{4c}}$; AC $=$ BC $= 2r\sqrt{\frac{3a\sqrt{3}}{4c\sqrt{4r^2-1}}}$

the transverse axe GD $= \sqrt{\frac{a\sqrt{3} \times \sqrt{4r^2-1}}{3c}}$, and the conjugate $= \sqrt{\frac{a\sqrt{3}}{c\sqrt{4r^2-1}}}$

as required.

Question 167, by *Mr. John Ashton, of Harrington, near Liverpool.*

If the wall of a house be 30 feet high, and a spout be fixed at the top thereof, of $2\frac{1}{2}$ feet in length from the wall; it is required to find the angle it must make with the plane of the wall, so that the water may fall into a reservoir, on an horizontal plane, at 10 feet distance from the bottom of the wall.

Solution, by *Mr. John Ryley.*

It has been proved by writers on hydraulics, that spouting water describes the curve of a parabola; therefore if we suppose BC (Fig. 12) to be the wall, or axis of the parabola, and EC the spout; we have given BC $= 30 = a$, the ordinate AB $= 10 = b$, $c = 2\frac{1}{2} = c$, and CV $=$ VD $= x$; then by Euc. 47. 1 ED$^2 = C^2 - 4x^2$, and by the property of the curve $x : c^2 - 4x^2 :: a - x : b^2$, and from hence $4x^3 - 4ax^2 - \overline{b^2 + c^2}.x = \frac{ac^2}{4}$; or in numbers, $x^3 - 30x^2 - \frac{425}{16}x = -\frac{375}{8}$; which equation being resolved, we get $x = .89227106$, and $2x = 1.78454212$. Hence per Trig. the angle ECD is found $= 44^\circ\ 27'\ 13.3''$ as required.

The same, by *Mr. John Dawes, Surgeon.*

Construction. Draw AC $= 30$ (Fig. 13) the height of the wall; also BC $= 5$* the perpendicular; set one foot of the compasses in the line AC, so that a semi-circle may be drawn through the points B and A, which will give *dc* the perpendicular height of

* *See Ferguson's Hydraulics.*

the fluid; lay off *cg* (given) to meet *dg* the perpendicular C*ab* : $\overline{BC}^2 \div Ac = dc = \frac{5}{6}$.
Otherwise, put $x = dc$, $b = BC$, $a = AC$, now B*k* (*kd*) $= \frac{a+x}{2}$ and *k*C $= \sqrt{\overline{\frac{a+x}{2}}|^2 - b^2}$ also *k*C $= \frac{a-x}{2}$, therefore $\sqrt{\overline{\frac{a+x}{2}}|^2 - b^2} = \frac{a-x}{2}$; now this equation reduced gives $x = \frac{b^2}{a} = \frac{5}{6}$ as before; then in the orthogonal triangle *cdg* are given *cg* and *cd*, to find *dcg*, which is 70° 31′ 46″.

Question 168, by *Mr. O. G. Gregory, Yaxley, in Huntingdonshire.*

If we admit that a musical chord, in length 20 inches, and weight 4.69097 grains, when stretched with a weight 8lbs. avoirdupois, will sound the note C-sol-fa-ut; required the weight of chords of the same length and tension, which shall sound the ditone, diapente, and diapason to the abovementioned note.

Solution, by *Mr. John Ryley.*

When the tension and length of the string are constant, the diameters will be as the times of vibration, and consequently the weights will be as the squares of the times; therefore, as the greater third, the fifth, and the octave are $\frac{4}{5}$, $\frac{2}{3}$, and half of the monachord; if 4.69097grs. the given weight of the string, be multiplied into the square of $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{1}{2}$, respectively, the result is, for E-la-mi, or ditone 3.00222grs. for G-sol-re-ut, or diapente 2.08487 grs. and for C-sol-fa, or diapason, 1.17274 grs. as required.

Question 169, by *Mr. John Rowbottom, West-Hallam, Derbyshire.*

Four men, A, B, C, and D, undertook a bargain of work for $26\frac{1}{2}$l. Now, A could finish it himself in 4 months, B in 6, C in 9, and D in 12 months. But B began to work a certain time after A, and C and D both began together a certain time after B; when the work was finished, A received 13l. 3s. $11\frac{21}{22}$d. more than C, and B and D received betwixt them 8l. 1s. $7\frac{7}{22}$d. How long did A work before B began, and B before C and D began; what did each person receive for his work; and how long was it in finishing?

Solution, by *Mr. John Ryley.*

As the whole work cost $26\frac{1}{2}$l. and B and C received betwixt them 8l. 1s. $7\frac{7}{22}$d. if this sum be taken from $26\frac{1}{2}$l. the remainder is 18l. 8s. $4\frac{15}{22}$d. = what A and C received; therefore as A received 13l. 3s. $11\frac{21}{22}$ more than C, $\frac{18l.\ 8s.\ 4\frac{15}{22}d. - 13l.\ 3s.\ 11\frac{21}{22}d.}{2}$ = 2l. 12s. $2\frac{8}{22}$d. = what C received, and 2l. 12s. $2\frac{8}{22}$d. + 13l. 3s. $11\frac{21}{22}$d. = 15l. 16s. $2\frac{7}{22}$d. = what A received. Now, as $26\frac{1}{2}$l. : 4 months, :: 2l. 12s. $2\frac{8}{22}$: $\frac{39}{44}$ parts of a month, the time C and D wrought; for by the question they both began together.

Moreover 12 months : $26\frac{1}{2}$l. :: $\frac{39}{44}$ months : 1l. 19s. $1\frac{17}{22}$d. = what D received; therefore 8l. 1s. $7\frac{7}{22}$d. — 1l. 19s. $1\frac{17}{22}$d. = 6l. 2s. $5\frac{12}{22}$d. = what B received; consequently $26\frac{1}{2}$l. : 6 months :: 6l. 2s. $5\frac{12}{22}$d. : $1\frac{17}{44}$ months, the time B wrought; therefore $2\frac{17}{44}$ — $1\frac{17}{44}$ = 1 month, the time A wrought before B began; and $1\frac{17}{44} - \frac{39}{44} = \frac{1}{2}$ a month, the time B wrought before C and D began. Now, as A wrought the whole time that the work was going forward, the time that the work was in *finishing* was $2\frac{17}{44}$ months.

Question 170, by *Mr. Thomas Simpson Evans, Odiham, Hants.*

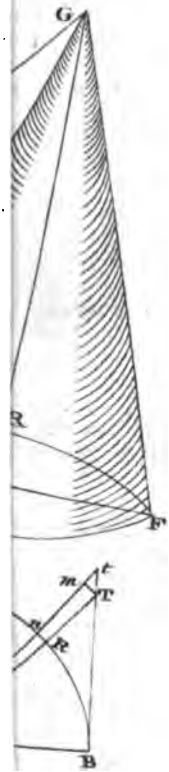
Two men, A and B, agree for 2s. to carry 2cwt. 2qr. 12lb. of wheat, 3 miles, on a pole 6 feet long. At their first setting out, the weight was 3 feet 4 inches from A; in which state they carried it 6 furlongs, where resting, they changed places, the weight continuing in the same place, and carried it $1\frac{1}{4}$ mile further, where resting again, the weight was by accident moved to 30 inches from B, in which situation it was carried the remainder of the way. How much of the money must each man receive in proportion to his trouble.

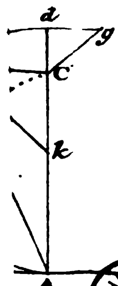
Solution, by *Mr. John Ryley.*

As the men carried the wheat 3 miles, or 24 furlongs, for 2s. or 24d. their hire is 1d. per furlong. Now, as they carried the wheat 6 furlongs, and then changed places, it is evident that they would be upon a par at the end of 12 furlongs, let the situation of the weight on the pole be what it would, and each person would have earned 6d. but they carried it four furlongs further before they rested the second time, when the weight was placed 32 inches from B, or 40 from A. Now, as the weight supported by each person is inversely as the distance, we have 72 : 4d. :: $\left\{\begin{array}{l} 32 : \frac{16}{9} = \text{A's share of 4d.} \\ 40 : \frac{20}{9} = \text{B's share of 4d.} \end{array}\right\}$. But after resting the second time, the weight was 30 inches from B, or 42 from A; therefore, 72 : 8d. :: $\left\{\begin{array}{l} 30 : \frac{10}{3} = \text{A's share of 8d.} \\ 42 : \frac{14}{3} = \text{B's share of 8d.} \end{array}\right\}$. Consequently $11\frac{1}{9}$d. is A's share of the whole money, and $12\frac{8}{9}$d. is B's share of the same.

☞ *This solution is given upon the hypothesis that the pole is without weight.*

Plate. 12.





Question 171, by *Tesvonnes*.

Given the velocity of a ball acquired by descending down the slant side of a cone, to determine the cone's dimensions, when its whole surface is a minimum?

Solution, by Mr. *James Stevenson, of Heath, near Chesterfield.*

It is well known that the cone will have the least superficies when the slant side is to the semi-diameter of the base in the ratio of 3 to 1; therefore, take $3x$ and x for them respectively; likewise, put v for the given velocity, and $m = 16\frac{1}{12}$ feet $= 193$ inches. Now the perpendicular altitude of the cone is $2x\sqrt{2}$; and the velocity of the ball, acquired by falling through this altitude, is the same (by Prop. 20, *Em. Introduction to Mechanics*) as that acquired by the ball descending down the slant side; therefore, by the laws of gravity, we have $2\sqrt{m \times 2x\sqrt{2}} = v$; hence $x = \frac{v^2}{8m\sqrt{2}}$; therefore the diameter of the base is $\frac{v^2}{4m\sqrt{2}}$, and the perpendicular altitude $\frac{v^2}{4m}$, as required.

Question 172, by *Tesvonnes*.

To divide a given right line into two parts, such, that their rectangle shall be to the square of their difference in the ratio of m to n?

Solution, by *Mr. J. Johnson, at Mr. Lowry's School.*

On the given line AB (Fig. 1) describe a semi-circle; bisect AB in F, take any distance FH, and draw HI perpendicular to AB, and make 2FH to HI :: $\sqrt{n} : \sqrt{m}$; draw FID, to meet the circle in D, demit the perpendicular DG, then AG, GB, are the lines required.

Demon. AG and GB $=$ DG² by the property of the circle, and because AB is bisected in F, FG is $=$ to half the difference of AG, GB, therefore AG $-$ GB $=$ 2FG : GD :: 2FH : HI :: $\sqrt{n} : \sqrt{m}$, therefore $\overline{AG - GB}^2 : GD^2 = AG, GB :: n : m$.

Question 173, by *Tesvonnes*.

ABC is a plane triangle, the sides AB, BC, and CA of which, are to each other as three given numbers r, s, and t, respectively. Required the said sides when the sum of their cubes is $=$ to n times the area?

Solution, by *Mr. J. Johnson.*

Let xr, xs, and xt, represent the sides of the triangle; then, because the ratio of the three sides is given, the ratio of the perpendicular is also given; let the perpendicular be px. Then $r^3x^3 + s^3x^3 + t^3x^3 = n.tpx^2$, or $\overline{r^3 + s^3 + t^3} \times x = ntp$; therefore, $x = \frac{npt}{r^3 + s^3 + t^3}$. Q. E. D.

Question 174, by *Tesvonnes.*

In a plane triangle are given the vertical angle and the sum of the including sides, to determine it, when the rectangle under the base, and the difference of the said sides is of a given magnitude?

Solution, by *Mr. J. Stevenson.*

Let m represent the given magnitude, and assume a and x for the half sum and half difference of the including sides respectively; also put v for the versed sine of the given angle, then (by Case V. pa. 124, *Em. Trig.*) the base is denoted by $\sqrt{4x^2 + 2v \times \overline{a^2 - x^2}}$ and per the question, we have $2x\sqrt{4x^2 + 2v \times \overline{a^2 - x^2}} = m$; the resolution of this equation, gives $x = \sqrt{\frac{m^2}{8.\overline{2-v}} + \frac{v^2a^4}{4.\overline{2-v}|^2} - \frac{va^2}{2.\overline{2-v}}}\Big|^{\frac{1}{2}}$; therefore, &c.

Question 175, by *Tesvonnes.*

There are three numbers in arithmetic progression, the sum of which is equal to their product, and the square of their sum is equal to the sum of their cubes. Required the numbers?

Solution, by *Mr. James Stevenson.*

If we assume y = the second number, and x = their common difference, then the required numbers will be expressed by $y - x$, y, and $y + x$. Now, by the nature of the question, we have $3y = y^3 - yx^2$, and $9y^2 = 3y^3 + 6yx^2$; from the first of these equations, we obtain $x^2 = y^2 - 3$, which value substituted in the second, gives $9y^2 = 3y^3 + 6y^3 - 18y$, by reducing this equation we get $y^2 - y = 2$, hence $y = 2$, then $x = \sqrt{y^2 - 3} = 1$; consequently, the required numbers are 1, 2, and 3.

Question 176, by *Mr. J. Collins, of Kensington.*

There are two numbers, whose sum added to their product equal 156.45, and the difference of their cubes equal 97323.022125. Query, the numbers?

Solution, by *Mr. James Stevenson.*

Put $a = 156.45$, $b = 97323.022125$; and let x denote the greater, and y the less of the required numbers; then by the question, $x+y+xy=a$, and $x^3-y^3=b$; from the first of these equations we get $x=\frac{a-y}{1+y}$, then by substitution the second equation will be $\left(\frac{a-y}{1+y}\right)^3-y^3=b$, hence $y=2.35$, consequently, $x=\frac{a-y}{1+y}=46$, as required.

Question 177, by *Mr. Collins.*

Suppose a ship from the Lizard, sailed between the South and West so far, until her departure from the meridian was in proportion to the distance sailed, as 5 : 9, and the sum of their squares equal 24077,9118985. I demand her course, distance, and the latitude and longitude she was then in, without the assistance of Algebra.

Solution, by *Mr. James Stevenson.*

As $\sqrt{9^2+5^2}$: $\sqrt{24077.911895}$ :: 9 : 135.6435, the distance sailed; and 9 : 5 :: 135.6435 : 75.3575 the departure. Now by the principles of sailing, as distance sailed, : radius :: departure : sine of the course 33° 45′, consequently she steered S. W. by S.—Again, as sine of course : departure :: co-sine of course : proper diff. of lat. 1° 53′, which subtracted from 49° 57′ (the latitude sailed from) gives 48° 4′ N. for the lat. come to.—Lastly, as co-sine course : merid. diff. lat. (172) :: sine course : 1° 55′ the difference of longitude, which added to 5° 14′ (the longitude left) gives 7° 9′ W. the longitude arrived at.

Question 178, by *Mr. James Stevenson.*

Given the vertical angle, and sum of the including sides, to determine the triangle, when the square of half the base is equal to the area.

Solution, by *Mr. R. Wood, Excise-Officer, Birmingham.*

Take AH (Fig. 2) on an indefinite right-line AI, and on it describe a segment of a circle to contain the given vertical angle; bisect AH in E, draw ED perpendicular to AH. and = AE and DL ∥ AH to meet the periphery in L; draw AL, which produce to G, and AG = the sum of the sides; make the angle AGB = half the given one; join GB to meet AI in B, make the angle GBC = angle G; join BC, then ACB is the required triangle. Draw CN perpendicular to AB the angle CGB = CBG; then CG = CB, and AC + CB = AG the angle ACB = angle G + angle CBG; by reason of parallel lines, AB : NC :: AH : ML, and as ML = half AH ∴ CN = half AB, then CN × ½ AB = ¼ AB². Q. E. D.

Note. The angle at the vertex must be equal, or less than a right one.

The same, by *Mr. J. Johnson.*

On any line AB (Fig. 3) describe a segment of a circle capable to contain the given angle; erect the perpendicular CD, and take GD $= \frac{1}{4}$ AB; draw GE parallel to AB, to cut the circle in E, join AE, BE, and the triangle AEB is similar to the required one.

Demonst. The angle AEB is equal to the given one by construction, and AD $\times$ FE $=$ area of the triangle AEB; but FE $=$ DG $=$ AD, therefore $AD^2 =$ to the area.

Question 179, by *Mr. Stevenson.*

Given the angle at the vertex, and the sum of the including sides of a plane triangle, to determine it, when the difference of the squares of the segments (made by the perpendicular falling from the said angle) of the base is equal to the area.

Solution, by *Mr. J. Johnson, of Mr. Lowry's School.*

On any line AB (Fig. 4) describe the segment of a circle capable to contain the given angle; bisect AB in D, and at any point G, draw GH perpendicular to AB, and equal to 4DG; draw DHF to cut the circle in F; join AF, BF, and AFB is a similar triangle to the required one.

Demonst. The angle AFB is equal to the given one by Construction; let fall the perpendicular FE; then by similar triangles, DG : GH :: DE : EF; but GH is $=$ 4DG, therefore EF $=$ 4DE. Moreover DE $=$ half the difference of the segments of the base, and AB $\times$ 2DE $= AE^2 - EB^2, =$ AB $\times$ EF $\div$ 2, and AB $\times$ EF $\div$ 2, equal to the area of the triangle. Q. E. D.

The same, by *Mr. R. Wood, Excise-Officer, Birmingham.*

On any indefinite right-line AH (Fig. 2) describe a segment of a circle to contain the given vertical angle, which bisect in E; draw LM to the periphery, and $\perp$ to AH, so that it may be equal to 4EM, then draw AL, which produce to G, and make AG $=$ the sum of the sides, the $\angle$ AGB half the vertical; draw GB to meet AC in B, and make the $\angle$ GBC $=$ CGB, join BC, then ACB is the Δ required. Bisect AB in O, draw CN perpendicular to AB, the $\angle$ CGB $= \angle$ CBG, then CB $=$ CG, and AC $+$ CB $=$ AG, the $\angle$ ACB $= \angle$ CGB $+$ CBG. As the Δs ALH, ACB are equiangular, AM : MH :: AN : NB :: ML : NC, and 2AH (4EH) : EH :: LM (4EM) : EM, by construction; by similar triangles, 2AB : BO :: CN : ON; then, 2AB $\times$ ON $=$ BO $\times$ CN (and by Cor. 2. 9. 2 B. *Simp. Geom.*) 2AB $\times$ ON $= AN^2 - NB^2$, then BO $\times$ CN $= AN^2 - NB^2$. Q. E. D.

PRIZE—Question 180, by *Mr. Joseph Waters, of Gravesland.*

To find three such cube numbers, that the product of any two of them being divided by the other, shall leave a cube number remaining.

Solution, by *the Proposer.*

Assume $2x$, $3x$, and $5x$ for the cube roots of the required numbers, and let x^3, a^3x^3, and b^3x^3, represent the cubical remainders, when $8x^3$, $27x^3$, and $125x^3$, are respectively made divisors; then by the question $(3375x^6 - x^3) \div 8x^3$, $(1000x^6 - a^3x^3) \div 27x^3$, $(216x^6 - b^3x^3) \div 125x^3$, or their equals $421x^3 + (7x^3 - 1) \div 8$, $37x^3 + (x^3 - a^3) \div 27$, $(216x^3 - b^3) \div 125$ are integers; but if $x \div 9$ leaves a, $x^3 \div 27$ will leave a^3, (a being within the limits of possibility, or a natural number whose cube is less than 27) for putting $(x - a) \div 9 = q$, $(x^3 - a^3) \div 27$, becomes $27q^3 + 9aq^2 + a^2q$; in a similar manner it may be demonstrated, that every value of x answering the conditions of $(15x - 1) \div 8$, and $(6x - b) \div 125$ (under the same restrictions) will also answer the conditions of $(7x^3 - 1) \div 8$, and $(216x^3 - b^3) \div 125$, and because the divisors 8, 9, 125, are prime to each other; the quotients $(15x - 1) \div 8$ $(x - a) \div 27$, $(6x - b) \div 125$, must all be integers when their sum $2x + (307x - 1125 - 1000a - 72b) \div 9000$ is an integer; to find it such, cancel $2x$, and multiply the remainder by 8443 (the least whole number (m) that makes the value of $(307m - 1) \div 9000$ a whole number); reject all multiples of 9000 from the product, and put the remainder $(x - 3375 - 1000a - 4896b) \div 9000 = n$, so shall x be found $= 9000n + 3375 + 1000a + 4896b$, in which a has two values (1, 2) and b four (1, 2, 3, 4). If $n = -1, 0, 1, 2$, &c. and $a = b = 1$, (or the three remainders are supposed equal) $x = 271$, 9271, 18271, &c. and the least numbers are 159220088, 537367797, and 2487813875.

Question 181, by *Mr. George Clifton.*

The three sides of a triangle are 48, 64, and 80 respectively; it is required to find the diameter of the inscribed circle, the side of a square inscribed in that circle, and the area of an isosceles triangle in that square?

Solution, by *Mr. John Ryley.*

As the three sides of the triangle are multiples of the numbers 3, 4, and 5, it is evident that the triangle has one angle right; therefore, $48 + 64 - 80 = 32$, the diameter of the inscribed circle, and $16\sqrt{2} = 22.62741699$, the side of its inscribed square; therefore, 512 is the area of the square, and 256 the area of the isosceles triangle inscribed in the said square.

Question 182, by *Mr. Newton Bosworth.*

Suppose a right cylinder and cone, whose altitudes are each 32 inches, and diameter of each at the base 24 inches, were cut by a plane entering at the edge of the base of each, and making an angle therewith of 48°: the favor is requested of Mr. *Whiting's* correspondents to determine the difference of the two ellipses thus formed.

Solution, by *Mr. John Ryley.*

Let ABCD (Fig. 5) be the cylinder, and ABE the cone; AK the cylindric section, and AI the conical one; then in the triangle ABK all the angles are given, and side AB, to find AK = 35.86744, the transverse axe of the cylindric section, whose conjugate is 24: Again in the triangle BEC two sides are given, and the angle C a right-angle, to find the angle BEC = ABE = 69° 26′ 38″; hence the ∠ AIB = 62° 33′ 22″. Now in the triangle AIB, all the ∠s and side AB are given to find AI = 25.3215, and BI = 20.09715; but by *Euc.* 47. 1. BE = 34.176; hence EI = 14.07885, and by similar triangles EB : BA :: EI : IF = 9.8868; and per conics, $\sqrt{AB \times FI} = 15.404$ = the conjugate axe of the section AI; therefore $35.86744 \times 24 - 25.3215 \times 15.404 \times .7854 = 376.80835$ the difference required.

Question 183, by *Mr. John Collins, Schoolmaster, Kensington.*

Three sides of a trapezium are 83, 72, and 55 chains respectively: required the 4th side which is the diameter of its circumscribing circle, and also the area?

Solution, by *Mr. John Ryley.*

Put AB (Fig. 6) = x, AD = a, DC = b, and CB = c; then because the angles in a semi-circle are right angles, by *Euc.* 47. 1. we have BD = $\sqrt{x^2 - a^2}$, and AC = $\sqrt{x^2 - c^2}$; also, by Prop. XXXII. B. IV. *Emers. Geom.* AC × BD = AB × DC + AD × BC; or in symbols $\sqrt{x^2 - a^2} \times \sqrt{x^2 - c^2} = bx + ac$: or, by squaring $\overline{x^2 - a^2} \times \overline{x^2 - c^2} = b^2x^2 - 2abcx + a^2c^2$; hence $x^3 - \overline{a^2 - b^2 - c^2} \times x = 2abc$; which put into numbers becomes $x^3 - 15098x = 657360$; hence $x = 140.616$, &c. Now, by Rule V, Pa. 72, *Dr. Hutton's Mens.* the area is found = 6308.8342, &c.

Question 184, by *Mr. James Stevenson, of Heath, near Chesterfield.*

Given the vertical angle equal to 50° 50′, and sum of the two including sides 100; to determine the triangle, when the biquadrates of half the difference of the said sides is equal to double the area.

Solution, by *Mr. John Ryley.*

Put x = half the difference of the sides, a = half the sum, and s = the sine of the vertical angle; so shall $a + x$ = the greater side, and $a - x$ = the less; also, per Trig. $\overline{a^2 - x^2} \times \frac{s}{2}$ = the area of the triangle, which by the question is $= x^4$; therefore $x^4 + \frac{sx^2}{2} = \frac{sa^2}{2}$, and by compleating the square, &c. $x = \sqrt{\frac{\overline{8sa^2 + s^2}|^{\frac{1}{2}} - s}{2}}$

$= 5.562175$, and $x^4 = 957.1469$, the area. Now put $a+x = 55.562175 = c$, $a-x$, $= 44.437825 = d$, and $n =$ the cosine of the vertical anglo, then the base will be expressed by $\sqrt{c^2 + d^2 - 2cdn} = 45.088237$, &c.

Question 185, by *Mr. Joseph Woollin, of Smalley.*

A gentleman having a garden in form of an equilateral triangle, in the midst of which stands a conical pillar, the diameter of its base 4 feet, and solidity 100 feet, and from the summit of the solid to the angle of the garden is 20 yards.——Now he would be obliged to any young student to tell him the area of the cultivated part of the garden.

Solution, by *Mr. John Ryley.*

First, the height of the pillar is $= 100 \div 4 \times 3.1416 = 23.873977$; which being squared and taken from the square of 60, the square root of the difference is $55.0457392 = AO$ (Fig. 7). Hence $CD = \frac{3}{2} AO = 82.5686088$, and $AD = \frac{1}{2} AO \sqrt{3} = 47.671007$. Therefore $AD \times DC = 3936.128728 =$ the area of the garden, from which deduct 12.5664 the area of the pillar's base, and there remains 3923.562328 feet $= 436$ yards, the area of the cultivated part of the garden.

Question 186, by *Mr. John Fildes, Schoolmaster, Liverpool.*

ABC is a triangular field, right-angled at B, the side AB being 154 yards, in which, at D 100 yards from A, stands a tree, and in the side BC there is another tree at E, 154 yards from C; now if AE, DE, and DC be drawn, the angle ACD and AED will be equal: Required the area of the field.

Solution, by *Mr. John Ryley.*

Const. Upon the end E (Fig. 8) of the given line EC, erect the $\perp$ EF $=$ a mean proportional between BA and BD; to O the middle of CE, draw FO; upon CE describe the semi-circle CTE, and draw the tangent TB to meet CE produced in B; $\perp$ to CB draw BA of the given length, join AC, and ABC will be the triangle required.

For, by the nature of the circle, $OB = OF$, and $BT = FE$, also $BC \times BE = BT^2 = FE^2 =$ by construction $BA \times BD$; therefore a circle will pass through the points A, D, E, C, and consequenty the $\angle AED = \angle ACD$.

Calculation. By *Eu.* 47, 1. $\sqrt{OE^2 + EF^2} = 119.34823 = FO = OB$; therefore, $CB = 196.34823$, and $\frac{1}{2} AB \times BC = 15118.8$ yards $= 3$ A. 1 R. 12.85 P. as required.

Question 187, by *Mr. J. Walson.*

Two numbers (47 and 59) prime to each other, being given; to find the least multiple of each of them, exceeding by unity a multiple of the other.

Solution, by the *Rev. Mr. Hellins, Teacher of the Mathematics and Philosophy.*

First, to find the least multiple of 59 that exceeds a multiple of 47 by 1: call the multiplier of 47 x, and that of 59, y, and we have $47x = 59y - 1$; therefore $x = \frac{59y-1}{47} = y + \frac{12y-1}{47}$, where it is evident that when x and y are whole numbers, $\frac{12y-1}{47}$ is a whole number. But $\frac{48y-4}{47}$, and $\frac{47y}{47}$ are whole numbers, and consequently their difference, $\frac{y-4}{47}$ is either a whole number or o, and since the conditions of the question require that the value of y shall be the least possible, it is evident that $y - 4 = o$, or $y = 4$; and then $x = 5$, and 236 is the least multiple of 59 that exceeds (235) a multiple of 47 by 1.

Secondly, to find the least multiple of 47 that exceeds a multiple of 59 by 1: Let x, and y, again stand for the multipliers, and we have $47x = 59y + 1$, from which equation, by an argumentation similar to that above, we prove that $\frac{y+4}{47} = 1$; and then we get $y = 43$, $x = 54$, and 2538, the multiple required, which exceeds (2537) a multiple of 59, by unity.

Another Solution, by the Rev. *Mr. John Garnons.*

For the multiplier of 59, put x, and for the difference of it and that of 47 put y. Then to find the least multiple of 59 that exceeds a multiple of 47 by unity, we have $59x = 47 \times \overline{x+y} + 1$, therefore $\frac{12x-1}{47} = y$; where it is evident that when x and $\overline{x+y}$ are the least possible whole numbers, (as the question requires) then y $\left(\frac{12x-1}{47}\right)$ is a whole number $=$ to 1, and $x = 4$, $\overline{x+y} = 5$, therefore 236 is the least multiple of 59 that exceeds (235) a multiple of 47 by unity.

Secondly. To find the least multiple of 47 that exceeds a multiple of 59 by unity: we have $59x = 47 \times \overline{x+y} - 1$, whence $x = 43$, and $y = 11$, and $\overline{x+y} = 54$; therefore, 2538 is the least multiple required, which exceeds (2537) a multiple of 59 by unity

Question 188, by *Mr. James Webb.*

What is the declination of that star which has the greatest altitude possible, $3^h\ 37^m$ after it has passed the meridian in latitude $51°\ 31'$ N.

Solution, by *Mr. J. Meritt.*

Describe the primitive circle AZPR (Fig. 9) to represent the meridian of the place, make ZP equal to the compliment of the given latitude, and draw the six o'clock hour circle PC. With the secant of 54° 15', the measure of the time from noon, describe the hour circle PS and through Z, describe the vertical circle ZSD to cut PS at right-angles: then will PS be the complement of the stars declination. For round Z, as a pole, let the almicanter ASB be described through S; and it is manifest, as the vertical circle ZSD is perpendicular both to the alimancanter ASB and hour circle PS, that the former touches the latter in the point S, consequently a star in that part of the hour circle is nearer to the zenith than any other which is in the same hour circle.

Hence, as the tangent of the latitude is to the cosine of the angle ZPS, so is radius to the tangent of PS the stars distance from that pole which is of the same name with the latitude: in the case given. As tangent 51° 31' : cosine 54° 15' :: radius : tangent 24° 54'¾.

Question 189, by *N. T.*

Sailing N. N. W.—I came in sight of two Islands, the one bearing N. and the other W. After running 8 miles I found myself equally distant from them, and when I had run 3 miles farther I was in a right line with them: it is required to find my distance from these two Islands at each time of setting them?

Solution, by *Mr. George Sanderson.*

Let C (Fig. 10) E, and D, represent the places of the ship, at the first, second, and third observations, A and B the two islands.

Because ACB is a right angle and AE = EB, if OE be drawn perpendicular to AB, then O is the center, and AB the diameter of a circle passing through the points ACB, and if CD be produced to meet it in P, and AP, BP be joined, the angles PAB and PBA are = to the given angles PCB and PCA, whence the angles CAB and CBA may nearly be determined by the following method:

On any right-line AB describe a circle, and make the angle BAP = 22° 30', draw AP to meet the circle in the point P, through which draw the indefinite right line LPE perpendicular to AB; and parallel thereto, through O, draw the indefinite line GHR; on AO, produced if necessary, take OK to OF as 8 to 3, and on FL take PL to PF in the same ratio of 8 to 3, through the points K and L draw the indefinite right lines MN and LN, parallel to OR, and AB to meet in the point N, with which, as assymtotes, through the point O, describe the equilateral hyperbola OCQ. to cut the circle in C; join the points AC, CB, and the angles CBA and CAB are equal to the angles which the North and West rhumbs make with a line joining the two islands.

Draw CP cutting GO and OB in the points E and D, alſo draw CZ parallel to GO, cutting OK in V, and LN in Z, and draw CI perpendicular to MN.

By a well-known property of the hyperbola the rect. CN = rect. ON, wherefore IC (KV) : KO :: OR (EL) : CZ (CV + FL), but KO : KF :: PL : FL (by conſt.) hence, KV : KF :: PL : CV + FL, and (by diviſion, &c.) KV : PL :: FV : FP + CV :: DF : FP (by ſimilar triangles); again (by alternation) KV : DF :: PL : PF :: KO : FO by conſt. hence (by div. and alter.) KO : FO :: VQ : DO :: CE : ED, by ſimilar triangles, but KO : FO :: 8 : 3 (by conſt.) therefore CE : ED :: 8 : 3. Q. E. D.

The ſame anſwered by *Apollonius.*

If the thing were done, and A (Fig. 11) and B the ſituation of the two Iſlands, C that of the ſhip when A bore due North and B due Weſt, D its ſituation when equally diſtant from them, and E its ſituation when in a right line with them: moreover if DI be drawn at right angles to the line AB which joins them; it is manifeſt that as DA = DB, IA will be = IB; and conſequently, by *Euc.* III. 31, the line joining the two Iſlands is biſected by a ſemi-circle deſcribed on DE; and from this conſideration, the following conſtruction is evident:

Draw the North rhumb CA, the N. N. W. rhumb CE, and the Weſt rhumb CB. In CE, take CD equal 8, and CE equal to 11 miles; and on DE deſcribe a ſemi-circle DIE. From D, as a center, deſcribe ſeveral concentric circles, cutting CA in *a*, A, a, &c. and CB in *b*, B, b, &c. join the correſponding points *a*, *b*; A, B; a, b, &c. with the right lines *ab*, AB, ab, &c. and biſect theſe lines in the points *i* I, i, &c. Then, if through theſe points the curve D *i* I i be deſcribed, cutting the ſemi-circle DIE in I, and through the points I and E the ſtraight line AB be drawn, cutting CA in A, and CB in B; it is manifeſt that A and B will be the ſituations of the two Iſlands.

Queſtion 190, by *Mr. J. Walſon.*

A parabola being given, and a point without it; to determine the ſhorteſt diſtance between that point and the curve.

Solution, by *Mr. George Sanderſon.*

Conſt. From the given point P, (Fig. 12) draw PA perpendicular to the axis (VH) meeting it in A, take VB equal to half the parameter of the axis, and biſect BA in E; erect the perpendicular EN = ¼ PA; with N center, and NV, the diſtance from the vertex, radius, deſcribe a circle, cutting the curve again in C, and PC is the diſtance required.

Demonſt. Draw the ordinate CG, and perpendicular to it ND; alſo produce PC to meet the axis in the point H.

$VE^2 + EN^2$ (DG^2) $= ND^2$ (EG^2) $+ CD^2$ (*Euc.* 47. I.) $\therefore VE^2 - EG^2 = CD^2 - DG^2$, or $VG \times \overline{VE - EG} = CG \times \overline{CD - DG}$ (*Simp. Geom.* B. 2, Theo. 7.) whence $VG : CG :: CD - DG$ ($CG - 2DG$) $: VE - EG :: VG \times 2VB . CG \times 2VB$, but $VG \times 2VB = CG^2$, by the property of the curve; $\therefore CG : 2VB :: CG - 2DG : VE - EG$, whence (by alter. and division) $CG : 2DG$ ($\frac{1}{2}PA$) $:: 2VB : 2VB - VE + EG = VB + AG$ (because $AE = EB$ by const.) hence, $CG : PA :: VB : VB + AG :: GH : AH$ ($AG + GH$) by similar triangles; therefore $HG = VB = \frac{1}{2}$ paramater, by const. Wherefore, by a well-known property of the curve, a tangent at the point C is perpendicular to PH; consequently PC is the shortest distance between the point P and the curve, as required.

N. B. This construction holds good when the perp. PA falls above the vertex on the axis produced.

Question 191, by *Astronomicus.*

The declinations and right ascensions of two stars being given; to find their distance from the meridian when their difference of azimuth is the greatest or least possible.

Solution, by *Mr. James Webb.*

Let ZP (Fig. 13) be an arc of the meridian, where Z is the zenith and P the elevated pole: also suppose S and *s* to be the places of the two stars when the difference of their azimuths is a *maximum* or *minimum*; the angles PZS, PZ*s* their azimuths, PS and P*s* their polar distances, and ZS and Z*s* their distances from the zenith.

Then, by what is done in answer to Question 10. $\dot{P} : \dot{Z} :: R^3 : \text{cos. } PZ \times R^2 - \text{sin. } PZ \times \text{cos } Z \times \text{cot. } SZ$ $\dot{p} : \dot{z} :: R^3 : \text{cos. } PZ \times R^2 - \text{sin. } PZ \times \text{cos. } Z \times \text{cot. } sZ$. Hence, because the fluxions of the two azimuths must be equal when their difference is a *max.* or *min.* $\text{sin. } PZ \times \text{cos. } PZs \times \text{cot. } sZ = \text{sin. } PZ \times \text{cos. } PZS \times \text{cot. } SZ$; or $\text{cos. } PZs \times \text{cot. } Zs = \text{cos. } PZS \times \text{cot. } ZS$. Therefore, when the difference of azimuths of two stars is a *max.* or *min.* the cosines of their azimuths are directly as the tangents of their altitudes.

Question 192, by *Numericus.*

Three school boys laid out equal sums of money in fruit—apples and oranges: they all paid the same price apiece, for their apples as well as their oranges; and yet the whole number of apples and oranges (together) purchased by the first boy was but 9; whilst the second had 18, and the third no fewer than 24: moreover, the difference between the number of apples and the number of oranges, purchased by the first boy, was the least that the question will admit of. What was the number of apples bought by each boy, and what did each sort of fruit cost them?

Solution, by *Mr. Thomas Todd.*

Let x, y, and v, represent the number of apples bought by the first, second, and third boys respectively; then will $\overline{9-x}$, $\overline{18-y}$, and $\overline{24-v}$ be the oranges. Let n be the price of an apple, and m that of an orange.

Then $nx + 9m = ny + 18m - my$; from which $y = 9 + \frac{9n}{m-n} + x$. Also, $nx + 9m - mx = nv + 24m - mv$; wherefore, $v = 15 + \frac{15n}{m-n} + x$. Now, it is evident that the greatest value of x must be taken to answer the problem, and that will be when $\frac{9n}{m-n}$ and $\frac{15n}{m-n}$ are the least whole numbers prime to each other, which will be the case when m is $=$ to 4 times n, or when $m = 4$, and $n = 1$, for then $\frac{9n}{m-n} = 3$, and $\frac{15n}{m-n} = 5$, and thence $y = 12 + x$, and $v = 20 + x$. Now, the limit, or greatest value of x is 3; in which case $y = 15$, and $v = 23$: for x cannot be 4, because v would then be 24, and so the third boy would have all apples, and no oranges, which is contrary to the question. Hence, if we put $m = 4$, $n = 1$, of farthings, halfpence, or any coin whatever, we shall have the values of A, B, and C, apples and oranges, as required.

The same answered by *Mr. Isaac Dalby.*

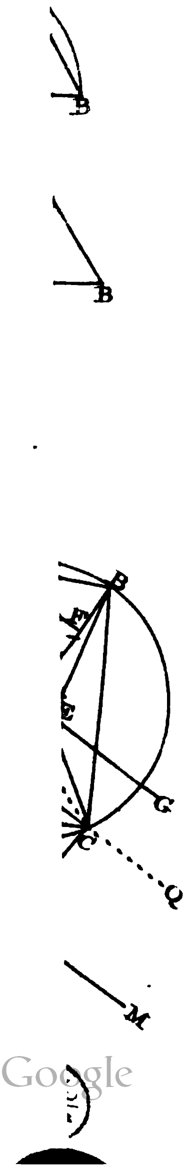
Let x, y, and z, represent the number of oranges bought by the first, second, and third boys respectively, then will $9-x$, $18-y$, and $24-z$, be the apples; let a be the price of an orange, b that of an apple, then by the question, $ax + \overline{9-x} \times b = ay + \overline{18-y}, \times b = az + \overline{24-z}, \times b$, or $ax - bx + 9b = ay - by + 18b = az - bz + 24b$, and putting $d = a - b$, we have $dx = dy + 9b = dz + 15b$, hence $x = y + \frac{9b}{d} = z + \frac{15b}{d}$. Now, it is manifest that when the terms $\frac{9b}{d}$, and $\frac{15b}{d}$ are the least possible whole numbers, and d the greatest common measure of the co-efficients, 9 and 15, d will be $= 3$, and $b = 1$; therefore $x = y + 3 = z + 5$. Here it appears that x must be greater than 5, for when $x = 5$, $z = 0$: therefore, $x = 6$; and, consequently, $y = 3$, and $z = 1$, the numbers of oranges; and 3, 15, and 23, are the numbers of apples bought by the first, second, and third boys, respectively.

Scholium. It is evident the prices a and b may be any numbers in the ratio of 4 to 1; thus if the price of an orange be a penny, that of an apple will be a farthing, &c.

Mr. John Blake

Substitutes x, y, and z, for the oranges bought by the first, second, and third boys re-

Plate. 13.



ſpectively, and r to 1 for the ratio of the price of an orange to that of an apple. Then, by a proceſs ſimilar to Mr. *Dalby*'s, he finds $r = 4$, $x = 6$, $y = 3$, and $z = 1$.

ARTICLE XXIII.

A DIALOGUE *between* PLATO *and* DIOGENES.

Communicated by Mr. JOHN WILLIAMS, *of London.*

DIOGENES.——Plato ſtand off——a true philoſopher as I was, is no company for a courtier of the tyrant of Syracuſe. I would avoid you, as one infected with the moſt noiſome of plagues, the plague of ſlavery.

Plato.—He, who can miſtake a brutal pride and ſavage indecency of manners for freedom, may naturally think that the being in a court (however virtuous in one's conduct, however free one's language there) is ſlavery. But I was taught by my great maſter, the incomparable Socrates, that the buſineſs of true philoſophy is to conſult and promote the happineſs of ſociety. She muſt not therefore be confined to a *tub* or a *cell.* Her ſphere is in ſenates, or the cabinets of kings. While your ſect is employed in ſnarling at the great, or buffooning with the vulgar, ſhe is counſelling thoſe who govern nations, infuſing into their minds Humanity, Juſtice, Temperance, and the Love of true Glory, reſiſting their paſſions, when they tranſport them beyond the bounds of Virtue, and fortifying their reaſon by the antidotes ſhe adminiſters againſt the poiſon of flattery.

Diogenes.—You mean to have me underſtand, that you went to the Court of the younger Dionyſius, to give him antidotes againſt the poiſon of flattery. But I ſay he ſent for you only to ſweeten the cup, by mixing it more agreeably, and rendering the flavour more delicate. His vanity was too nice for the nauſeous common draught; but your ſeaſoning gave it a reliſh, which made it go down moſt delightfully, and intoxicated him more than ever. Oh! there is no flattery half ſo dangerous to a prince as a fawning philoſopher!

Plato.—If you call it fawning, that I did not treat him with ſuch unmannerly rudeneſs as you did Alexander the Great, when he viſited you at Athens, I have nothing to ſay. But, in truth, I made my company agreeable to him, not for any mean ends which regarded only myſelf, but that I might be uſeful both to him and to his people. I endeavoured to give a right turn to his vanity; and know, Diogenes, that whoever will ſerve mankind, but more eſpecially princes, muſt compound with their weakneſſes, and take as much pains to gain them over to virtue, by an honeſt and prudent complaiſance, as others do to ſeduce them from it, by a criminal adulation.

Diogenes.—A little of my ſagacity would have ſhewn you, that, if this was your purpoſe, your labour was loſt in that court. Why did not you go and preach chaſtity to Lais? A philoſopher in a brothel, reading lectures on the beauty of continence and decency, is not a more ridiculous animal than a philoſopher in the cabinet, or at the table of a tyrant, deſcanting on liberty and public ſpirit! What effect had the leſſons of your famous diſciple Ariſtotle upon Alexander the Great, a prince far more capable of receiving inſtruction than the younger Dionyſius? Did they hinder him from killing his beſt friend, Clitus, for ſpeaking to him with freedom, or from fancying himſelf a God, becauſe he was adored by the wretched ſlaves he had vanquiſhed? When I deſired him *not to ſtand between me and the ſun*, I humbled his pride more, and conſequently did him more good than Ariſtotle did by all his former precepts.

Plato.—Yet he owed to thoſe precepts, that, notwithſtanding his exceſſes, he appeared not unworthy of the empire of the world. Had the tutor of his youth gone with him into Aſia, and continued always at his ear, the authority of that wiſe and virtuous man might have been able to ſtop him, even in the riot of conqueſt, from giving way to thoſe paſſions that diſhonored his character.

Diogenes.—If he had gone into Aſia, and had not flattered the King as obſequiouſly as Hæpheſtion, he would, like Calliſthenes, whom he ſent thither as his deputy, have been put to death for high treaſon. The man who will not flatter, muſt live independent as I did, and prefer a tub to a palace.

Plato.—Do you pretend, Diogenes, that, becauſe you were never in a court, you never flattered? How did you gain the affection of the people of Athens, but by ſoothing their ruling paſſion, the deſire of hearing their ſuperiors abuſed? Your cynic railing was to them the moſt acceptable flattery. This you well underſtood, and made your court to the vulgar, always envious and malignant, by trying to lower all dignity and confound all order: you made your court, I ſay, as ſerviley, and with as much offence to virtue, as the baſeſt flatterer ever did to the moſt corrupted prince. But true philoſophy will diſdain to act either of thoſe parts. Neither in the aſſemblies of the people, nor in the cabinets of kings, will ſhe obtain favour by fomenting any bad diſpoſitions. If her endeavours to do good prove unſucceſsful, ſhe will retire with honour, as an honeſt phyſician departs from the houſe of a patient, whoſe diſtemper he finds incurable, or who refuſes to take the remedies he preſcribes. But if ſhe ſucceeds; if, like the muſic of Orpheus, her ſweet perſuaſions can mitigate the ferocity of the multitude, and tame their minds to a due obedience of laws and reverence of magiſtrates; or if ſhe can form a Timoleon, or a Numa Pompilius to the government of a ſtate, how meritorious is the work! One king, nay one miniſter, or counſellor of ſtate imbued with her precepts, is of more value than all the ſpeculative, retired philoſophers, or cynical revilers of princes and magiſtrates that ever lived upon earth.

Diogenes.—Don't tell me of the muſic of Orpheus, and of his taming wild beaſts. A wild beaſt brought to *crouch* and *lick the hand of a maſter*, is a much viler animal than he was in his natural ſtate of ferocity. You ſeem to think that the buſineſs of philoſophy is *to poliſh men into ſlaves*; but I ſay, it is to teach them to aſſert, with an untamed and generous ſpirit, their independence and freedom. You profeſs to inſtruct thoſe who want to *ride* their fellow creatures, how to do it with an eaſy and gentle rein; but I would have them thrown off and trampled under the feet of all their deluded or inſulted equals, on whoſe backs they have mounted. Which of us two is the trueſt friend of mankind?

Plato.—According to your notions all government is deſtructive to liberty; but I think that no liberty can ſubſiſt without government. A ſtate of ſociety is the *natural* ſtate of mankind. They are impelled to it by their wants, their infirmities, their affections.—The laws of ſociety are rules of life and action neceſſary to ſecure their happineſs in that ſtate. Government is the due enforcing of thoſe laws. That government is the beſt which does this moſt effectually, and moſt equally; and that people is the freeſt, which is moſt ſubmiſſively obedient to ſuch a government.

Diogenes.—Shew me the government which makes no other uſe of its power than duly to enforce the laws of ſociety, and I will own it is intitled to the moſt abſolute ſubmiſſion from all its ſubjects.

Plato.—I cannot ſhew you perfection in human inſtitutions. It is far more eaſy to blame them than it is to amend them: Much may be wrong in the beſt: but a good man reſpects the laws and the magiſtrates of his country.

Diogenes.—As for the laws of my country, I did ſo far reſpect them, as not to philoſophiſe to the prejudice of the firſt and greateſt principle of nature and of wiſdom, ſelf-preſervation. Though I love to prate about high matters as well as Socrates, I did not chuſe to drink hemlock after his example. But you might as well have bid me *love* an ugly woman, becauſe ſhe was dreſt up with the gown of Lais, as *reſpect* a fool or a knave, becauſe he was attired in the robe of a magiſtrate.

Plato—All I deſired of you was, not to amuſe yourſelf and the populace by throwing dirt upon the robe of a magiſtrate, merely becauſe he wore that robe and you did not.

Diogenes.—A philoſopher cannot better diſplay his wiſdom than by throwing contempt on that pageantry, which the ignorant multitude gaze at with a ſenſeleſs veneration.

Plato.—He who tries to make the multitude *venerate nothing* is more ſenſeleſs than they. Wiſe men have endeavored to excite an awful reverence in the minds of the vulgar for external ceremonies and forms, in order to ſecure their obedience to religion and government, of which theſe are the ſymbols. Can *a philoſopher* deſire to defeat that good purpoſe?

Diogenes.—Yes, if he ſees it abuſed to ſupport the evil purpoſes of ſuperſtition and tyranny.

Plato.—May not the abuſe be corrected without loſing the benefit? Is there no difference between *reformation* and *deſtruction*?

Diogenes.—*Half measures* do nothing. He who desires to *reform* must not be afraid to pull down.

Plato.—I know that you and your sect *are for pulling down every thing that is above your own level.* Pride and envy are the motives that set you all to work. No one can wonder that passions, the influence of which is so general, should give you many disciples and many admirers.

Diogenes.—When you have established *your republic*, if you will admit me into it, I promise you to be *there* a most *respectful* subject.

Plato.—I am conscious, Diogenes, that *my republic* was imaginary, and could never be established. But they shew as little knowledge of what is practicable in politics, as I did in that book, who suppose that the liberty of any civil society can be maintained by the destruction of order and decency, or promoted by the petulance of unbridled defamation.

Diogenes.—I never knew any government angry at defamation, when it fell upon those who disliked or obstructed its measures. But I well remember that the thirty tyrants at Athens called opposition to them *the destruction of order and decency.*

Plato.—Things are not altered by names.

Diogenes.—No—but names have a strange power to impose on weak understandings. If, when you were in Egypt, you had laughed at the worship of an onion, the priests would have called you an Atheist, and the people would have stoned you. But I presume that, to have the honour of being initiated into the mysteries of that reverend hierarchy, you bowed as low to it as any of their devout disciples. Unfortunately, my neck was not so pliant, and therefore I was never initiated into the mysteries either of religion or government, but was feared or hated by all who thought it their interest to make them be respected.

Plato.—Your vanity found its account in that fear and that hatred. The high priest of a deity, or the ruler of a state, is much less distinguished from the vulgar herd of mankind, than the scoffer at all religion, and the despiser of all dominion.——But let us end our dispute. I feel my folly in continuing to argue with one, who in reasoning does not seek to come at truth, but merely to shew his wit. Adieu, Diogenes; I am going to converse with the shades of Pythagoras, Solon, and Bias. You may jest with Aristophanes, or rail with Thersites.

F I N I S.

THE

Poetical Delights.

CONTAINING

ENIGMAS, CHARADES, REBUSES, &c.

WITH THEIR ANSWERS;

SELECTED FROM AN EXTENSIVE CORRESPONDENCE,

BY

THOMAS WHITING,

MASTER OF KEPPEL-HOUSE SEMINARY,

FULHAM-ROAD, *near* BROMPTON.

Editor of the SCIENTIFIC RECEPTACLE.---Author of SELECT EXERCISES.---
A KEY TO DITTO, &c. &c. &c.

LONDON:

PRINTED for Messrs. WHITING, DAVIS, GALE, GLENDENNING, and KEMMISH;

And sold by T. N. LONGMAN, Paternoster-Row.——1797.

POETICAL DELIGHTS.

ÆNIGMAS TO BE ANSWERED.

Enigma 1, by *Mr. Gregory, of Yaxley.*

THE full orb'd moon, refulgent lamp of night,
With majesty ſerene, ſhed forth her light;
For pow'rful ſol, the glitt'ring god of day,
Had to the weſtern ocean bent his way,—
And ſilence reign'd—ſave where the diſtant rill
In bubbling murmurs glides adown yon hill;
Or where in Zula's wood the lofty Trees
Reflect in hollow ſounds the eaſtern breeze;
Or, from the ivy-mantled tow'r (devote
To father Time) iſſues the ſcreech-owl's note.
When lo! Cimene appears with ſullen gait
And clouded brow, lamenting his hard fate:
For he, alas! the butt of fortune's ſport,
His miſeries no longer could ſupport,
And dark deſpair had ſeiz'd his troubled mind,
So he reſolv'd by me relief to find.
Becauſe poor mortals when involv'd in grief,
Have lately thought that I could give relief:
Ah! fatal error—to my tale attend,
And gain, as Cimene did, a better friend.
Determin'd by my aid his woe to eaſe,
He raſhly rais'd his arm; but it did pleaſe
Th' interpoſing pow'r t' avert the ſtroke,
Reaſon appear'd, and thus to him ſhe ſpoke.
"Forbear, vain Man! ſay, why would'ſt thou thus gain
A wide extended ſea of endleſs pain;
For by thus acting, woes thou wilt inſure,
That will to all eternity endure;
'Tis cowardice by which thou'lt be undone,
Deſiſt! take my advice, that monſter ſhun."
With horror ſtruck at what within his breaſt
By reaſon's voice had thus been well expreſs'd,
He then exclaim'd—' Good Heaven, how I view
The dire attempt;—but ſhould my woes anew
Come pouring on, in future I will truſt
On *ONE*, omnipotent, all-wiſe, all-juſt.'
This ſaid, with haſty ſteps he homeward goes,
Reſolv'd, with reaſon's help, me to oppoſe;
Tho' oft unfortunate, yet he ſeems content,
Nor ne'er in public does his ſorrows vent:
He lives in hopes that he ſhall always be
Poſſeſs'd of fortitude to keep from me.

Enigma 2, by *Mr. J. Walton.*

YE fair ſoft authors of delight attend,
While I deſcribe the virtues of a friend,
Whoſe lib'ral favors all the juſt obtain;
Yet, oh! ye fair, the wicked I diſdain.
How many men devoid of me lament,
When all they have is in profuſion ſpent!
How many, whom paternal care poſſeſs,
Would gladly leave me to their wanton race.
But if you long to ſee me ſtill more plain,
By two ſhort hints you'll ſurely know my name;
My head lopp'd off, ſome prodigals adore,
And prize me more than e'er they did before.
Tho' if you further ſtill abridge my name,
You'll find what noted gameſters often blame.

Enigma 3, by *Mr. Olinthus Gregory, of Yaxley.*

BEHOLD, dear gents, in myſtic dreſs,
I deign to ſhew my face;
And, tho' in dark diſguiſe I gueſs,
You'll give to me a place.
With eager haſte and clatt'ring din,
I bring myſelf to view;
But now the miracles begin,
Which I'll rehearſe to you.
Four eyes have I, two mouths alſo,
I can extend them wide;
And I would likewiſe have you know,
They open at my ſide.
But I'm no monſter I declare,
Tho' oft I ſwallow men
And women too nay do not ſtare,
They'll all come out again.
Tho' in my ſtomach they remain
For hours and ſometimes days,
They have no reaſon to complain
Of my accuſtom'd ways.
When to my journey's end I come,
I void them all alive;
And when they're gone, if I find room,
To ſwallow more I ſtrive.
But change the ſcene, and turn your eye
To yonder rural ſhed;
It's dwellers all on me rely,
To earn their daily bread.
Led on by me the whiſt'ling hind
With pleaſure ſows the corn;
Content and peace poſſeſs his mind,
Nor ne'er is he forlorn.
Bleſt with my aid, each learned ſage,
Has knowledge gain'd in ſtore
Whilſt genius help'd—and many a page,
They grac'd with charming lore.
If from my face you wiſh to take
This enigmatic veil;
Apply but me, the charm will break,
For I ſcarce ever fail.

Enigma 4, by *Mr. J. Becket.*

O female tyranny! that heart
Is ſure conſign'd t' eternal ſmart,
And loſt to ev'ry peaceful hour,
That's ſway'd by thy deſpotic pow'r!
For me, I'm doom'd to tortures ſore,
Nor e'er muſt hope for comfort more:
Tho' once but now for ever flown,
I happier, gentler hours have known.
Each night at opera or play,
I ſhone in brighteſt colours gay;
But when ſtern age, with every eaſe,
Depriv'd me of the pow'r to pleaſe;
An armed female tore my ſkin,
And then conceited me too thin;
Yet fed me with ſuch taſteleſs ſtuff,
She ſoon perceiv'd I had enough.
But were I, reader, to declare
The ſtabs which I am forc'd to bear,
And thoſe from ſuch as moſt pretend
To be my guardian or my friend,
You'd think me falſe as are the hearts
Of thoſe, the ſcource of all my ſmarts:
Hence, then, I will my woes conceal,
Till you my myſtic name reveal.

Enigma, by *Mr. Swift, of Stow.*

LADIES, a fav'rite I appear,
And your dear hands I kiſs, ye fair;
My ſhape's an oblong ſeen to be,
And dreſs is handſome you'll agree;
I'm dreſs'd in red, with ſpangled gold,
And hence my name you may unfold;
In mourning I ſometimes am clad,
But never known to do what's bad;
The title that I with me bring,
Makes me receiv'd, ev'n by the King;
All ranks with me, it is well known,
Have bent ſubmiſſive lowly down;
When virgins me they come before,
Hymen—they bluſh—I'll ſay no more.

Prize Enigma 6, *by Mr. Thomas Nield.*

THE coachman cracks his whip and crys ge-ho,
Then cuts his horſes—ſtamps his foot below,
The rattling ſound, together with the ſmart,
Makes every horſe to prick his ears and ſtart,
But without me, he cracks his whip in vain,
His horſes move, no more than does the ſlain.
See! Hodge the farmer on old Dobbin ride,
Tommy give way, the farmer loudly cried,
My horſe is mad, and feign would ſhew his ſpeed,
His high flown mettle—worth and noble breed;
Swift flies old Dobbin, and the farmer's eye
Looks down with ſcorn, on all he paſſes by,
But by degrees I did his Dobbin leave,
Which ſlack'd his foot, and did the farmer grieve;
Anon my whole entirely from him fled,
The farmer whipt, and Dobbin fell for dead.
Ceaſe! Farmer ceaſe! nor riſe again thy hand,
He's ſhort of me, and me, thou can't command;
But ere awhile when he recovers breath,
He'll have enough of me, to quit the earth,
Then mount, and ride but gently I employre,
Leſt he drops dead, then I return no more.
On learned Newton in Queen Ann's bleſt reign,
I gave my influence in a copious ſtrain,
Thro' me he brought each myſt'ry dark to light—
Each doubt he clear'd, and rectify'd aright.
Ingenious Arkwright, all the world agree,
Completeth every ſcheme by help of me;
Nor cou'd brave Rodney, if I'd quit his force,
Drove the proud Dons, with ſhame to their own courſe.
Pitt holds me dear—with him I'm all in all,
Shou'd I but leave him, he would quickly fall.
The awful Being who in all things pry,
And ev'ry mortal's inward thoughts deſcry,
Without my aid, wou'd know no more than you,—
Strange this appears! but you will find it true,
When you find me, till then, dear gents adieu.

☞ Whoever anſwers the above Enigma, before the 1ſt of Auguſt next, will have a chance of winning a ſet of the DELIGHTS.

CHARADES TO BE ANSWERED.

Charade 1, by *Mr. Bowyer.*

HAIL bleſſed firſt! thou balm of human mind,
Thrice happy he who's ſure thy aid to find;
Hence uſeful next, and ſcan the ſhiv'ring gale,
Or Engliſh commerce ſounds a woful tale:
Ariſe my whole, for at thy heavn'ly birth,
All nations ſure will find a heaven on earth.

Charade 2, by *Mr. Nield.*

MY firſt is black, and of infernal hue;
The robber with my next accoſteth you;
My whole is uſed in ſchools,—aſſiſts in trade,
So ken, my boys, the name of my Charade.

Charade 3, by *R. Thomas.*

IN pity, alas! fable first
From darkness uncover my tale—
O'er disdain—of most passions accurst,
O, let not thy silence prevail.

From the beams of bright Sol's vivid ray,
Alphonso, kind victim of love;
To my second his steps bore away,
The cares of his breast to remove:—

But, unable his grief to support,
My whole he unhappily 'spy'd;
Which, despairing, he eagerly caught,
He eat—he sigh'd—and he died!

False *Mira!* for ever now mourn
The act thy unkindness provok'd;
Can peace to your bosom return?
No!—for ever that blessing's revok'd.

Say, canst thou his virtues forget,
His goodness and meekness of heart;
How often in pain, at thy feet,
Affection he strove to impart.

Ah! view his sad cot on the lawn,
Of happiness once the retreat;
Where chearfulness rose with the morn,
Each symptom of care to defeat:

Neglected, forsaken, and left,
No swains now its pastimes explore;
Each scene of all pleasure's bereft,
Since *Alphonso*, its chief, is no more!

Charade 4, by *Mr. Thomas Nield.*

OH! beware that your next is not brought to my first,
Or alas! it will shorten your days;
But your body from troubles will then be at rest,
And your toil and your labours will cease.
My whole you'll oft meet with in almost all schools,
Believe me I tell you no lies;
Tho' in fact I shou'd say it as stupid as mules,
And their master's good counsels despise.

Charade 5, by *Mr. William Burdon.*

RESPLENDENT first, when thou thy aids display,
My next begins to shew the world its way;
But, oh! be wise, revere my whole lest you
In sad remorse repent the direful cue.

Charade 6, by *Mr. Gregory.*

HOW swiftly life does wear away,
Of death the grave's my first I say,
(We all must die you know)
My next's an herb, in Wales 'tis fam'd,
In England too, 'tis often nam'd
In pottage for to go:
These join'd will name a plant, which seldom grows on ground,
Tho' 'twixt the earth and sky, it often may be found.

NEW REBUSES.

Rebus 1, by *Mr. Thomas Nield.*

TO three-fourths of affection be pleased to add,
Two-thirds of a snare, and no more;
Then the name of a fair one will quickly be had,
A fair one I do much adore.

Rebus 2, by *Mr. Gregory.*

From a word in the English language, denoting a very necessary part of the furniture of an house, take away the initial, there will remain a kind of covering; from this also take away the initial and there will remain a notable fluid. Required the names of all three?

ENIGMAS ANSWERED.

1 Suicide. 4 Cushion.
2 Grace. 5 Prayer-book.
3 Diligence. 6 Power.

CHARADES ANSWERED.

1 Friendship. 4 Blockhead.
2 Inkstand. 5 Sunday.
3 Nightshade. 6 Houseleek.

REBUSES ANSWERED.

1 Lovet. 2 Chair.

SOLUTIONS to the PRIZE ENIGMA.

1. *By Mr. Philip Norris, of Liverpool.*

YOUR prize, kind Sir, I'd fain transmit
Unto a future age;
But O! I fear some greater wit
Will push me from the page:
Yet had I strength and *pow'r*, I'd then,
In spite of such have place;
But since all hope rests with my pen,
Alas! I doubt my case;
Ah! merit, could I call thee mine,
How blest would be my fate;
With pleasure to implore thee mine,
When fame would ope her gate.
T. Whiting then would aid my lays,
And give me true delight;
By entitling me unto the bays,
As to my lawful right.
But since, alas! I'm uninspir'd,
Such favor cannot be desir'd.

2. *By Mr. John Rimmer, of Liverpool.*

DEAR Whiting and ye bards divine,
Who on Parnassus' summit stand,
Be kind, should I obtrude a line,
Drawn with a weak untutor'd hand.
Your songs such *pow'r* of parts display,
And geniusses so sublime,
That from the task I shrink away,
And oft retract my feable rhyme.
But now my muse more bold is grown,
And strives the mystic prize t'expound;
And should success my efforts crown,
A grateful friend I shall be found.

We received very ingenious general answers to the Enigmas, Charades, &c. from Mess. Davis, Gregory, Griffith, Hewitt, Nield, Noris, Rimmer, and Webb, which we should have been glad to have inserted if we could have found room. The prize of one set of Delights has fallen to the lot of Mr. John Rimmer, of Liverpool.

ENIGMAS TO BE ANSWERED.

Enigma 1. (7) *by Mr. John Rimmer.*

IF your attention, bards, I can but gain,
Before you here I will myself explain:
I've got two arms, two legs, a head likewise,
And body too, of a moderate size:
My belly sometimes holds all kinds of dirt,
By which my constitution's seldom hurt;
But when I'm fill'd, immoveable I stand,
Unless my master takes me by the hand;
And then, which is stranger than the rest,
My body's weight upon my head is press'd,
Which mostly by an iron hoop is bound,
For when I move, alas! it must turn round;
When my belly's full and hunger allay'd,
In front of my master see me display'd;
But when I'm empty, and want something more,
Behind him I move with the wrong end before.

Enigma 2. (8) *by Mr. O. Gregory.*

YE British youths, give ear awhile,
And on your friend indulgent smile:
Your happy state depends on me,
And you would in confusion be,
Your youthful beauty would be gone,
If once from you my aid were flown.
From most great towns I long have fled,
In rural huts to hide my head;

For noisy mirth I much detest,
Yet thund'ring plaudits please me best.
Ye sons of Bacchus now attend,
I'm sure you'll own me as your friend;
'Tis I support your fav'rite tun,
Unpropp'd by me its use is none,
And all the liquor it contains
Would never fuddle your poor brains,
If I my useful help withdrew,
And, on my honour, this is true.
Whene'er a pois'nous draught is made
I then am sure to lend my aid;
And when misfortunes on us frown,
I make the nauseous cup go down.
In eloquence I'm always found,
Where truth and music do abound;
And all its use would futile be,
If it for once were robb'd of me.
When (from the parent tree just torn)
Some dainty fruit the board adorn,
(Without the help of magic wand)
I always in their centre stand.
But hold—these hints my name will shew,
So, for awhile, dear gents—adieu!

Enigma 3. (9) *by Mr. T. Stephens.*

I AM fat, I am slender, I'm pale as a sheet;
  My nose is as warm as a toast:
No cloak I require, save to screen me from heat;
  I may sweat, but I never can roast.
Like a hero in battle, tho' sure to expire,
  I never once flinch from my station;
One gut, like a bear, and no more, I require,
  And I die of the gut-inflammation.

Enigma 4. (10) *by Mr. Stephens.*

BEFORE creating nature will'd
  That atoms into form should jar,
By me the boundless space was fill'd,
  On me was built the first-made star.
For me the saint will break his word;
  By the proud Atheist I'm rever'd:
At me the coward draws his sword,
  And by the hero I am fear'd.
Scorn'd by the meek and humb'e mind,
  Yet often by the vain possess'd;
Heard by the deaf, seen by the blind,
  And to the troubled conscience rest.
Than wisdom's sacred self I'm wiser,
  And yet by every blockhead known;
I'm freely given by the miser,
  Kept by the prodigal alone.
As vice deform'd, as virtue fair,
  The courtier's loss, the patriot's gains;
The poet's purse, the coxcomb's care.
  Read—and you'll have me for your pains.

Enigma 5. (11) *by Mr. Stephens.*

I'M an eye that never had sight,
When alive I am buried, when dead brought to light.
I'm admir'd by the rich, not so much by the poor,
And belong to a man who's a very great whore.

Prize Enigma 6 (12) *by Mr. Lewes.*

SCENE A WOOD.

Three Witches performing Incantations.

FIRST WITCH.

HASTE! sisters haste! exert your baleful power;
'Tis now the witching, the propitious hour!
Bind well the charm in dark impervious shade;
Shall mortals dare our mystic rights invade!

CHORUS.

About, about, about,
From every eye shut out;
In a magical round
We will dance on the ground:
See how swiftly we go!
Rise up quick from below.
[*The charm rises.*

FIRST WITCH.

GO, loathsome sight, with thy discolour'd face!
Hence to the *gallies*, and assist the *chase!*

Tho' *black* thy masters' deeds, or *gore-red* dy'd,
Thou ne'er ſhalt quit him till he's *juſtify'd.*

SECOND WITCH.

Fly, noxious taylor! know 'tis my command,
Cloſe to the *gallows* thou ſhalt take thy ſtand:
Laid on the *coffin* thou the *form* ſhalt view;
The laſt, laſt *pull,* and ſee the *form renew.*

THIRD WITCH.

Without the pedant's aid, in ſchools untaught,
Go, ſkill'd in *letters,* and with *ſcience fraught:*
Whate'er in *learning* may *obſcure* appear,
By thy aſſiſtance ſhall be render'd *clear.*

CHORUS.

The charm is now done!
Then fly ere the ſun
Shall peep o'er the hills, or the plains:.
The mortal that tells
The contents of our ſpells,
A prize he ſhall have for his pains.

CHARADES TO BE ANSWERED.

Charade 1. (7) *by Mr. O. G. Gregory.*

NO houſe I think has e'er been made,
Except my firſt did give its aid;
Nor ſcarce a reg'ment has there been,
But in my ſecond it was ſeen:
My whole's a bird in general known,
Whoſe name in holy writ is ſhewn.

Charade 2. (8) *by Mr. O. G. Gregory.*

TO the tales of lovers my firſt does attend,
And ſometimes it hears the voice of a friend,
Tho' that's not ſo often the caſe;
When Colin and Lucy in marriage are join'd,
My ſecond is brought and ſhe is inclin'd
On her finger to give it a place:
My whole in the head-dreſs of ladies is ſeen,
From the wife of a tradeſman, to princeſs or queen.

Charade 3. (9) *by Mr. Rimmer.*

UPON my firſt the harmleſs infant ſee,
Reclin'd to reſt from care and ſorrows free;
My next's a faithful ſervant to mankind,
But we my whole an uſeleſs play-thing find.

Charade 4. (10) *by Patrick O'Sullivan, Eſq.*

My firſt affords much barb'rous ſport
When in the hunter's view;
My ſecond ſometimes tears their ſkins
While they my firſt purſue:
Then cruſh my whole and drink my blood,
When to exceſs you've run,
Perhaps I may digeſt your food,
And eaſe your bodies ſoon.

Charade 5. (11) *by Mr. Norris.*

WHEN Cuſtine heads his Sans Culottes,
My firſt leads up the van;
And when they fly from Cobourg's ſhots,
'Tis foremaſt in the train.
Each youth's my next, in merry mood;
Or, when in ſplendor dreſt;
My whole, my firſt, eſteems good food;
And's plac'd oft in the breaſt.

Charade 6. (12) *by Mr. Norris.*

MY firſt is much us'd for the making of holes,
But internally taken's the joy of good ſouls;
My next, like the earth round its centre revolves,
When impell'd on the green, on our pleaſure reſolves;
My whole is moſt pleaſing well fill'd with my firſt,
To the toper, whoſe wailings are always of thirſt.

REBUSES TO BE ANSWERED.

Rebus 1. (3) *by Mr. R. Thomas.*

TO find an invention, (as authors relate)
By the sister of Mars, Bellona the great,
Take four-sevenths of a town in Suffolk well known,
And three-eighths of a warrior, by Paris o'erthrown:
If after this, ladies, I'm left to repose,
Your fore-finger end my name will disclose.

Rebus 2. (4) *by Mr. R. Thomas.*

OF a female attire, display'd on the head,
Two-thirds—with the particle, in;
Will name the first mortal by envy misled,—
The first of the fratricide train.

Rebus 3. (5) *by Mr. Norris.*

THE god of silence, and the wife of Orpheus,
A warrior fam'd, the son of Peleus;
A marsh, where Hercules the Hydra kill'd,
He, whom a peace with Romulus fulfill'd;
A river fam'd where Orpheus' head was thrown,
When, by the Cycones he was rent and torn:
These initials join, and they will straightly shew
A greater blessing than all else below.

Rebus 4. (6) *by Mr. Norris.*

The half of a part, and one-third of a bone,
Two-fifths of a circle, with one-fourth of a tone;
Will shew a companion to those fond of talk,
That can bear with all nonsense and suffer a joke.

Anagram, *by Mr. Norris.*

TRANSPOSE a well-known measure,
And you've the sportsman's joy;
Then please behead this pleasure,
And lo! the gamesters toy;
Transpose the whole again, sirs,
'Twill shew your guardian friend:
Now lop from this the tail, sirs,
A seat of state 'twill stand.
Now gents unveil your worthy friend,
And may it all your haunts attend.

Ode to Friendship, *by Mr. Norris,*
(Answering the Charades and Rebuses.)

SWEET Friendship, purest of our joys!
Too oft, alas! we thee despise,
Unconscious of thy worth!
May I presume, with my *Inkstand,*
With pen and paper at command,
To set thy beauties forth?
O'er deadly *Nightshade's* baneful pow'r,
Or *Blockhead's* scorn in envy's hour,
Thy virtues high shall soar.
As *Sunday* frees us from our toil;
So thou in trouble yields thy smile,
To cheer the heavy hour!
The *Houseleek*, some a balm may call,
Compar'd with thee, its use is null;
Such is thy influence!
Miss *Lovet* or fam'd Nield may tell,
Whither betwixt them thou dost dwell;
And there thy sweets dispense.
Thy worth may justly claim the *Chair*,
Where to preside o'er each affair,
True judgement to dictate!
To hurl down envious rebels base!
And by the *Hair* expose their race,
In *Air*, to head the state!
From such deceivers bare and false,
Under thy mask, may truth repulse,
And humble in the dust!
That so thy influence may be known
In Britain's isle, and still be shewn
To all who're truly just.
So may great George long time preside,
Blest with thy aid to be our royal guide.

ENIGMAS ANSWERED.

1 WHEELBARROW.
2 LETTER U.
3 CANDLE.
4 NOTHING.
5 POPE'S-EYE in a Leg of Mutton.
6 *Printer's* DEVIL.

CHARADES ANSWERED.

1 SPARROW.
2 EARRING.
3 LAPDOG.
4 BUCKTHORN.
5 NOSEGAY.
6 PUNCHBOWL.

REBUSES ANSWERED.

1 NEEDLE.
2 CAIN.
3 HEALTH.
4 PARROT.

ANAGRAM.

ACRE, RACE, ACE, CARE, CAR.

SOLUTIONS to the PRIZE ENIGMA.

1. *By Mr. Gowler.*

MUCH of our country's discord and distress,
Springs from the too great freedom of the press;
Nor can we wonder at this source of evil,
While every *printer* keeps in pay a *devil.*

2. *The origin of the Printer's Devil, By Mr. Brown.*

WHEN Faustus at first did his printing begin,
A boy he employ'd and confin'd him within;
Lest, perchance, if abroad he were suffer'd to stroll,
'The gaff he might puff,' and discover the whole.
Now those who had seen the poor lad thro' a chink,
All over begrim'd with dirt, paste, oil, and ink;
Declar'd 'twas the *devil*; since no one but he
Could make copies so nice, to a tittle agree.
Nay, some e'en went so far as to say that they saw
The horns on his head, and his devilship's paw.
So 'twas held at that time, that whate'er was in print,
Must be done by the *devil*, and the *devil* was in't.
Thus the name was establish'd.—And now, sirs, adieu;
But for this information, give the *devil* his due.

3. *By Mr. R. Tattam.*

OH! murder! murder! Type's wife cry'd;
I ask'd the cause of strife:
Why don't you hear? a wag reply'd;
It is the printer's wife.—
Not so my friend, quoth I; I fear
You quite mistake the evil;
'Tis not the printer's wife we hear,
'Tis sure the *printer's devil.*

4. *Description of the Art of Printing, by Mr. L. Alexander.*

HENCE, ye midnight hags, away,
Nor dare to meet the coming day.
By your incantations vile,
Wretched mortals you beguile;
From your cursed spells may flow
Discord, murder, toil, and woe;
But the objects of your charm
Tho' you raise, you cannot harm.
From their *case* doth wisdom spring;
The *letters* to their *stick* they bring;
Then from the *gallies* to the *chase,*
Where each pregnant *page* they place;
And next with *furniture* surround,
Till all complete and tight is found.

Now to the *press* the weighty *form's* con-
vey'd;
And near the *gallows* on the *coffin* laid.
The well stuff'd *balls* the jetty *ink* supply;
Then clean'd afresh with the corrosive *lye*:
Till *pull'd*, and *pull'd*, at length the snow-
white *sheet*,
With learning and instruction is replete.
From this unravelling, 'tis plainly seen,
The *art* of *printing* is the *charm* you mean.

We are sorry room will not permit us to insert the ingenious general answers to the Enigmas, Charades, &c. given by Messrs. Davis, Gregory, Griffith, Hewitt, Nield, Norris, Rimmer, and Webb.—The Prize of a set of Delights has fallen to the lot of Mr. Brown.

ENIGMAS TO BE ANSWERED.

Enigma 1 (13) *by Mr. M. Mitcham.*

A MEAGRE figure, tall and thin,
Without a nose, or eyes, or chin,
Most humbly begs you would admit
His name within your book of wit.
Yet tho' no eyes or chin I boast,
I've got a head oft rules the roast.
'Tis true, indeed, I have no brains,
Yet much you'll find my pate contains.
I in my office work so fast,
No running footman makes more haste;
Nor is his body in more heat,
Nor bears it larger drops of sweat:
And tho' as good a servant I
As ever master yet come nigh,
If my swift pace I ever slack,
He's such a huffing, saucy Jack,
That, with some dirty hussey's aid,
He drives me faster—Truth, indeed!
But tho' these hardships I endure,
I'm fed most nobly, to be sure:
Sometimes with good fat goose or fowl,
You'll see your servant cheek-by-jole.
At other times I taste a chine,
A pig, or eke a rich surloin;
Often, indeed, I fancy hare,
But very seldom pudding share.
So much already's said, no doubt,
Your wit, so keen, will find me out;
But, lest you should mistake the clue,
One hint, and then I'll bid adieu.
A female tyrant, once, 'tis said,
(With horrid thought and mischief led)
Tho' hard I labour'd for her gains,
Took me, and bound me fast in chains;
The reason why, she us'd me so,
When you have found me, well you'll know.

Enigma 2 (14) *by Mr. Beckett.*

THO' man may boast unbounded pow'r,
Know, I, in less than one short hour,
Can make the stubborn mortal bow,
And the most haughty humble low:
And yet, which doubtless will surprise,
I never was beheld by eyes;
Nor can the most attentive ear,
My never ceasing motion hear.
Hence take another hint or two,
And then 'tis meet I bid adieu.
Not one 'twixt here and the world's end,
Or man or beast, can I call friend;
But ever doom'd to bear the weight
Of every living creature's hate:
What wonder is it, then, that I
Am every creature's enemy;
And, all-revengeful, in my turn,
Make all of every sex oft mourn?
But as it is man's constant prayer
To be deliver'd from my care,
That you may know I'm not all spite,
For once I'll humour you—good-night.

Enigma 3 (15) *by Mr. Nield.*

E'ER trees were cloth'd in leaves of velvet
green,
Or vegetation sprang, or even seen,

I had my being,—had my dreary birth,
Deep in the bowels of the firm-set earth;
Hid from the piercing,—prying eyes of man,
Till he grew wise, then wisdom form'd the plan,—
He found me out,—exposed me to the world,
Plung'd into water soon I then was hurl'd,
Where I endur'd malignity, and cold
Beyond description,—not fit to be told;
But very soon, revers'd my fortune was,
Thro' scorching flames, I then was forc'd to pass;
Pass! nay confin'd and forc'd for to endure,
A kind redress from none, cou'd I procure,
Until his will,—'till this barbarian chief
Released me, for to encrease my grief;
Then see! my tears, like twinkling stars descend,
O! angels, from his wrath do me defend,
Stop now his blows, I sink beneath their weight,
Still, heavier still, and thicker too they light.
He's done! and form'd me to his fell desire,
To live on flames, and feed on scorching fire;
Alas! my troubles now are past relief.
Woe leads to woe, and grief succeeds to grief.
Each night, when sable curtain veils the day,
And plumy songsters, sleep upon the spray,
I'm handed forth; and plac'd upon the plain,
And like a corps, am in a coffin lain;
At head and feet, two little spectres rise,
Like jack-a-lanthorns, meet your wand'ring eyes.
By them, you oft direct your weary way,
Sigh, moan, or cry, dance, laugh, or skip and play.
But e'er anon, without my kind correction,
They'll dull appear, and seem in great dejection;
Some social friend, a friend to love and peace,
Who's fond to please, and see a cheerful face,
Raises me up, the spectres I behead,
(Again enurn'd, again appear as dead),
Yet they are happy, free from pain, or fear,
And even then, more cheerfully appear;
They smile applause, nor think I've done them harm,
In th'room of which, as if by magic charm,
Fresh ones arise; I then renew the fight,
And eager snatch them from the realms of light;
And like a cannibal, do on them feed,
Such is my nature, such the food I need.
By ennodation now, explode my name,
And place my merits in your page of fame.

Enigma 4 (16) *by Celia.*

MEN to the specious sacrifice,
The real disregard;
Each glittering toy attracts their eyes,
And gains a safe reward.
But is there none whose mental sight,
Can dissipate the shade;
O'er error's mist induce the light,
And leave the trifler's trade!
Yes—some there are, and these will own,
How bright my merit shines;
In wisdom's eyes I'm purer shewn
Than all Golcanda's mines.
By me extended commerce reigns,
And rolls from shore to shore;
I mark the pole in azure plains,
Nor dread the tempest's roar.
Relying on my friendly aid,
The sailor smiles serene;
When clouds the blue expanse o'erspread,
And sun's arise in vain.
Yet small my form, and low my birth;
No gaudy tints I shew;
Drawn from my fertile mother earth,
Thro' purging fires I go.
Till fashion'd by the artist's skill
He ties the marriage chain;
When I my destin'd ends fulfil,
And still my love retain.

Prize Enigma 5 (17) *by Mr. O. G. Gregory.*

COME, my muse, and aid my song,
With me briskly trip along,
Let us prance across the vale,
Over hill, and over dale;
Then with mirth let us be seen,
Dancing on th' enamell'd green:
Help me then to tell my case,
And my birth and lineage trace:——
Cast your eye to yonder mead,
Where the sons of Io feed,
Amongst them is my father seen,
In stature stout and sturdy mien:
Where I to tell you how he's us'd,
How by men he is abus'd,
How he's tortur'd, put in pain,
And, at length, alas! is slain:
If you could but me believe,
Your hearts, if not of stone, would grieve.
When from my parent's side I'm torn,
Thro' various stages I am borne,
I'm sometimes low, and sometimes high,
Immers'd in wet, then hung to dry;
And when each change is undergone,
My name and use is then well known.
My stomach then does full appear,
But still my lot is quite severe;
Instead of good, substantial fare,
They feed me commonly with air!
But change the scene, and view yon plains,
Where the jocund country swains,
Are in martial order drawn,
Widely spreading o'er the lawn:
See! the contest is begun,
Perchance to last till setting sun;
Mark the heat, and arduous pace,
With which they follow up the chase:
Nor should you wonder thus to see,
The close pursuit made after me;
For know, more rivals for me strive,
Than for the fairest nymph alive,
But then with sad contempt I meet,
And oftentimes they me ill treat;
And tho' for me there's such a rout,
'Tis only to thump me about.
Thus they often spend the day,
'Till its light is chas'd away;
Then sit down and tell the tale,
O'er a jug of nut-brown ale,
And as fresh jorums of the nappy,
Make them feel content and happy,
I must not discontented be,
Tho' they ne'er think of toasting me.
Then come ye enigmatic train,
In darkness let me not remain,
Chase away the shades of night,
Bring me forth before the light,
My name ere this you've long descri'd,
So—draw this flimsy veil aside.

CHARADES TO BE ANSWERED.

Charade 1 (13) *by Mr. Gregory.*

OH! when my Maria does smile upon me,
My first does my bosom o'erspread;
And food for my next the reader will be,
When he's number'd amongst the dead:
My whole may be seen almost ev'ry night,
In hill, dale, or meadow, or grove;
Where it often shines forth in splendour quite bright,
Imitating the bodies above.

Charade 2 (14) *by the same.*

SELDOM have you at dinner been,
But my first is on the table seen;
And sometime when a person's vex'd
He'll to another give my next:
My whole is made my first to hold,
So now I pray it's name unfold.

A new Paradox, *by Mr. John Rimmer.*

A PART of ev'ry creature's frame,
Four letters will display;
And yet but one behind remains,
If one you take away.

ENIGMAS ANSWERED.

1. Spit.
2. Pain.
3. Snuffers.
4. Compass.
5. Foot Ball.

CHARADES ANSWERED.

1. Glow Worm. | 2. Salt Box.

The PARADOX—Is Bone.

Enigmas answered *by Mr. Nield.*

AN ELEGY.

On the Death of Wm. Turton, Esq.

Of Knolton-Hall.

MOURN! mortals, mourn! and let your sorrow shew;
Your heart-felt grief in tears of purest woe.
Exclaim his worth to Heaven's great awful throne,
There tell his deeds, his character make known.
To him what orphans made their daily prayer:
To him what widows dropt their woeful tear;
To him what fathers bow'd their aged head;
To him what beggars blind and halt were led;
With generous hand his purse he open'd wide,
And showers of comfort pour'd on either side.
The naked cloth'd—the hungry fill'd with bread,
The aged warm'd—the little infant fed.
Grieve wives and children, grieve, but ah! in vain,
Will sorrow bring cold *death* to life again?
A long like gilded *snuffers*, now he's lain,
Conquer'd by time,—by time alas! he's slain.
Within his heart no *spite* did e'er reside,
Like *bladder* puff'd, nor fill'd with empty pride,
Like th' *compass* steering. *mariners* so brave,
He'd scourge the villain, and correct the knave;*
But all the good, he us'd with tender care,
Distinguish'd them, like *glow-worms* from afar,
Then *box* with *salt* beef, often times he's fill'd;
And *bones* of mutton sent, when sheep he † kill'd.
But now alas! his charity is o'er,
No more in crowds, the indigent and poor
With uplift hand, will flock about his door.
He's gone! farewel! and may his generous soul
In *glory* live,—in sweets *eternal* roll.

* When a Justice of Peace.

† He has given this Winter, before his Death, to the poor of this parish and neighbourhood 700l. in meat, coals and cloathing.

Another Answer *by Mr. R. Humber.*

Since *pain*, *disease*, and *death* doth stalk,
Around this earthly *ball*,
May I within due *compass* walk—
(Ere long the Lord will call,)
The spark of life will be *snuff'd* out, SNUFFERS
A *spit* of earth cast on my head,
My rhyming o'er, without a doubt,
And I be number'd with the dead.

ENIGMAS to be ANSWERED.

Enigma 1, *by Mr. John Griffith, of London.*

MY whole great numbers wish to get,
But disappointments often meet.
Take off my head, then will appear
What ladies don't refuse to wear:
Once more lop off the head, and then,
You'll see what's priz'd by gaming men.

Enigma 2, *by Mr. John Gorton, of London.*

E'er aged Snow top'd time his pilgrimage commenc'd,
Or animating Sol his genial warmth dispens'd;
E'er hoary-headed Ocean in his confines groan'd,
Or silver shooting Cynthia, queen of night was thron'd;

E'er call'd from empty nought the willing earth obey'd,
I long existence knew, and pow'rful charms display'd.

Gay and resplendent Phœbus o'er th' Ætherial plain,
His nimble footed coursers taught I to restrain,
To guide the fiery chariot and ensure the rein.
When brilliant constellations first from chaos sprung,
And with surpassing lustre Heav'ns blue concave hung.
Inspir'd by me, they mov'd, and each her matchless maker sung.
Of this creation, fair I drew the mighty plain,
And in idea form'd that noble being man.

Nor can my pow'r decrease, by conflagration spent,
When nature dreary scenes of ruin shall present;
When to profound oblivion sinks the gaudy sun,
And corm'rant time himself his stated race hath run;
When pageantry, when pride, and when stern war shall cease,
And devastation wrap the jarring world in peace:
Unmov'd thro' ev'ry shock I shall my state maintain,
While glories circling round my head can never cease to reign.

Enigma 3, *by Mr. Thomas Nield, Master of a Boarding-School, in Pen—y—lan.*

Behold! my sire, with head aloof he stands,
And like a chieftain, he the train commands,
Commands my mother, who in stately pace,
Goes slowly on, adorn'd with every grace,
In peace and happiness, parades the plain,
And has in number, oft a numerous train.
Sweet peace prevails, while I am by her side,
But when snatch'd forth, her peace it is destroy'd,
For some short time, until her pain be o'er,
Then I'm forgot, and never thought of more.
Useless at present; 'till some skilful hand,
With form acute, he gives me full command.
Now I become the little school-boy's plague,
Assists Diana in her plann'd intrigue,
Perplexes,—pleases,—teazes; such is my fate,
The project's fram'd, and I then move in state
With nodding head, and gentle graceful mein
I lead the van, and form the amusing train.
Sometimes, all mirth and joy from me take place,
But ere anon, with sorrow'll fill your face,
Well bound around with human flesh and bone,
I've oft strange faces, and great wonders shown;
Deluded some, (I speak this to my shame;
And by my works, I some have rais'd to fame.
But stop! no more I need myself expose,
Or I'm found out, and all my merits lose.

Enigma 4, *by Mr. Gale, Edgware-Road.*

Though I assist the riddling train,
I don't remember to have seen
One hitherto, who's wrote a line
On me; either in prose or rhyme:
Yet they in me most pleasure find,
Neglect is then the more unkind:
If I did not attend their call,
There is not one would write at all;
Or even count, by line or rule,
In fact he'd be an empty fool.
When I am not, the human mind
Is one vast void, all dark and blind.
It's I that maketh science shine,
And human kind almost divine.
Though I am pent in narrow hold,
Wonderful things I do unfold:
And strange to tell in my small womb,
All things therein find ample room.
Nor love, nor triumph without me,
Could of the least duration be.
Now what I am, I beg you'll tell,
Wherein so many objects dwell.

Enigma 5, *by Mr. Gale.*

Ye rhymers rare, your thoughts prepare;
My name for to unfold;
I guide you all, both great and small,
Likewise the young and old.

Of fancy born, and to my scorn,
Folly's my chief support,
I'm brought from far, in time of war
Or peace; I'm found at court.
You'll think it strange, my shape I change,
Just to my maker's will,
By ladies sought, and oft am brought,
In being by their skill.
Without my aid, there'd be no trade,
For I'm what's most admir'd;
Being the thing, that's by the king,
And subjects most desired,
Now find my name, and then with fame,
You shall be rewarded;
Besides by me, you then will be,
Esteemed and regarded.

Enigma 6, *by Adolescens.*

I have ever thought Enigmas to commence,
To be more difficult than t' explain their sense:
This way, and that, in vain you try,
Till out of patience, you implore and cry
The Muse's aid, which if they'll not grant
With passion great, you rave, and rant:
Elbow the table, bite the feather'd quill,
Your forehead strike, your ink perhaps spill;
At length a lucky thought with joy you spy,
Begin to write, and so will I:—
Behold the tradesman, and perhaps his son,
Sometimes repair to it when business done,
To beguile the time, relieve the mind,
Pleasure and instruction both to find.
It is a mirror thro' which you may see,
What you are, and what you ought to be.
Sometimes you're struck with horror sad,
And all your faculties are in sorrow clad;
But jocand mirth soon comes to your relief,
And dispels quickly the dark clouds of grief,
To great perfection we have brought it
Tho' from other nations we have sought it,
Genius, and wit, it delights to cherish,
For without it faith it would perish,
It supports learning and by learning is supported,
So for its great use it may be courted;
But more than I meant I have already said,
Then of this Enigma I will break the thread,
To discover it I'm sure you'll have no pains,
For it is not in alley's, or in dirty lanes,
On the contrary open to your sight,
But of this no more I chuse to write.

Prize Enigma, *by Mr. J. Becket.*

One day in town, I chanc'd to meet
A thing that mov'd along the street;
Tho' it had neither legs nor feet,
Nor wings,—nor head,—nor tail;—and yet
Of feather'd kind, and black as jet.
It could not fly, and yet I found
It touch'd not in its course, the ground!
By two twin brothers 'twas directed,
And each fire passengers protected.
It mov'd on slow, yet kept before,—
What I admired ten times more.
Now say, what was this thing so black,
And you'll oblige your servant Jack.

☞ *The Printer is exceeding sorry, that he (through the hurry of business) left out Mr.* Davis's *name in the Title; it certainly is a great oversight, but he has no doubt but Mr. D's goodness will pardon it, or any other error that might escape him, or the Editor, in this Number, and assures Mr. D. and the rest of the partners of this Work, that the strictest attention shall be paid in future.*

AN ANAGRAM.

By Mr. James Gale.

DEAR gents if you please to take,
Three letters from the alphabet
Join them with care, and they will name,
An animal of ancient fame;

Or instrument of war explore,
Much us'd, 'tis said, in days of yore;
If you transpose these letters three,
A word of mischief then you'll see;
Transpos'd again you will find,
A thing most useful to mankind

CHARADES to be ANSWERED.

Charade 1, *by Mr. Thomas Nield.*

With azure blue, behold! my first appear;
My next is gay, and free from care and fear;
My whole doth in my first, exulting rise,
And charms my ear, with his melodious noise.

Charade 2, *by the same.*

All in my first, fair Celia she is dress'd,
And plac'd my second, in her lovely breast;
To make her charms, and beauty more appear,
I'll cull my whole, and give it to my dear.
Join then my whole, and second, and you'll see,
Two lovely flowers, of no low degree.

Charade 3, *by Mr. R. Humber.*

'Tis right of precedence—stands prior,
Ah! awful next—(kind muses aid the lyre)
Alas! how soon, the fleeting moments glide!
Swift as th' billows—of yon rapid tide,—
Charming whole,—if innocence attends
Th' active village school-boy, oft befriends.

Charade 4, *by Adolescenes.*

My first in magnitude, and immensity of space,
Of any thing beneath the heavens must take place;
Its boundless wealth, its matchless store,
Would take whole ages to be counted o'er,
A small part of which only to obtain,
Of adventurers many, has been the bane.
My next, tremendous assails the coward's ears,
Fills him with dread, and multitude of fears,
How unlike the hero! who rejoices at his name,
Where renown is got, and everlasting fame:
Of my whole at present, you may sometimes read.
But you know enough, away I'll haste with speed.

Charade 5, *by the same.*

Once more, dear Sirs, I your attention crave,
Which favour pray do not refuse;
My Charade to hear, but time to save
Is not by commenting, but to lose.

An insect is my first, and small in size,
That to do good to mankind delights;
They join themselves by friendly ties,
And toil both days and nights.
My next is of toil the well known cell,
To which doth bring his share,
Where idleness could never dwell,
Nor sloth to enter dare.

My whole in my last is fully told,
So seek no more to know,
One word more, and then I'll hold;
Man's to my first a foe.

A NEW REBUS.

By Mr. James Gale.

FIRST take a coin, and to it join,
A title of renown,
Next to my wish, two thirds o'a fish,
And then a Seaport Town.
Join these with care, and I've no fear,
But you will soon disclose,
The best relief for pining grief,
That interrupts repose.

ENIGMAS ANSWERED.

1. Place.
2. Wisdom.
3. Pen.
4. Brain.
5. Fashion.
6. Drama.
7. Lady's Muff.

CHARADES ANSWERED.

1. Sky Lark,
2. White Rose.
3. Pastime.
4. Sea Fight.
5. Bee Hive.

The ANAGRAM—Is Ram, Mar, and Arm.

The REBUS—Is Sleep.

The Prize Enigma, *answered by Mr. John Murgatroyd, of Mixenden.*

ON RETURNING SOME NUMBERS OF DELIGHTS TO THE REV. J. SHACKLETON.

MY thanks to you for loan of the delights:
Unrivall'd science here her stores unites.
For Whiting's perseverance too I'll pray,
Forwarding still the knowledge of the day.

The same by the Rev. J. Shackleton Thorton, near Bradford, Yorkshire.

What Jack admired, I think was Stella's Muff,
If I am right, then, Sir, I've said enough.

Mr. O. G. Gregory's Address to his Pupils, in answer to the Prize Enigma, No. 5, Delights.

Recollect, my dear Boys, as the School you draw near
That to all my instructions you ought to give ear,
Not a moment in School time should you idly spend,
But in each branch of learning should strive to amend;
When your hours of attendance are vanish'd away,
You may then for amusement with each other play,
For exercise brisk, to your health will conduce,
But then, pray take care that you use not abuse;
For tho' you at Cricket or *Foot ball* engage,
You ne'er should give way to that foul monster rage.
Nay, believe me, dear boys, you will find that through life,
There is nought to be gotten by rancour and strife:
So of each vicious passion I'd have you beware,
And strive to avoid it as you would a snare.
But religion and virtue, I assure you will tend
To make you respected and gain you a friend;
Be these your companions, and none can prevent
Your enjoying the best of all blessings—Content.

Ingenious answers were also given by Messrs. Clifton Davis, Gale, Hewitt, Humber, Ward and Webb.

Enigma 1, *by Mr. John Ward, Schoolmaster, Atwick, near Beverly, Yorkshire.*

WITH God I reign perpetual without end,
And at fix'd times, I all mankind befriend.
No stranger I, each morn I call on you,
Like a near friend, to ask you how you do:
But more than this, for I ne'er on you call,
But I you serve.—I servant am to all;
Except some vile, who from my presence steal,
Who love me not, for I their works unveil.
My Parent's black, but I'm as white as snow,
Rule half the sphere, and every climate know.
Take one hint more, then me you'll clearly see,
Where I am not, my parent there must be.

Enigma 2, *by Mr. O. G. Gregory, of Yaxley.*

Make room for me.—"For you, for what?"
Nay my good friend, don't be too hot;
I'm for the rich an epithet,
Transpose aright, nor do not fret,
For on my word, there then will be
A thing we at the fire place see.

Cut off the head and then I say,
There will be left what you oft pay;
Transpose this too, and then is shewn,
What is full well by Grocer's known;
Transpose again, for then you'll view,
What has been sometimes shed by you.
Take off the head of this, and then
There's left an useful part of men;
Transpose but this, and there will be
A plural verb for you to see.
Take the tail from what's oft shed,
Transpos'd you'll see the use of bread;
If not transpos'd it would display,
What's mostly taken twice a day.
From what's oft paid take off the tail,
And the remainder will not fail
To be, direct, a reptile bold—
Revers'd, of use to young and old.
Then come ye worthy British youth,
Whose science is to search for truth,
With ease you will discern my name,
In Whiting's page record the same.

Enigma 3, *by Mr. John Ward.*

I'm first and I'm last, nor beginning nor end,
I in both sexes reign, to all I'm a friend;
I reigned in Heaven before all things there—
I reigned in Eden, nor knew grief or fear;
When all flesh had sinned, 'twas I that brought down,
Blest Jesus to suffer, to purchase a crown
For penitent sinners, and so to restore,
An Eden more happy than that lost before.
'Twas I paid the ransom, 'tis I that make whole,
Give sure hope to the Christian, and gladden his soul;
I cover his failings, give new life and breath,
And cause him to joy o'er sin, sorrow and death.
I'm the radience of glory, Eternity's Air,
The sweet breath of heaven, and reign in all there.

Enigma 4, *by Mr. Joseph Jones.*

I'm both an enemy and friend,
I bless and curse without an end;
For, every hour of every day,
Life's saved by me and took away,
I'm long and short, I'm round and square;
I'm found on earth, yet ride in air:
I've fabled residence in hell,
And strength and weakness in me dwell.
Who want me, bitterly complain;
Who have me, treat me with disdain;
For me full many a league men stray,
Yet take me with them all the way:
I can the fondest pair divide,
Or bring the lover to his bride.
I'm both an antidote, and bane
Of dearth and plenty on the plain.
Men, beasts, and fowl, feel my effect,
And Earth and Air my course direct:
And who to live without me try,
Repent their folly oft, and die.

Enigma 5, *by Celia.*

A faithful friend, a steady guard,
A constant watch without reward;
I check the bold, the rash, the rude,
Nor suffer villains to intrude.
My guardian care the rich commend;
The wretched claim me for a friend—
My power prevails when force retires,
My aid the lover's heart inspires.
To me his rest the miser owes;
From me the public safety flows:
E'en justice owns my greater might,
To save untouch'd the private right;
And law, and truth, and wealth were vain,
Without my all-protecting reign.

Enigma 6, *by Mr. Gale.*

No doubt, for me, all will agree
A place I ought to have
In Whiting's page, where youth and age
Inſtruction may receive.
Of all things rare, I pray take care,
Of me your choiceſt prize,
For if ill us'd and oft abus'd,
May leave you by ſurprize.
I am belov'd and much approv'd
By all in whom I dwell,
And without me, there's none wou'd ſee,
The Hermit in his cell.
Often I'm ſhort, but with effort
I lengthen out my time;
And it's no pun, hazards are run
For me, by the divine.
And yet 'tis true, men me purſue,
Often in dark diſguiſe,
He's the beſt man, who forms a plan,
To take me by ſurprize.
Though ſtrange to tell, I'm favor'd well
And ev'ry one's my friend,
Being the boon, but ah! too ſoon
You'll ſay I find an end.
And now good gents, from theſe few hints,
You'll find my name with eaſe,
Then if I can, he that's the man,
Shall wear the earned bays.

Enigma 7, *by Lintonienſis.*

Ladies and gents, tho' ne'er, 'till now,
I've dar'd to ſtand before ye;
Your well known goodneſs prompts to crave
Attention to my ſtory.
Our family you've count'nanc'd oft,
And kindly, too, reſpected;
Tho' in the end, to ſpeak the truth,
We're all thrown by neglected.
And I, no doubt, that fate muſt ſhare,
As others have before;
I, too, ſhall be, perplex, be known,
Deſpis'd, and be no more.
Our family are all born cheats,
Deceit is what employs us:
No wonder, then, who finds us out,
Immediately deſtroys us.
Our dreſs is black; we thereby gain
Reſpect, firſt ſight—(you'll find
We've brethren in iniquity,
To many, 'mongſt mankind.)
My name is of ſuch general uſe,
Whatever ſchemes on foot,
Tho' never known, it ſurely points
The dark contrivance out.
No more: ſuch vengeance follows thoſe
Who ſecrets thus betray;
I ſhouldn't be myſelf again
Were I much more to ſay.

Prize Enigma, *by H. Thomas, Eſq.*

I am myſelf a perfect whole,
Altho' I am but half;
I bear of mighty names the ſcroll,
A ſort of cenataph.
I am both ſexes, paſt diſpute,
And ſtill Sir am of neither;
But am, tho' not in vaſt repute,
Companion fit for either,
As many heads as hands have I;
But what may more ſurprize,
No teeth, no ſmell, no ſight—yet why?
I've noſes, lips, and eyes.
A hero bold, and goddeſs free,
I'm yet held cheap and vile;
Some beg to have me piteouſly,
Yet have me all the while.
Altho' a hero, void of crime,
I ne'er was known to quarrel;
I ne'er cou'd fight, or ſing, or ryhme,
Yet ever wore the laurel.

A CHARADE.

By Mr George Clifton, of Peterboro'.

My firſt with ſenſe and reaſon is endow'd;
My ſecond is the cuſtom that's allow'd:
My whole they ſtrew, upon our Engliſh ground,
T' improve the grain; for which, it is renown'd.

A REBUS.

By Mr. J. Clifton.

A common liquor much in uſe,
A beaſt that oft receives abuſe;
The Engliſhman's devout deſire,
A fiſh that's chiefly in the mire;
The initials found will appear,
To be a name that reigneth here.

A PARADOX.

By Mr. Gregory.

Amid the gay parterre I'm often found,
Diffuſing pleaſing odours all around.
Six letters fully will my name diſplay,
Yet ſix are left, if four you take away:
If five inſtead of four away be ta'en,
You then will find that nothing does remain.

AN ANAGRAM.

By Mr. G. Clifton.

The tyrants character explore,
Which when tranſpoſed is a ſore;
Tranſpoſe again it will explain,
What all in trade wiſh to obtain.

Another by Mr. O. G. Gregory.

If the name of a ſea fiſh you nicely tranſpoſe,
Of a ſhoe, or a boot you a part will diſcloſe:
From this take the head, and I'd have you beware
For then you'll diſcover before you a ſnare:
If this ſnare you reverſe you ſurely will find,
That you've only a ſhare or a piece left behind.
From the part of a boot if the tail had been ta'en
What ſhines bright each eye when tranſpos'd would remain,
Reverſe this with care and you will have expreſt
Some vermin which frequently houſes infeſt:
From theſe take the tail and revers'd will be ſhewn,
The name by which ſeamen in England are known;
Tranſpoſe this with ſpeed and you'll then have in view,
What when ſolving this puzzle is uſed by you.
From the kind of a ſnare away take the head,
And what's left would perhaps make our knuckles look red;
If this be revers'd there will then be diſplay'd,
A kind of equality well known in trade.
Now you youths who in ſolving ſuch riddles delight,
An anſwer to this to our friend Whiting write,
That when the next number is publiſh'd he may,
In his famous Delights your ſolution diſplay.

Lines intended for a School Piece.

All you my friends who now expect to ſee
A piece of writing, thus perform'd by me:
Caſt but a ſmile on this my mean endeavour,
I'll ſtrive to mend and be obedient ever.

ADVICE

To certain profound Philoſophers *who have diſcovered that* "Death is an eternal Sleep."

Ye who, in *Reaſon's* and *Religion's* ſpite,
Preſume to *ſleep* through an *eternal night:*
From frightful *Dreams* 'twould guard the wiſeſt head,
With a *good conſcience* to retire to *bed.*

J. G.

ENIGMAS ANSWERED.

1. LIGHT.
2. GREAT.
3. LOVE.
4. GOLD.
5. LOCK.
6. LIFE.
7. ENIGMA.
8. HALFPENNY.

CHARADE—MANURE.

REBUS—GALE.

PARADOX—VIOLET.

ANAGRAM 1, CRUEL, &c.
——— 2, SPRAT, &c.

The PRIZE ENIGMA, *answered by Mr. John Tayler,* Commercial School, *Mansfield-Place, Kentish-Town.*

YE rich, on whom gay fortune smiles,
And plenty show'rs her copious stores,
Nor poverty, nor hunger's trials,
Nor piercing cold, nor care are your's—
Oh haste, the poor man's wants supply,
And wipe the tear from sorrow's eye!

Let not ambition's haughty crest
The gen'rous deed of mercy blight—
Let pity soothe the troubled breast,
Nor spurn the wretched from your sight:
Bid iron-hearted av'rice fly,
Nor grudge the sordid *Halfpenny.*

Relieve the sons of sad distress,
Be-dry the grief-worn widow's tears—
The weeping orphan's lull to rest,
And calm the hoary goodman's cares.
So shall you blessings reap on earth,
And joys eternal after death.

The *same, by Mr. R. Humber, of Brighton.*

A room! a room! ye gents sublime,
For *no* adept at making rhyme,
While I the Esqr's prize unfold,
Which is not silver, lead, or gold—
Its worth in copper is but small,
The *half a penny* buys it all.

The *same, by Mr. Collins, of Kensington.*

Permit me, sir Thomas, the whole truth to unfold,
You've coppied verbatim, exceedingly bold;
For I'll lay you a *halfp'nny*, I show you the theme
Which you sent the Delights, (where it now may be seen).
It was printed at Sherborne, by *Goady* and *Co.*
Where the answer was sent about nine years ago.
Now I pray, good sir, when you scribble again,
To compose what you send, not get hist for your pain;
For no laurel can ever your noddle intwine,
While you rob brother scribblers of jingling rhyme.

ENIGMAS, &c. to be ANSWERED.

Enigma 1, *by Mr. R. Humber, of Brighton.*

AS five to one the diff'rence ran
When the disorder first began—
And ere the darken'd shades of night
Succeed the sweet declining light,
A strange phenomenon appears,
Which myriads of revolving years
Had never brought to light before,
And surely never will no more:
All the angelic host on high,
With men below the azure sky,
Must stand astonish'd—well they might—
For God himself ne'er saw the like,
Until this period (thus decreed),
The consequence is, numbers bleed—

But mark, this wonderous event
Was through the boon of one dear saint—
The course of nature was turn'd round!
By this methinks the thing you've found;
If not—another help I'll lend—
It happen'd in the promis'd land;
The time the sacred penmen say,
Doth in between the Adams lay—
The stars of heav'n were not in sight,
Nor was it seen by sable night:
Th' above I pray you ponder well,
Then sure, what thing I am, you'll tell.

Enigma 2, *by Mr. G. Clifton.*

I hope you'll think it no disgrace,
  When you unfold this mystic lore;
Nor yet refuse it a small place
  Amongst the number men explore.
Tho' from a beast, behold I spring,
  Yet man's perfection I receive—
Then I am us'd by prince and king,
  And other persons I relieve.
Altho' I often cross the main,
  From Russia's dominions—
From Turkey I do not refrain,
  Nor yet from Spanish regions.
To rich and poor my aid I lend,
  For a shield against cold weather;
All sorts of learning I defend,
  Yet, a poor defence to either.
Of divers colours I am made,
  For a different use design'd—
But like the rose, in time I fade,
  The property of all mankind.
So now the hints, that I give here,
  You may find out at your leisure,
And to the world, let them appear
  'Mong our friend (the author's) treasure.

Enigma 3, *by Mr. John Tayler, Mansfield-Pl... Kentish Town.*

Bards, tho' I am your constant slave,
I nothing at your table crave—
Tho' at your service I attend,
And my assistance you b. friend—
Nothing from you do I receive,
But stripes and blows, without my leave:
When stormy wind tempestuous roars,
And soaking rain, descending pours,
Then my enlivening warmth you ask,
(In imitation of the mask)
In journeys too, you post away,
I'm sure to bear you company,
And step by step thro' mud and mire,
Am bound to shield your gay attire;
I own these favours are your due,
But not from any love to you,
Do I the cheering comfort lend,
'Tis force that makes the stubborn bend.
Thro' Albion's isle to yonder throne,
My genuine usefulness is known:
Then learned gents display my fame,
Stern winter dignifies my name—
Be sure you give the honor due,
For now, perhaps, I succour you.

Enigma 4, *by the same.*

Hell-born race of Pluto's cavern,
  Ever known to teaze mankind;
Swarming, near where rolls the Severn,
  And great Thames, you them may find—
Sov'reign George, our lord and master,
  Is not from their malice free;
Many a fair-one in disaster,
  From their venom'd arrows flee—
I myself have felt their hatred,
  Tho' in humble, low estate;
Their hungry stomachs often sated,
  Curs'd as oft my wretched fate.

In features ugly as the Devil,
Sharp as envy is their bite;
Like most other things of evil,
Chusing darkness rather 'n light;
In holes and crevices abiding,
Thinking safe to keep from man;
Yet, at times no fear nor chiding,
Without murder! tame them can.
With such characters before ye,
Soon, ye gents, you'll know their fame—
To friend *Whiting* tell the story,
Show the world their loathed name.

PRIZE—Enigma 5, *by Mr. J. Collins, School-master, of Kensington.*

Ye learned gents, whose piercing eyes,
Can soon unfold th' abstrusest prize;
I come conceal'd, t' employ your wit,
Which soon, too soon, I fear you'll hit.

Well known was I in days of yore,
In Noah's time, and long before;
And shall for everlasting be,
From time unto eternity.

In straw-thatch'd sheds where peasants dwell,
I am, and some say too in Hell;
In city, town, and country too,
In dungeons dark you may me view.

In palaces I may be seen,
Where dwells our noble king and queen;
Where dire destructive cannons roar,
You'll see my ghastly visage soar.

At early dawn you'll see me rise
Toward the vaulted azure skies;
From thence you'll see me head-long hurl'd
Through diff'rent regions of the world.

Where topers quaff their nut-brown ale,
Enjoy their pipe and cuff a tale;
There, valu'd bards, I may be seen,
Amongst the Bacchanalian train.

Besides, amongst the numbers past,
I'm found more useful at the last,
By country oaken farmers, when
Pomona's crops begin to spring—

When nauseous insects would it pest,
'Tis then they do my aid request,
For to dethrone the num'rous swarm
That would the luscious blossoms harm.

Ye noble youths, mark what I've penn'd,
I now shall strive to gain my end,
So let this hint my theme compleat,
I often help to dress your meat.

More might be said my name t' obtain,
But, surely, I have spoke too plain:
Now gents, till next we'll bid adieu,
Lest I impose on time and you.

CHARADE 1, *by Mr. R. Humber, of Brighton.*

My first, in records we are taught,
Against a warlike general fought;
Propitious next, a welcome friend,
Which does by day assistance lend—
My pleasing whole, with aspect bright,
Oft cheers the darksome shades of night.

Charade 2, *by Mr. J. J. Thompson, of Brighton.*

How dreaded is my first by sinners all,
The righteous keep my next against God's call;
My whole's a warning, as men say,
T' reclaim the sinner from his erring way.

An Anagrammic Charade 3, *by Mr. John Collins, of Kensington.*

Ye valiant hero's stout and bold,
Who know your task, my first unfold;
Next, gents, select a scripture name—
My whole's a rope, explain the same:
A letter change, transpose it right,
Another rope you'll have in sight—
A letter change, invert once more,
Implies delight—the same explore;
Two letters drop*, the change repeat,
You'll find it is a thong complete—
Change it again, a husk you'll find;
Transpose once more, if your'e inclin'd,
A scripture name you will disclose;
One letter drop, again transpose,
A once-fam'd judge 'twill then descry—
Again to change it if you try,
To reconcile you will declare;
Transpose it right, with art and care,
You will display a scripture name,
Stand forth ye bards, explain the same—
One letter change, transpose again,
Shews what with fools doth often reign;
Invert it right, ye Britons free,
'Tis practicable then you'll see:
Cut off my tail, you'll have exprest,
What all must do e'er they exist—
Now lastly change it, without doubt,
A sort of bev'rage you'll make out.

Charade 4, *by the same.*

Ye noble gents, and British fair,
Who can the darkest themes declare,
No doubt my first, in early days,
Your craving appetites did please.
Now o'er the sacred writings scan,
('Tis pleasure to a scripture man)

* From the last rope.

A name from thence you'll quickly trace,
Well known of Adam's num'rous race.
These parts select, and them connect,
And then the answer you'll detect—
The which you may *T. Whiting* send,
And tell him 'tis the printer's friend.

Charade 5, *by Mr. J. Tayler, of Mansfield-Place, Kentish Town.*

Search the rules of mensuration,
  Where old Euclid's laws abound,
Or the scales of computation,
  I with either them am found:
Hail, ye sires, with age grown hoary,
  Frequent you my second are;
Yet, perchance, I may before ye
  Struggling, quit this vale of care.
Virtue, sense, and clear-ey'd reason,
  Sprightly nymph! adorns my whole;
Many an eve in winter season,
  Hast thou cheer'd my pensive soul.

Charade 6, *by Mr. R. Humber, of Brighton.*

My first not near,
My next oft dear—
My whole amount
Of small account—
From this charade
Remove the shade,
You'll quickly see
What thing I be.

ANAGRAM, *by the same.*

A thicket, gents, if you transpose,
Reveals those berries which there grows;
Now change again, and Mary knows
What oft she does to Billy's clothes.

ENIGMAS ANSWERED.

1 Sun.
2 Leather.
3 Umbrella.
4 Bugs.
5 Smoke.

CHARADES.

1 Moonlight.
2 Deathwatch.
3 Halter.
4 Paper.
5 Inchbald.
6 Farthing.

ANAGRAM.

Shaw, Haws, and Wash.

Answers to the Prize Enigma.

1. *By Mr. John Hindſon, of Lincoln.*

OH! come ſweet peace, erect thy olive wand,
Bid diſcord ceaſe and commerce bleſs our land;
Oh! let the *ſmoke*-fraught cannons ceaſe to roar,
And war, dread war, be baniſhed from our ſhore.

2. *By L. W. D. of Brighton.*

In Whiting's *Delights*, Sirs, beneath a ſly veil,
Shrewd Collins has told you a myſtical tale;
But fairly ſet too, and the veil's quickly broke,
And all Collins's efforts ſoon vaniſh in *ſmoke*.

3. A Sonnet, *addreſſed to* Sleep, *by Mr. John Tayler, Manſfield-Place, Kentiſh Town.*

Come gentle Sleep, thy lulling pinions ſpread,
And fold my ſenſes in the arms of reſt;
Why fly my couch, and leave my aching head
To combat thoughts that tear my tender breaſt?
Ah, ſtupifying friend, make me neglect
Fair Mira's frowns that harrow up my ſoul—
Her beauteous form and ſprightlineſs forget,
And every anxious thought of her controul.

In Lethe waves my boiling paſſion veil—
O ſtill my ſoul—ye balmy ſlumbers deign;
As all entreaties, ev'ry plaintive tale,
She weighs as *ſmoke*, light curling o'er the plain.

Stay then thy care—nor grant her wonted reſt,
That ſhe may know to pity the diſtreſt.

General Answer to all the Enigmas,

By Mr. R. Humber, of Brighton.

MORNING.

The *moon* retires, and *ſol* appears, 1
And dawning light the country cheers;
The *bug*le horns re-echoing ſound, 4
" And gladden'd nature ſmiles around."
Behold yon hamlet teems with *ſmoke*— 5
Hark! the hoarſe Ravens loudly croak;
See how the blooming village maid,
(Unmindful of th' *umbrella*'s ſhade) 3
With cheerful ſteps purſues her way,
To labor thro' the ſultry day:
The ruſtic too, with home-ſpun coat
And *leather*n bottle, leaves his cot— 2
Acroſs the lawn, with peaceful brow,
Prepares to guide the furrowing plough;
The wounded earth receives the corn—
Anon! mid-day abſorbs the morn.

NEW ENIGMAS.

1. *By Mr. John Tayler, Manſfield-Place, Kentiſh Town.*

INGENIOUS youths that myſtic tale
Ne'er crept beneath enigmas veil,
Was e'er more open laid before ye,
Than this my theme and ſimple ſtory.

To tell from who, or whence I came,
Or where I am, or what's my name:
All nations know that I have been
Yet certain 'tis I ne'er was ſeen.—
Thouſands thro' life doth wait for me,
Yet wait in vain my form to ſee.
In vain old Gripus hoards his ſtore,
And rake-all ſpendthrift ſues for more,
To cloy their over-craving mind,
And ſeek enjoyments none can find:
For me their ſtrenuous toil extend,
In hopes with me their cares will end—
Such wild-gooſe chaſe, ſuch vain purſuit,
Miſtaken brethren ne'er will do it.
Each morn preſents I boldly ſay,
What I ſhall be, was yeſterday—
How ſtrange ſoe'er it may appear,
When fartheſt off you think I'm near.
The guilty priſoner pleads in vain,
'Till I arrive reſpite to gain;
Ah, could he get ſuch pardon penn'd,
His days would laſt 'till time ſhould end:—
Myriads diſcourſe hourly of me,
As many dread the miſery,
The throbbing pangs, the aching hearts,
The bleeding wounds, the piercing ſmarts
I cauſe amongſt the throng to rove,
Tho' my proceedings they approve—
Many I cauſe to break their reſt,
While anxious thoughts corrode their breaſt;
Yet was I ne'er with them nor you—
Your humble ſervant—Gents adieu.

2. *By Mr. R. Humber, of Brighton.*

——————— "Propitious nine."
Diffuſe thy rays, and ſhed thy beams benign;
Aſſiſt for to enwrap my direful name,
In pitchy night, amid ſeraphic flame:
My origin, tremendous to relate,
Such is the tale of my relentleſs fate;
Down ſubterraneous caverns, diſmal ſhades,
But faintly where the glimmering light pervades;
'Mid frightful caves involv'd in gloomy night,
'Till by ſome conq'ring hand I'm brought to light;
Borne from the earth, do not ſpontaneous riſe,
Ye gods propitious! hear my rending cries;
Doom'd to th' flames, and furious blows, for gains,
Perchance may bind the monſter for his pains!
The vanquiſh'd victor oft parades in me;
(Kind is the friend who ſets the wretched free)—
In doleful dungeons, where the priſoner groans,
'Mid darkſome cells where yonder captive moans!
A truce with theſe unpleaſing ſeats of woe;
I'll now deſcribe thoſe brighter ſcenes I know.—
——————— "Hail! lovely maid,"
Once more I do invoke thy friendly aid
To tell my worth, which is immenſe oft known;
Careſs'd by ſov'reigns on the lofty throne.
How brilliant does my dazzling luſtre vie,
With burning orbs of yon beſpangled ſky:
The charming fair would not my glories hide,
Hence takes me near, and ſeats me by her ſide—
And when Clorinda deigns to ope the dance,
I with the gentle nymph do friſk and prance:
And ſhould you chance to ramble o'er the lawn,
At ſetting eve, or ere the breaking dawn,
I move before—your ev'ry ſtep attend,
And prove a conſtant, and a faithful friend:
Should you deſire to know how minutes fly,
To aid your kind reſearch I'm ever nigh;
And at your head do take my wonted ſtand,
When ſhades of night pervade the gloomy land.
Come, rend thoſe lines, and burſt the myſtic veil,
My name, O gents, ſome future day reveal.

3. *By Mr. J. Collins, Schoolmaſter, Kenſington.*

Hail! fam'd Apollo, haſte I pray,
The ſummit of thy temple quit,
And Oh! beſtow one gentle ray,
And aid my feeble muſe to write.

When mounted on Pegaſus' wing,
No dread or danger need I fear;

Like learned Darwin would I sing,
  Like Pindar would I strive to steer.

I to the learned am well known—
  As are my useful kindred all;
Tho' much unlike, as has been shown,
  For some are curv'd, some straight and tall.

I am the youngest of the kin,
  But I such curious nature boast,
That never one is with me seen,
  Except, it is to end a roast.

And then but one amongst a score
  Can e'er admitted be with me;
We two then substituted are,
  Straight in the stead of seven and three.

And when alone I three supply,
  Tho' strange to you it may appear;
Gents, on the truth you may rely,
  The lawyers all do know it clear:

To them my use is much confin'd,
  E'er they their grov'ling fees can crave;
May all sweet peace and union find,
  Nor their assistance want to have.

I aid the printer and the sage,
  And the geometrician gay;
No longer let my pen engage,
  But let me now be brought to day.

Stay—one hint more, I must declare,
  My comic figure to describe;
Tho' not an oval, round, nor square,
  Many are of my num'rous tribe.

Such absurd shape I'm known to wear,
  You may not ever think it true,
Take this last hint, I'm serpentine,
  Now till next year will bid adieu.

CHARADES.

1. *By Mr. John Tayler, Mansfield Place, Kentish Town.*

On yonder green where flowers are seen,
  And nature's bloom ascends;
Virtue and bliss with happiness,
  My humble first attends.

Ye British fair my next declare,
  Nor imitate her ways—
Love cleanliness tho' in distress,
  And you will merit praise.

Now, wedded youth, to speak the truth,
  I wish you free from cares—
My whole pray shun, let wives alone,
  And mind your own affairs.

2. *By Mr. Samuel Barrow.*

When men go out to seek for prey,
  My first are often seen;
My second, with a woman may
  In yonder vale have been:
My pointing whole you plain may see,
  And trembling say, go past!
For fear a first yourself may be,
  As sure as be a last.

3. *By Mr. J. Collins.*

Enlighten'd bards, who often scan
Friend *Whiting's* philosophic plan—
Accept the thesis of a swain,
Who veils his mystic truths in vain:—

My first is an ingredient good,
It often helps digest our food:
A female is my next when shown—
Join these, my charade will be known.

4. *By Mr. R. Humber.*

I am a flow'ret of the plain,
Amid the goddefs Flora's train—
Should you a fault'ring ftep but make,
My fecond is the dire miftake;
Th' whole my bellowing firft devours,
Alas! for thefe ill-fated flow'rs.

5. *By Mr. J. Collins.*

An infect now ye gents defcribe,
And one of Adam's num'rous tribe—
Select the parts, and them unite,
And fend it to friend *Whiting*, right.

6. *By Mr. Samuel Barrow.*

Behold my firft, when dawn appears
O'er yonder craggy mount;
My next you'll find to fpinners dear,
If you're inclin'd to count.
My whole's an ornament you'll find,
That much adorns the female kind.

7. *By Mr. J. Collins.*

A liquor, pray, with care felect,
To it a fcripture name connect;
And you will then expofe to view,
An offspring of my former two.

8. *By the fame.*

An accident my firft implies,
My fecond is a game well known;
Connect the parts ye wits of fame,
My accidental whole is fhown.

REBUS—*By Mr. John Tayler.*

An element, gents, firft prefent,
A friend to haughty man;
A paffion name, held in great fame,
Thro' Britain's happy land:
What priefts difplay when chriftians pray,
Eternal joys to find:—
What often refts in human breafts,
And preys upon the mind—
The initials take and them connect;
One—Oh, forbear, from me keep clear
Of the tormenting fect.

ANAGRAMS.

1. *By Mr. J. Collins.*

To Smithfield market ftraight repair,
Tranfpofe what's often felling there,
You'll inftantaneoufly declare
What oft offends the lovely fair.

2. *By Mr. R. Humber, of Brighton.*

Tranfpofe a fault
Reveals a halt—
Invert with care,
Behold a fnare:
Tranfplace the lay,
A fchool-boy's play;
Within thefe lies
What fwiftly flies.
Now name them gent,
And I'm content.

3. *By Mr. John Tayler.*

Beyond Heav'ns fpangled high concave,
Refides my firft, fupreme in blifs;
His name tranfpofe my next to have,
An emblem of true faithfulnefs.
One letter drop you will behold
A prince, by Ifrael's children flain,
Combine, another to unfold,
A beaft delicious tho' unclean.

4. *By Mr. J. Collins.*

A mifer if you right tranfpofe,
An helplefs ftate you will difclofe.

ENIGMAS ANSWERED.

1 Tomorrow.
2 Chain.
3 Character &.

CHARADES.

1 Cotage.
2 Marksman.
3 Father.
4 Cowslip.
5 Mother.
6 Topknot.
7 Brother.
8 Haphazzard.

ANAGRAMS.

1 Straw, Warts.
2 Stop, Pots, Tops, &c.
3 God, Dog, Og, Hog.
4 Churl and Lurch.

General Answer to the Enigmas,

By Mr. Richard Humber.

TO-MORROW and *S*,
The centre a *chain*,
Doth all th' Enigmas,
I think right explain.

NEW ENIGMAS.

1. *By Mr. D. Griffith, Keppel-House.*

On the wave's ærial dance,
Sportive, on a summer's eve;
Sailing through the vast expanse,
The hero of my theme perceive.
Son of pleasure's empty train,
Light and airy, vague and vain—
Take thy pleasure, take thy way.
Vain thy course and short thy stay!

Random wand'rer! roving 'round;
Tending to no point or place:
Atom in the vast profound!
Idler ranging boundless space!
Wand'ring idler! sport away:
Wanton out thy short-wing'd day.
Thought nor care thy course annoys:
Thought nor care thy breast employs.

Like vain pleasure's airy band,
Floating far and floating near:
No end in view—no means in hand—
No compass points—no pilots steer.
Son of an hour! spread the sail—
Yield before the driving gale:
An inch of space—a moment more,
Thy flight is flown, thy course is o'er!

Far into the future ken;
His random life's for ever o'er:
But, vain man! remember, then,
Thine begins to end no more!
Son of an hour!—immortal soul,
While eternal ages roll!
Mite amid the boundless vast!
Mind that must for ever last!

EPIPHONEMA.

Sons of Pleasure! is it so?
Heirs of endless bliss or woe!
Seize the moment—catch the gale—
Steer aright, and spread the sail:
Now all your care, and skill employ,
Make the port of endless *joy*.
See th' imperial city rise:
See her towers climb the skies:
See the radiant banners wave—
See bright gold the city pave:

I

See the galling crofs laid down:
See, affum'd, the brilliant crown!
See eternal glory bright—
See foft ravifhing delight—
See fweet blifs inwrap the foul:
See full raptures fpread the whole:
See your Saviour chide your ftay;
Beck your fluggifh fouls away!

☞ *This is not inserted for any* enigmatical *Merit, but the Moral it is intended to convey.*

2. *By Mr. J. Collins, of Kenfington.*

Ingenious bards, of everlafting fame,
To whom belongs a meritorious name;
Blame not a fimple and illiterate youth,
Who ftrives in vain to veil a myftic truth:
Friend *Whiting*, fir, I hope you'll deign to fpare
A corner of your valu'd annual care;
And on the fame, I beg *M. K.* will print it,
As carriage free, I've to you, worthies, fent it.

Now from apologizing I fhall turn,
And tell from whence my origin firft fprung:
I like all mortal beings had a birth,
And then was nourifh'd by old mother earth;
Such chuff and furly cur am I in grain,
If man would turn me, he'd attempt in vain;
But if an animal fhould pafs me by,
'Tis known full well, if bid, he'd make me cry—
Would drive me wherefoe'er his mafter chofe,
And durft not, for a drubbing to refufe.
I in the woods and lanes am known to ftray,
And thro' the fields I often find the way:
But when gay fummer bids farewel to you,
And winter comes with cold and rimy dew—
When purling rills are bound in icy chains,
And flaky fnow conceals our native plains;
I then retire beneath a ftraw-thatch'd fhed,
And there I flay, and plenteoufly am fed.
But, reader, ceafe! while I the tale refume:
I am admitted in a better room—
But, oh! fuch tortures then I'm doom'd to feel,
I'm flead—hung up, ah! fhocking to reveal!
I then become the prefent myftic fcene,
And foon, too foon, I fear it will be feen.

3. *By Mr. R. Humber.*

Ye hallow'd nine, affift mine artlefs lay,
In myftic ftrains to fing my natal day;
Attune my ftrings, with fympathetic glow,
And let each line in pleafing numbers flow.—

Sought for by man, on Ganges golden fhore,
Where water-falls in diftant echos roar—
Beneath yon feas or margin of the wave,
Where limpid ftreams in peaceful filence lave—
Till by a wretch, borne from my native bed,
Then thro' a firy path I'm bafely led;
And next, by curious art, men me prepare—
Happy's the man who can my favors fhare.

For me ambitious men, convulfe the world;
Nations and ftates, are into mifery hurl'd.
Th' embattl'd chief, with blood-ftain'd banner rear'd,
Where din of war 'midft clafh of arms is heard;
And veins that with athletic ardor glow,
And purple tide's in copious torrents flow.

Known to the aftronomic's bright furvey,
Who can thro' fhades, celeftial figns portray—
What philanthropic men do oft impart,
To cheer the gloom of forrow's bleeding heart;
Sometimes a wreath from Flora's grateful train—
Anon, for me, devoted men are flain.——

Known to the pauper and the clown, and found,
The brow of Britain's fire t' encircle round:
Part of the human frame, yet ftrange to fing,
Fit only for an emp'ror or a king.
Unfold ye pearly gates of blifsful day,
My fparkling luftre there I do difplay;
'Midft radiant glory beam celeftial fires,
Whilft lift'ning throngs attune their golden lyres.